Probabilistic Estimation of Near Surface Winds in Tornadic Vortices

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Motivation

- Tornadoes as concentrated vortices - "simple" dynamics
- Existence of data - VORTEX2
- Partial observability - most damaging winds not observable by radar!

Tornado Models

- Navier-Stokes Equations
- Conservation of Energy
- Turbulence Parameterization
- Moisture, etc...
- Can we use a simplified set of equations and prior info to extract other velocity fields?

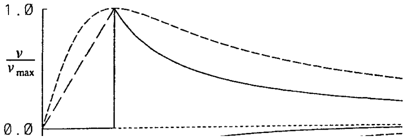
Tangential Velocity Models

- Tangential component estimated from data

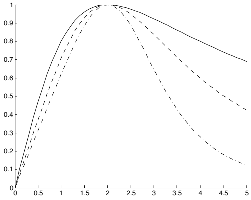
$$v(r,z) = v_c \phi(r;\vec{q}_r)\psi(z;\vec{q}_z)$$

$$\phi_{ww}(x;n,k,x_c) = \frac{n x_c^{n-k} x^k}{(n-k)x_c^n + k x^n}$$

- Generalization of Rankine vortex model
- Diagnose v: equations of motion are now overdetermined



From *Fiedler* (1994)



Wood-White vortex profiles ($n = 2, 3, 5$)

Inverse (Least-Squares) Error Probability Density Function

- To fit v we can minimize the sum of least squares function:

$$J(v_c, \vec{q}_r, \vec{q}_z) = \sum_{i=1}^{N_{\text{obs}}} \left(v_c \phi(r_i; \vec{q}_r) \psi(z_i; \vec{q}_z) - \hat{v}_i\right)^2$$

- We can turn this into a probability density function:

$$p(v_c, \vec{q}_r, \vec{q}_z) = \kappa e^{-J}$$

- Optimal parameters $\rightarrow$ Maximum likelihood estimator
- Uncertainty $\rightarrow$ sample parameter space, retrieved velocities weighted by probabilities

The Mathematical Problem Statement

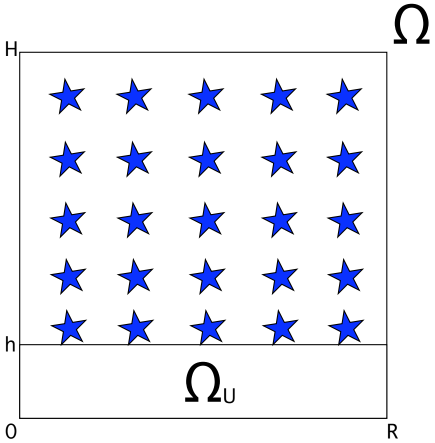
GOAL: Solve the system

$$\zeta u - \eta w = \nu(\zeta_r - \eta_z)$$
$$\frac{1}{r}(ru)_r + w_z = 0$$

on the domain

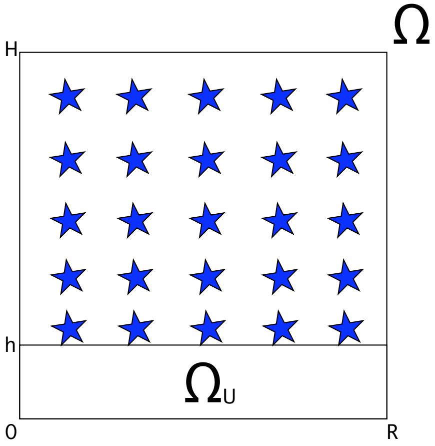
$$\Omega_u = (0, R) \times (0, h) \subset (0, R) \times (0, H) = \Omega,$$

and $\zeta = \frac{1}{r}(rv)_r$ and $\eta = -v_z$ are "known" (estimated from data) on $\Omega \backslash \Omega_u$.



Interesting Questions

- Sensitivity of u and w to errors in v
- Sensitivity to noisy data, quantity of data
- Sensitivity to unknown boundary conditions at $r = R$



Solution by Method of Characteristics

Solve first equation for u or w and plug in to continuity equation
$\Rightarrow$ hyperbolic equations (and characteristic equations):

- u equations:

$$\eta u_r + \zeta u_z + \eta \left(\frac{1}{r} + \left(\frac{\zeta}{\eta} \right)_z \right) u = \nu \eta \left(\frac{\zeta_r - \eta_z}{\eta} \right)_z$$

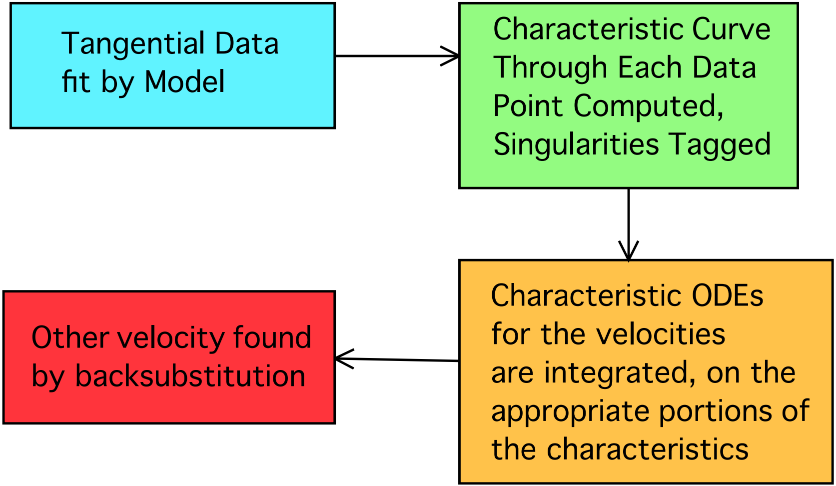
$$\Rightarrow \frac{dr}{dt} = \eta, \frac{dz}{dt} = \zeta, \frac{du}{dt} + \eta \left(\frac{1}{r} + \left(\frac{\zeta}{\eta} \right)_z \right) u = \nu \eta \left(\frac{\zeta_r - \eta_z}{\eta} \right)_z$$

- w equations:

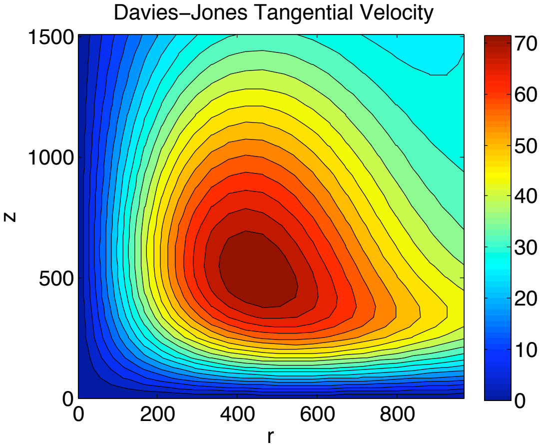
$$\eta w_r + \zeta w_z + \frac{\zeta}{r} \left(r \frac{\eta}{\zeta} \right)_r w = -\frac{\nu \zeta}{r} \left(r \frac{\zeta_r - \eta_z}{\zeta} \right)_r$$

$$\Rightarrow \frac{dr}{d\tau} = \eta, \frac{dz}{d\tau} = \zeta, \frac{dw}{d\tau} + \frac{\zeta}{r} \left(r \frac{\eta}{\zeta} \right)_r w = -\frac{\nu \zeta}{r} \left(r \frac{\zeta_r - \eta_z}{\zeta} \right)_r$$

Solution Methodology Flowchart



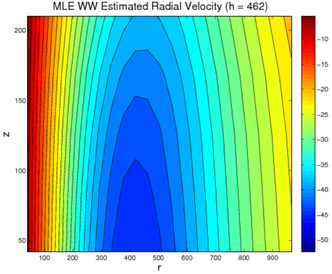
Testing the Methodology



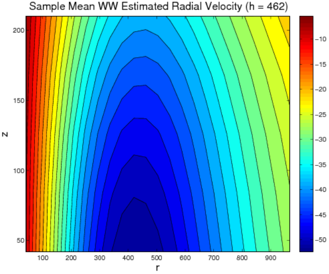
- Model output from Davies-Jones (2008) idealized thunderstorm/tornado cyclone model
- Take data for u and w from model output at $z = h$ $\Rightarrow$initial conditions for characteristic equations (perfect data).

Radial Velocity Estimates ($0 \leq z \leq 210$, $0 \leq r \leq 966$)

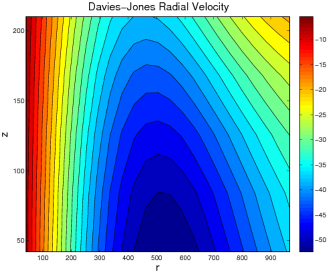
MLE U



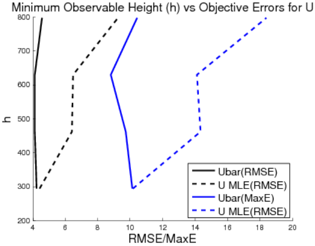
Sample Mean U



Davies-Jones U

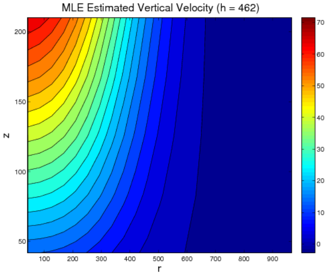


Error Measurements

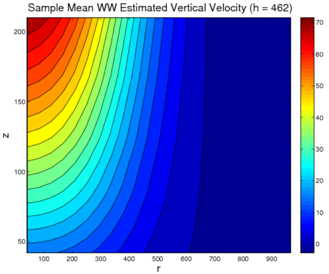


Vertical Velocity Estimates ($0 \leq z \leq 210$, $0 \leq r \leq 966$)

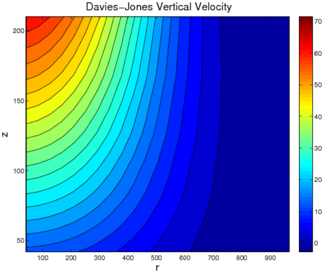
MLE W



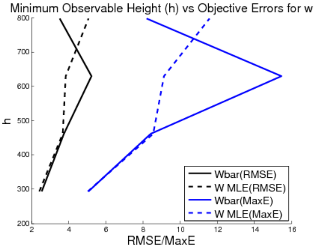
Sample Mean W



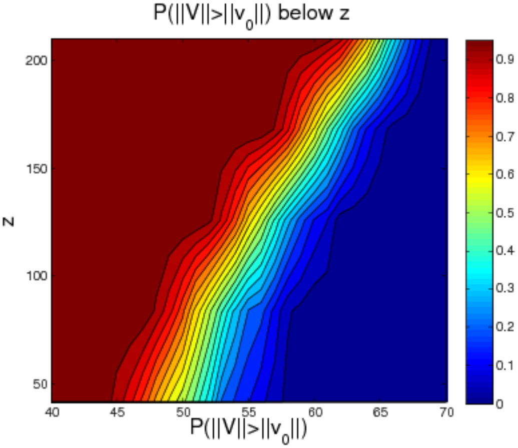
Davies-Jones W



Error Measurements



Random Variables from the Samples



Maximum Absolute Horizontal Winds

Thank You!