Aerodynamic resistance parametrization for heterogeneous surfaces using a covariance function approach

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Motivation

- Meso-scale models have a typical resolution of order 1 km – 100 km

→ Many important scales of surface heterogeneity not resolved

→ Large inaccuracies for distinct surface heterogeneities (mountainous areas, semi-arid forests, …)
Motivation

Δ$x$ = 0.5 km

Δ$x$ = 2 km

Yatir forest, Israel

Subgrid scale parametrizations needed
Introduction: Monin-Obukhov similarity (MOST)

MOST used for most subgrid scale parametrizations

\[
\text{surface properties (} z_0, u_*, w'\theta'_0, \ldots \text{)} \quad \rightarrow \quad \text{MOST} \quad \rightarrow \quad \text{flow quantities at } z (\bar{u}_i(z), \bar{\theta}(z), \bar{q}(z))
\]

⚠️ MOST developed for homogeneous surfaces

→ Direct application of MOST to heterogeneous surfaces (bulk similarity) only for weakly heterogeneous scenarios

→ Other methods of applying MOST to heterogeneous surfaces “Discrete” approaches
Introduction: Discrete approaches / tile method

- Fragmentation of grid cell into land surface types
- Application of MOST to every surface type individually
- Aggregation of patches to determine effect of grid cell $\rightarrow$ flux aggregation
- Tile approach: Weighting factors from percentage of coverage

$$F_{\Sigma} = \sum_i v_i F_i$$

weighting factors

No spatial organization of tiles
Introduction: Bulk/tile aerodynamic resistance

Aerodynamic resistance for heat transfer:

\[ r = \frac{T_0 - \bar{T}(z)}{w'T'_0} \]

- Bulk approach: Direct application of MOST:

\[ r_{\text{bulk}} = \frac{1}{\kappa^2 U} \int_{z_{0m}}^{z} \frac{d\bar{z}}{\bar{z}} \phi_m(\bar{z}/L) \int_{z_{0h}}^{z} \frac{d\tilde{z}}{\tilde{z}} \phi_n(\tilde{z}/L) \]

- Tile approach: Using \( r_{\text{bulk}} \) for every surface patch to calculate \( w'T'_i \)

\[ w'T' \Sigma = \sum_i v_i w'T'_i \]

\[ \left( \langle T \rangle_0 - \langle T \rangle(z) \right) / r_{\text{tile}} = \sum_i v_i \left( \langle T \rangle_{i,0} - \langle T \rangle(z) \right) / r_{\text{bulk},i} \]
Theory: Covariance function approach

Bulk & tile approaches do not respect all scales of heterogeneity

→ Derivation of a novel parametrization from covariance functions
→ Better representation of turbulence characteristics
   (Kolmogorov, Townsend)

**Def:** Covariance function for two flow variables $V_1$ and $V_2$

\[
E_{V_1V_2}((d,z), \hat{d}) = V'_1(d, z) V'_2(d + \hat{d}, z)
\]

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Theory: homogeneous \( \rightarrow \) heterogeneous

Correlate homogeneous \((E^h_{V_1V_2})\) and heterogeneous \((E^{nh}_{V_1V_2})\) covariance functions

\( \rightarrow \) Plot \(E^{nh}_{TT}\) from large-eddy simulation (LES) of circular heterogeneity:

\((\Delta x = \Delta y = \Delta z = 4 \text{ m}, N_x \times N_y \times N_z = 2000 \times 2000 \times 400, U_{bg} = 1 \text{ m s}^{-1})\)

\(E^{nh}_{TT}\) consists of two functions

- Maximum at \(\hat{d} = 0\)
  \(\rightarrow E^h_{TT}\)

- Surface heterogeneity, shifted by \(-\hat{d}\)
  \(\rightarrow \) Heterogeneity map \(\chi\)

\[E^{nh}_{TT}(\hat{d}, z) = \chi(\hat{d} + \hat{d}) E^h_{TT}(\hat{d}, z)\]
Theory: $r_{cf}$ derivation from covariance functions

\[ E_{TT}^{nh}((d,z),\hat{d}) = \chi(d + \hat{d}) E_{TT}^{h}(\hat{d},z) \]

- **Kolomogorov spectra**
- **Covariance function approach to calculate $r_{cf}$**
  - Eddy anisotropy for non-neutral stratification (Katul et al, 2011)
  - Analytical derivation, neglecting dispersive fluxes and advection
- **Heterogeneity corrections for $r_{bulk}$**

\[ r_{cf} = \frac{1}{\kappa^2U} \int_{z_{0m}}^{z} d\tilde{z} \frac{\phi_{m}^{cf}(\tilde{z}/L)}{\tilde{z}} \int_{z_{0h}}^{z} d\tilde{z} \frac{\phi_{h}^{cf}(\tilde{z}/L)}{\tilde{z}} \]
Theory: Heterogeneity corrections

\[ r_{sf} = \frac{1}{\kappa^2 U} \int_{z_{0m}}^{z} d\tilde{z} \frac{\phi_m^c(\tilde{z}/L)}{\tilde{z}} \int_{z_{0h}}^{z} d\tilde{z} \frac{\phi_h^c(\tilde{z}/L)}{\tilde{z}} \]

\[ \phi_m^c = \phi_m I_{ww} \]

\[ \phi_h^c = (\phi_m + (\phi_h - \phi_m)/I_{TT}) I_{ww} \]

Heterogeneity correction factors: \( I_{ww}, I_{TT} \)

Bulk similarity

Meso-scale model grid

Surface heterogeneity map \( \chi \)
Results: LES cases and meso-scale model grids

LES of surface heterogeneities:
- Case 1: Circular constant
- Case 2: Circular random
- Case 3: Downscaled Yatir forest

Six investigated grid resolutions (different meso-scale model grids):
- $\Delta x = 0.1 \times D_{nh}$
- $\Delta x = 0.2 \times D_{nh}$
- $\Delta x = 0.4 \times D_{nh}$
- $\Delta x = 0.8 \times D_{nh}$
- $\Delta x = 2.0 \times D_{nh}$
- $\Delta x = 4.0 \times D_{nh}$
Results: Root-mean-square error (RMSE) plots

Findings:
- Mainly $\text{RMSE}_{\text{cf}} < \text{RMSE}_{\text{tile}} < \text{RMSE}_{\text{bulk}}$
- For small $\Delta x$:
  - $\text{RMSE}_{\text{case1}} > \text{RMSE}_{\text{case2}}$
  - $\text{RMSE}_{\text{case3}}$ largest
- For large $\Delta x$:
  - Errors of same size
  - Errors approach each other for $\Delta x = 4 \times D_{nh}$
- $\text{RMSE}_{\text{cf}}$ smallest in most heterogeneous case
Summary & Conclusions

- Analytic derivation of subgrid-scale aerodynamic resistance parametrization from covariance-function approaches
  → correction factors to bulk similarity
- Correction factors depend on meso-scale model grid and heterogeneity map $\chi$.
- Comparison of $r_{cf}$ against $r_{bulk}$ & $r_{tile}$ (reference $r_{LES}$) for three test cases of surface heterogeneities (circular constant, circular random, Yatir forest)
- Covariance function approach shows smaller deviations from LES than bulk and tile approaches.
- Future work:
  
  - Calculation of $\chi$ from satellite data for realistic applications (here $\chi$ from input maps for LES)
  - Investigation of advection and flux divergence contributions
Thanks for your attention!!