Errors in Turbulence Statistics: The Hurst Phenomenon Approach

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Outline i

Introduction

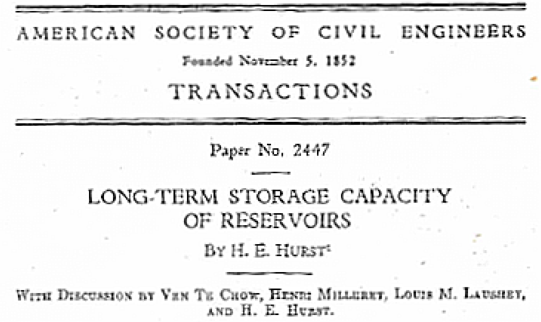
Key points ii

- So far, random errors of turbulence moments have been connected to the existence of an integral scale, as proposed by Taylor (1935).
- The integral scale is (next to) impossible to estimate from a finite record of (say) 30–60 min. We will present some evidence that it may not exist at all in some cases.
- Still, the random errors can be estimated. They are somewhat larger than previously thought.

For details, see Dias et al. (2018).

Origins iii

The Hurst phenomenon is named after H. E. Hurst's "Long-Term Storage Capacity of Reservoirs" (Hurst, 1951). Hurst had been interested in the design of reservoirs for the Nile River which should be able to meet a certain target demand.



AMERICAN SOCIETY OF CIVIL ENGINEERS

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TRANSACTIONS

Paper No. 2447

LONG-TERM STORAGE CAPACITY OF RESERVOIRS

By H. E. Hurst[1]

With Discussion by Ven Te Chow, Henri Milleret, Louis M. Laushey, and H. E. Hurst.

Background

Main statistics for "R/S" analysis iv

Let $x(t, \omega)$ be a **stationary**, stochastic process. ω is the "index" of a realization.
At scale Δ and beginning at time t, we define:

The sample mean: $$\widetilde{x}_\Delta(t) = \frac{1}{\Delta}\int_t^{t+\Delta} x(t')\,\mathrm{d}t',$$

The sample stdev: $$s^2_\Delta(t) = \frac{1}{\Delta}\int_t^{t+\Delta} [x(t') - \widetilde{x}_\Delta(t)]^2\,\mathrm{d}t',$$

The adjusted range: $$R^*_\Delta(t) = \max_{0\le\delta\le\Delta}\left[\delta\left(\widetilde{x}_\delta(t) - \widetilde{x}_\Delta(t)\right)\right] - \min_{0\le\delta\le\Delta}\left[\delta\left(\widetilde{x}_\delta(t) - \widetilde{x}_\Delta(t)\right)\right],$$

The rescaled range: $$R^{**}_\Delta(t) = \frac{R^*_\Delta(t)}{s_\Delta(t)}.$$

Hurst's discovery

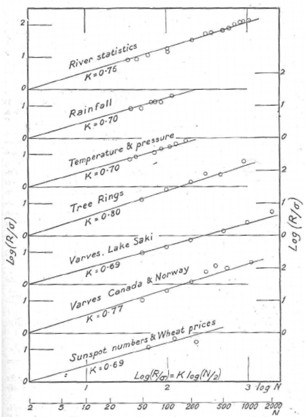
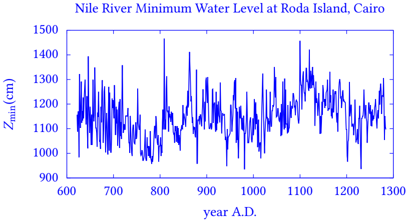


Fig. 4.—Relation Between Range of Summation Curve R, the Standard Deviation σ, and the Years of Record N



Hurst's Law:

$$\left\langle \frac{R^*_\Delta(t)}{s_\Delta(t)} \right\rangle = c\Delta^H$$

- $H = 0.5$ was expected *.
- Instead, Hurst found $H = 0.72 > 0.50$ in geophysical time series.

$*$ Mandelbrot and Wallis (1968): "The Brownian domain of attraction"

Turbulence data exhibit the Hurst phenomenon! vi

- Sutton (1932) (almost 20 years before Hurst!!)
- Laushey (1951) (discussion of Hurst's paper)
- Nordin et al. (1972) (lab flumes; Missouri and Mississipi rivers)
- Helland and van Atta (1978) (grid turbulence)

The connection with the integral scale

$$\mathscr{T} = \int_0^\infty \rho(\eta)\,\mathrm{d}\eta$$

and its practical uses (random error estimates), however, had not been made.

For

$$\rho(\eta) \sim \eta^{-q},$$

the shape of the autocorrelation function determines if the Hurst phenomenon is present:

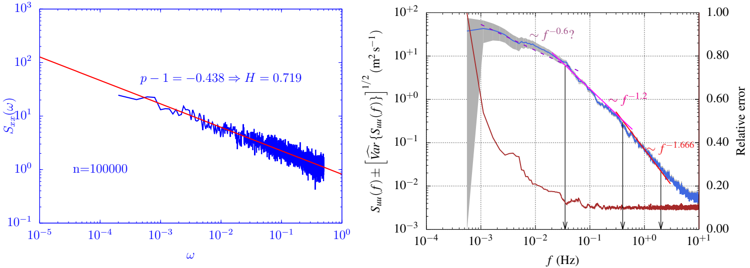
exponential decay or $1 < q < 2 \Rightarrow$ "no Hurst" (fast decay with Δ)
$0 < q < 1 \Rightarrow$ "Hurst" (slow decay with Δ).

From $\rho(\eta)$, one can find analytically (Lumley and Panofksy, 1964; Dias et al., 2018):

$$\mathrm{MSE}(\widetilde{x}_\Delta) = c\Delta^{-p},$$

where:

$0 < q < 1$: $\mathrm{MSE} \sim \Delta^{-q}$; $p = q$; $\mathrm{p} = 2 - 2\mathrm{H}$,
$q > 1$: $\mathrm{MSE} \sim \Delta^{-1}$; $p = 1$.



The Hurst phenomenon is associated with the lowest frequencies in the spectrum: large errors; it is in practice impossible to find the power law exponent.

Turbulence errors and the Hurst phenomenon

A power law is at the core of all methods discussed here ix

$$\mathrm{MSE}(\widetilde{x}_\Delta) = c\Delta^{-p}, \qquad \mathrm{RMSE}(\widetilde{x}_\Delta) = c^{-1/2}\Delta^{-p/2}.$$

With

- **LPM** (Lumley and Panofsky's Method; Lumley and Panofksy (1964)) $c = 2\mathscr{T}\,\mathrm{Var}\{x\}$; $p = 1$.
- **FIM** (Salesky et al. (2012)'s Filtering Method) fix $p = 1$; adjust c for a range of Δ's.
- **RFM** (Relaxed Filtering Method current work) adjust c and p for a range of Δ's; the values of p found empirically reveal the Hurst phenomenon in turbulence Surface-Layer data.

Experimental sites

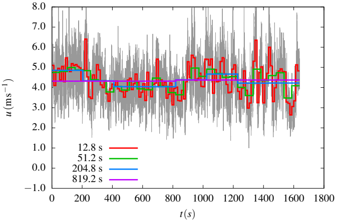
Three sets: Tijucas do Sul (short grass), Missal (Itaipu Lake) and AHATS (Kettleman City, CA)
Quality control:

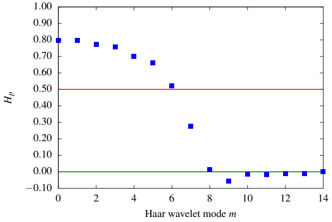
- Spikes.
- Physical limits.
- Weak turbulence.
- Nonstationarity of 1^{st}- and 2^{nd}-order moments.

The last is important because in practice it eliminates the possibility that the Hurst phenomenon is being caused by nonstationarity in the data sets.

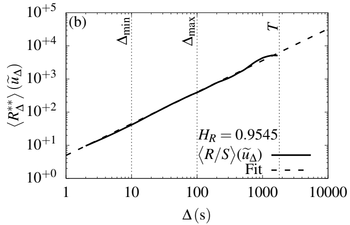
Effects of filtering on the Hurst coefficient H xi

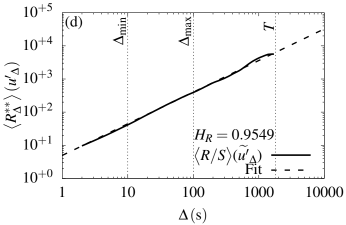
Haar Wavelet filtering:





Linear Detrending:





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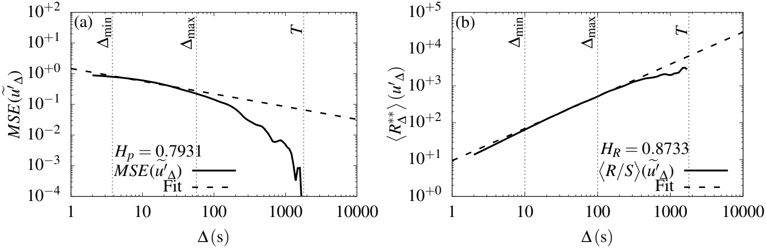
MSE *versus* R/S analyses

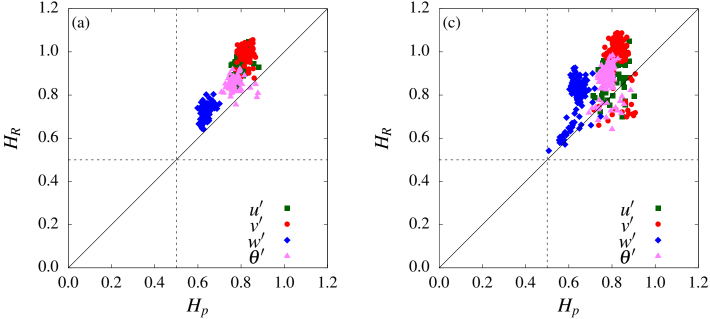
Datasets are of the type

$$x = u', \qquad x = w', \qquad x = w'u', \qquad x = w'\theta', \ldots$$



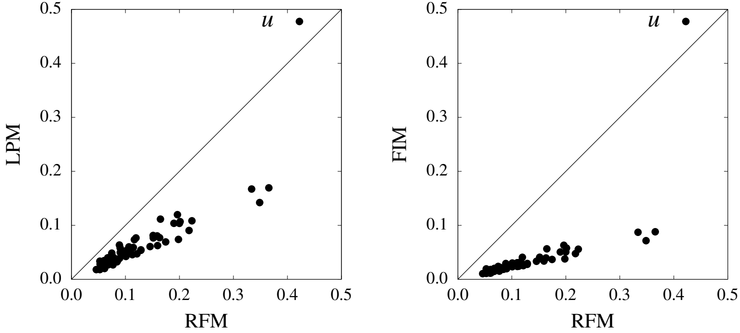
MSE plot at the heart of FIM (fix $p = 1$) and RFM (let both c and p vary freely).

Missal data (left) and AHATS data (right)



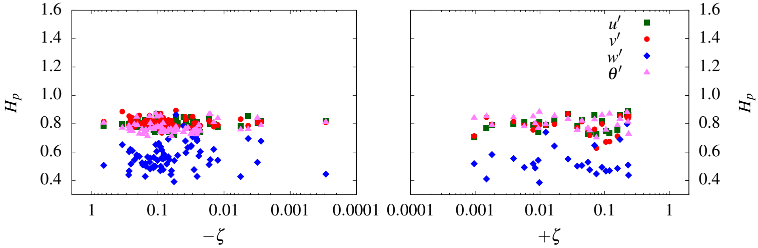
Overall effect on error estimates

Tijucas data: RMSE($\overline{u}$) ($\mathrm{m\,s^{-1}}$)



The Hurst phenomenon is outside the scope of Monin-Obukhov Similarity Theory xv

H_p versus Obukhov's stability variable ζ, Itaipu Lake, u' data.



(Very long range, very low frequencies)

Conclusions

Conclusions

- Hurst's phenomenon is ripe in surface-layer turbulence.
- In real turbulence, spectra, autocorrelation functions and structure functions devised to "see" inertial-range behavior have difficulty "seeing" Hurst's phenomenon.
- Taylor's integral time scale $\mathscr{T}$ often does not exist in surface-layer turbulence.
- This does not prevent error estimates from being possible, but errors may be somewhat larger than we thought, because of the lower decay of RMSE($\widetilde{x}_\Delta$) with Δ.
- H_R (from R/S) and H_p (from MSE) are **different** estimators (they don't yield the same H).
- The Hurst phenomenon is (very likely) outside the scope of Monin-Obukhov Similarity Theory. This is expected, due to the very long-range nature of Hurst's phenomenon.

Many thanks

. . . for your attention!

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