2.1 Can a gust front tilt horizontal vortex lines to produce a tornado?

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1. Introduction

The tilting of storm-relative environmental streamwise vorticity by an updraft explains updraft rotation and mesocyclone formation aloft (e.g., Lilly 1982; Davies-Jones 1984). However, in an environment devoid of vertical vorticity at the surface, rotation next to the ground does not seem to develop without a downdraft present nearby (Davies-Jones 1982a, b, 2008). In this paper we address whether, without a downdraft playing a role, environmental vortex lines can be tilted abruptly upward by a gust front, leading to strong vertical vorticity very close to the ground that can be stretched into a tornadic vortex. This process is called hereafter the gust-front mechanism. In this regard, Simpson (1982) proposed that a waterspout may form as a result of a steep density current gust front scooping up a bundle of horizontal vortex tubes from the sea surface and connecting these tubes to a mesocyclone that has extended downwards to the base of an overlying convective cloud (Fig. 1). We should mention, however, that this hypothesis did not appear in the later stage of this paper (Simpson et al. 1986). Davies-Jones (1982a,b) pointed out that vorticity tilted by an updraft alone acquires a vertical component only as it rises away from the surface. At the same time as it is being produced, vertical vorticity is being advected away from the ground. Thus, it seems that in the absence of a downdraft vertical vorticity can be present very near to the ground only if vortex lines near the surface are turned abruptly upward by intense gradients of upward velocity. Adlerman et al. (1999, p. 2045) claimed that this is highly improbable without either a strong vortex being present already at low levels to provide strong upward pressure-gradient forces or a gust front. Davies-Jones et al. (2001) attempted to rule out the gust-front mechanism by pointing out that mesocyclones form in numerical simulations that do not have the fine grid spacing necessary to resolve the abrupt upward turning of vortex lines. This argument is inductive. Below, we use theory, supported by a numerical simulation, to provide physical reasons why, even in extreme environmental shear, the gust-front mechanism fails to produce significant vertical vorticity in the lowest few hundred meters of the atmosphere.

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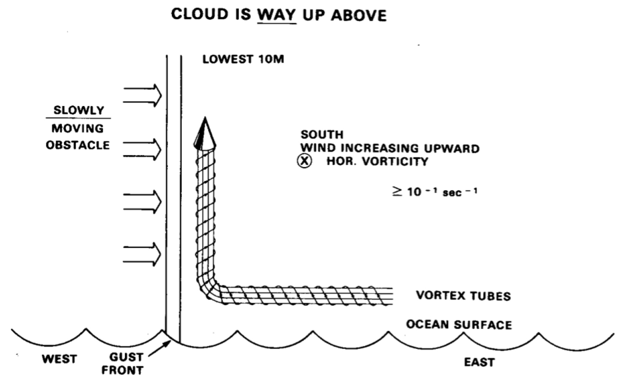

FIG. 1. Hypothesized upward tilting of vortex tubes in lowest boundary layer by gust front associated with cumulonimbus. Low-level vorticity associated with rapid upward increase of southerly wind just above ocean surface. Resulting cyclonic whirling postulated to connect with mesocyclone at cloud base producing visible funnel by rapid condensation. [Figure and caption from Simpson (1982).]

2. Vorticity in two-dimensional, three-directional flow

We start by considering vorticity in a simple flow that reveals how horizontal vortex lines are tilted upward at an "obstacle" such as a gust front. This flow is two-dimensional ($\partial/\partial y = 0$), inviscid, isentropic flow. The flow is three-directional to provide a component of horizontal vorticity that can be tilted. The y momentum in this slab-symmetric flow is equivalent to the angular momentum in an axisymmetric flow. Coriolis forces are omitted to eliminate ambient vertical vorticity. The momentum, mass continuity and entropy equations are

$$\frac{d\mathbf{v}}{dt} = -c_p\theta\boldsymbol{\nabla}\pi - \boldsymbol{\nabla}(gz) \quad (1)$$

$$\frac{d\alpha}{dt} = \alpha\boldsymbol{\nabla}\cdot\mathbf{v} \quad (2)$$

$$\frac{d\theta}{dt} = 0, \quad (3)$$

where the position vector is $\mathbf{x} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, the velocity vector is $\mathbf{v} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$, $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit eastward, northward and upward vectors, t is time, α is specific volume, p is pressure, $\pi = (p/p_0)^\kappa$ is the nondimensional pressure, T is temperature, $\theta = T/\pi$ is potential temperature, $\boldsymbol{\nabla} \equiv (\partial/\partial x, 0, \partial/\partial z)$, and $d/dt \equiv \partial/\partial t + u\partial/\partial x + w\partial/\partial z$.

The constants are g, the acceleration due to gravity, R, the gas constant for dry air, c_p $(= 7R/2)$, the specific heat of dry air at constant pressure, $\kappa = R/c_p$ $(= 2/7)$, and p_0, a standard pressure (1000 mb). The equation set is closed by the ideal gas law

$$\alpha = \frac{RT}{p} = \frac{R\theta\pi}{p_0\pi^{c_p/R}}. \tag{4}$$

For slab-symmetric flow, the vorticity is defined by

$$\boldsymbol{\omega} = \xi\mathbf{i} + \eta\mathbf{j} + \zeta\mathbf{k} = \left(-\frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x}\right). \tag{5}$$

From (1) and (2), the vorticity equation is

$$\begin{aligned}\frac{1}{\alpha}\frac{d(\alpha\boldsymbol{\omega})}{dt} &= (\boldsymbol{\omega}\cdot\boldsymbol{\nabla})\mathbf{v} + c_p\boldsymbol{\nabla}\pi\times\boldsymbol{\nabla}\theta \\ &= \frac{\partial(u,v)}{\partial(z,x)}\mathbf{i} + \frac{\partial(\pi,c_p\theta)}{\partial(z,x)}\mathbf{j} + \frac{\partial(w,v)}{\partial(z,x)}\mathbf{k},\end{aligned} \tag{6}$$

where the Jacobian $\frac{\partial(u,v)}{\partial(z,x)} = \frac{\partial u}{\partial z}\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial z}$, etc.

We obtain the integral of (6) by introducing Lagrangian coordinates (X, Y, Z, τ) where $\tau = t$, τ_0 is the initial time, and (X, Y, Z, τ_0) are the initial coordinates of the parcel currently at (x, y, z, t). The specific volume and velocity of this parcel at τ_0 are α_0 and (u_0, v_0, w_0), respectively. The parcel's initial vorticity is

$$\boldsymbol{\omega}_0 \equiv (\xi_0, \eta_0, \zeta_0) = \left(-\frac{\partial v_0}{\partial Z}, \frac{\partial u_0}{\partial Z} - \frac{\partial w_0}{\partial X}, \frac{\partial v_0}{\partial X}\right). \tag{7}$$

Note from $\mathbf{j}\cdot$ (1) and (3) that v and θ are conserved following a parcel, i.e.,

$$\begin{aligned} v(x,y,z,t) &= v_0(X,Y,Z,\tau_0) \\ \theta(x,y,z,t) &= \theta_0(X,Y,Z,\tau_0).\end{aligned} \tag{8}$$

Since v does not appear in any other equation in the set, it is a passive scalar. In other words, the flow in the x-z plane is unaffected by v. The Lagrangian continuity equation for the symmetric flow is

$$\frac{\partial(z,x)}{\partial(Z,X)} = \frac{\alpha}{\alpha_0} \text{ or } \frac{\partial(Z,X)}{\partial(z,x)} = \frac{\alpha_0}{\alpha}. \tag{9}$$

From (6), (8), and (9), the vorticity equation in Lagrangian coordinates is

$$\begin{aligned}\frac{1}{\alpha_0}\frac{\partial(\alpha\boldsymbol{\omega})}{\partial\tau} - \frac{\partial(u,v_0)}{\partial(Z,X)}\mathbf{i} &- \frac{\partial(w,v_0)}{\partial(Z,X)}\mathbf{k} \\ &= \frac{\partial(\pi,c_p\theta_0)}{\partial(Z,X)}\mathbf{j} \\ &= \frac{\partial[\pi-f(\theta_0),c_p\theta_0]}{\partial(Z,X)}\mathbf{j}\end{aligned} \tag{10}$$

The function $f(\theta_0)$ is superfluous for unsteady flows, but necessary for steady flows (section 3). After defining $\Pi = \int_{\tau_0}^{\tau}[\pi(\hat\tau) - f(\theta_0)]d\hat\tau$, which is the integral of π following a parcel, and using the identities $\partial x/\partial\tau = u$, $\partial z/\partial\tau = w$, $\partial v_0/\partial\tau = 0$, and $\partial\theta_0/\partial\tau = 0$, we can rewrite (11) as

$$\frac{\partial}{\partial\tau}\left[\frac{\alpha\boldsymbol{\omega}}{\alpha_0} - \frac{\partial(x,v_0)}{\partial(Z,X)}\mathbf{i} - \frac{\partial(\Pi,c_p\theta_0)}{\partial(Z,X)}\mathbf{j} - \frac{\partial(z,v_0)}{\partial(Z,X)}\mathbf{k}\right] = 0. \tag{11}$$

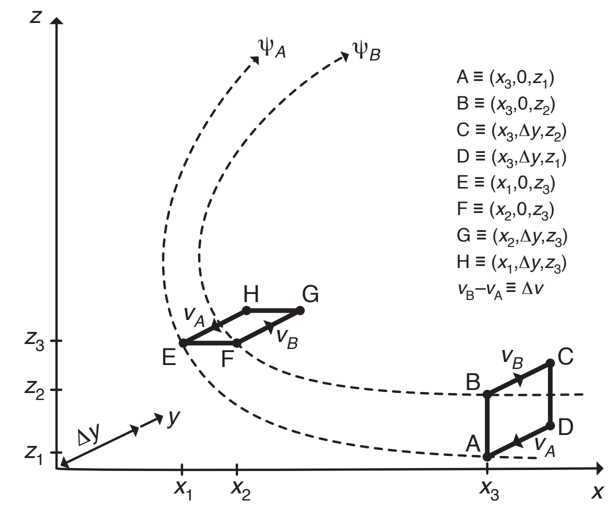

FIG. 2. Schematic of two streamlines ψ_A and ψ_B in the $y = 0$ plane in an in-up-and-out two-dimensional steady flow with v, the wind in the third direction, increasing monotonically with height. Points A and E lie on ψ_A, and points B and F lie on ψ_B, as shown. Points C, D, G, H are displaced from A, B, E, F, respectively, by a distance Δy in the direction of symmetry. The y-velocity is conserved, and is v_A and v_B on ψ_A and ψ_B, respectively. The vertical rectangle ABCD lies in the inflow where the streamlines and vorticity are horizontal, and the horizontal rectangle EFGH lies in the updraft. The circulation around ABCD is $\Delta v\ \Delta y$ where $\Delta v \equiv v_B - v_A$. This is also the circulation about the horizontal circuit EFGH. However, this much circulation around a horizontal circuit can only be realized in a rising flow at heights greater than the height of B because of v conservation.

Integration, application of the initial condition (7), and reuse of (9) yields the vorticity formula

$$\boldsymbol{\omega} = \left[-\frac{\partial v_0}{\partial z}, \frac{\alpha_0}{\alpha}\eta_0 + \frac{\partial(\Pi,c_p\theta_0)}{\partial(z,x)}, \frac{\partial v_0}{\partial x}\right]. \tag{12}$$

Note from (12) that v_0 serves as a "streamfunction" for the $(\xi, 0, \zeta)$ vector field. Thus, the vortex lines lie in surfaces of constant v_0.

In simulations of convective storms, the initial state often is an unperturbed horizontally homogeneous environment. In such cases, $v_0 = v_0(Z)$ and there is no initial vertical vorticity. From (12) it is evident that

$$\boldsymbol{\omega}\cdot\boldsymbol{\nabla}Z = \frac{\partial[Z, v_0(Z)]}{\partial(z,x)} = 0, \tag{13}$$

so the vortex lines lie in constant-Z surfaces (which coincide here with constant-v_0 surfaces, and the isentropic surfaces as well if the environment is stably stratified). It follows from (13) that

$$\zeta = \left(\frac{\partial z}{\partial x}\right)_Z \xi. \tag{14}$$

This suggests that significant vertical vorticity could be produced near the ground by abrupt uplifting of a constant-Z surface. However, a circulation argument indicates that without a downdraft, this mechanism fails to produce a significant rotation about a vertical axis within a few hundred meters of the ground. To see this, consider a vertical rectangular circuit ABCD in the inflow and a horizontal rectangular circuit EFGH in the updraft formed by points on the same streamlines as shown in Fig. 2. Because of symmetry and conservation of v, the circulation around both circuits is the same. However, because the flow is rising, the horizontal circuit can only be at a greater height than the top of the vertical one. For example, let the environmental shear in the y-direction be a constant 2×10^{-2} s^{-1}, corresponding to a maximum vertical difference in y-velocities of 10 m s^{-1} in the lowest 500 m of the inflow. Then in a purely rising flow, the difference in v in the updraft cannot exceed 10 m s^{-1} at the 500-m level, with proportionally lower limits at lower levels. In contrast, a downdraft, by transporting y-momentum downward, can create large circulations around horizontal circuits that are very close to the ground.

3. Vorticity in *steady* two-dimensional, three-directional flow

We can make further insights when the flow is assumed to be steady. Let q be any positive-definite conserved variable. Since $\boldsymbol{\nabla}\cdot(\mathbf{v}/q\alpha)=0$, we can introduce a streamfunction ψ defined by

$$u \equiv -q\alpha\frac{\partial\psi}{\partial z},\ \ w \equiv q\alpha\frac{\partial\psi}{\partial x}. \tag{15}$$

When $q=1$ (a valid choice), q disappears from the definition of ψ. At the ground ($z=0$) $w=0$ so ψ is a constant there. Since ψ contains an arbitrary constant, we may set $\psi=0$ at $z=0$. For steady flow, the subscript 0 refers, not to initial values, but to the uniform conditions that exist along a streamline far upstream from a storm where the inflow is purely horizontal, and Z equates with z_0, the height of the streamline at upstream infinity. Furthermore, trajectories and streamlines coincide, and the advections of conserved variables vanish, which implies

$$v=v_0(\psi),\ \ \theta=\theta_0(\psi),\ \ q=q_0(\psi). \tag{16}$$

Derivatives with respect to ψ and z_0 are linked by

$$-\frac{d}{q_0\alpha_0 d\psi}=\frac{d}{u_0 dz_0}. \tag{17}$$

Velocity and vorticity components are related by

$$(\xi,\eta,\zeta)=\frac{1}{q_0\alpha}\frac{dv_0}{d\psi}(u,0,w)+[0,-\boldsymbol{\nabla}\cdot(q_0\alpha\boldsymbol{\nabla}\psi),0], \tag{18}$$

or

$$(\xi,\eta,\zeta)=\frac{dv_0}{d\psi}\left(-\frac{\partial\psi}{\partial z},0,\frac{\partial\psi}{\partial x}\right)+[0,-\boldsymbol{\nabla}\cdot(q_0\alpha\boldsymbol{\nabla}\psi),0]. \tag{19}$$

Note that the projections of the vortex lines and streamlines onto the x-z plane coincide. Furthermore, when the conserved quantity $q_0^{-1}dv_0/d\psi$ is positive, the maximum values of $\alpha\zeta$ and w on a streamline are collocated.

The momentum equation in the x-z plane is

$$\boldsymbol{\nabla}B=\mathbf{v}\times\boldsymbol{\omega}+c_p\pi\boldsymbol{\nabla}\theta_0, \tag{20}$$

where B is the Bernoulli function

$$B\equiv c_p\theta\pi+gz+\frac{u^2+v_0^2+w^2}{2}. \tag{21}$$

The dot product of (20) with $\mathbf{v}$ shows that B is constant along a streamline in isentropic flow. Evaluation of the Bernoulli function far upstream gives

$$\begin{aligned} B(\psi) &= c_p\theta_0\pi+gz+\frac{u^2+v_0^2+w^2}{2}\\ &= c_p\theta_0\pi_0+gz_0+\frac{u_0^2+v_0^2}{2}. \end{aligned} \tag{22}$$

The pressure variation along a streamline is therefore given by

$$\pi-\pi_0=\frac{g(z_0-z)+(u_0^2-u^2-w^2)/2}{c_p\theta_0}. \tag{23}$$

From (18), the Lamb vector is

$$\mathbf{L}=\boldsymbol{\omega}\times\mathbf{v}=\left(q_0\alpha\eta-\frac{1}{2}\frac{dv_0^2}{d\psi}\right)\boldsymbol{\nabla}\psi. \tag{24}$$

Since B and θ are functions of ψ alone, (20) reduces to

$$q_0\alpha\eta-\frac{1}{2}\frac{dv_0^2}{d\psi}=-\frac{dB}{d\psi}+c_p\pi\frac{d\theta_0}{d\psi}. \tag{25}$$

When $q_0=1$ and $v_0=0$, this equation is the dry version of one derived by Lilly [1979; see his (3.6)] while reviewing theoretical work on squall lines by Moncrieff and Green (1972) and Moncrieff (1978). According to (25), the difference dB in the Bernoulli function between close streamlines is equal to minus the Lamb vector $\mathbf{L}$ plus $T\,dS$, where dS is the difference in the streamlines' entropies.

Subtraction of the upstream evaluation of (25) from (25) itself gives

$$\begin{aligned} \eta &= \frac{\alpha_0\eta_0}{\alpha}+c_p(\pi-\pi_0)\frac{d\theta_0}{q_0\alpha d\psi}\\ &= -\frac{1}{2}\frac{du_0^2}{q_0\alpha d\psi}+c_p(\pi-\pi_0)\frac{d\theta_0}{q_0\alpha d\psi}. \end{aligned} \tag{26}$$

Substituting for $c_p(\pi-\pi_0)$ from (23) and using (17) then yields

$$\eta=\frac{\alpha_0\eta_0}{\alpha}+\frac{\alpha_0}{\alpha}g\frac{d\ln\theta_0}{dz_0}\left(\frac{z-z_0}{u_0}+\frac{u^2+w^2-u_0^2}{2gu_0}\right) \tag{27}$$

where the first and second terms on the rhs are the barotropic and baroclinic vorticity, respectively. Generally, wind speeds are moderate enough to satisfy $|u^2+w^2-u_0^2|/2 \ll g|z-z_0|$ so that

$$\eta=\frac{\alpha_0\eta_0}{\alpha}+\frac{\alpha_0}{\alpha}\frac{g}{u_0}\frac{d\ln\theta_0}{dz_0}(z-z_0), \tag{28}$$

which is a special case of Moncrieff and Green's (1972) vorticity equation [their equation (12)]. In the Boussinesq approximation [$\alpha=\alpha_0=1$, $q_0=1$, $\theta_0=$ constant ($\equiv\theta_c$) except when multiplied by g] that is valid for shallow flows, (27) reduces to (Davies-Jones 2006)

$$\eta=-\nabla^2\psi=\eta_0+\frac{g}{\theta_c}\frac{d\theta_0}{dz_0}\frac{(z-z_0)}{u_0};\quad q_0=1. \tag{29}$$

We can deduce the effects of environmental stratification from (28). The first term on the rhs is exactly the barotropic y-vorticity, η_{BT}, and the second term is approximately the baroclinic vorticity, η_{BC}. In a branch of flow where warm air enters horizontally from the east ($u_0 < 0$) with positive shear ($\eta_0 > 0$), rises, and exits to the east (so that $z - z_0 \geq 0$), η_{BT} is positive. Because there is no stretching or tilting of y-vorticity, it changes only as a result of dilatation α/α_0. In the same branch, η_{BC} has the opposite sign of $d\theta_0/dz_0$, the environmental stratification. We rule out the $d\theta_0/dz_0 < 0$ case on the grounds that the inflow would be unstable and break down into convective rolls. As in the simulations of Markowski et al. (2003), stable stratification should be less favorable for vertical-vorticity production than neutral stratification. This is evident from the factor $-(z - z_0)d\theta_0/dz_0$ in the η_{BC} term. This factor is equal to the linearized restoring force when parcels are displaced vertically in a stably stratified environment (Dutton 1986, p. 71). The resistance to lifting weakens the circulation in the x-z plane and consequently decreases vertical vorticity because w is proportional to ζ in (18).

4. Numerical simulation of tilting of strong environmental vorticity by a powerful density current

We ran a simple numerical simulation to see if the abrupt upward-turning of horizontal vortex tubes at a gust front could produce significant vertical vorticity very close to the ground. The simulation is three-dimensional, but there are no gradients in the y-direction. In our attempt to make this happen, we chose an extreme case (or "worst-case scenario") with a very strong cold pool and an environment with very large shear and no static stability. Except for the addition of vertical shear in the y-direction, the simulation is similar to ones made by Rotunno et al. (1988; see their Figs. 19 and 20).

The dry version of the Bryan cloud model version 1 (CM1), release 16, is used (Bryan and Fritsch 2002). The model equations are discretized on a C-grid (Arakawa and Lamb 1977) having dimensions of 50 km $\times$ 10 km $\times$ 20 km. The domain has rigid, free-slip top and bottom boundaries, open west and east boundaries, and periodic north and south boundaries. The horizontal and vertical grid spacing is 50 m. The advection scheme is fifth-order, which has implicit diffusion. No additional artificial diffusion is included. Eddy viscosities are determined from the prognosed turbulent kinetic energy and a mixing length scale (Deardorff 1972). There are no surface fluxes, Coriolis force, or radiative transfer.

The simulation is initialized with a 5-km-deep block of cold air within the westernmost 10 km of the domain. The minimum potential temperature perturbation (found at the surface) within the cold-air block is -12 K. The potential temperature perturbation decreases linearly with height within the cold air. The environment is otherwise neutrally stratified. The environmental vorticity available for tilting, $\xi_0 = -dv_0/dz$, is -0.02 s^{-1} (Fig. 3d). This corresponds to a southerly wind shear of 20 m s^{-1} per km (Fig. 3b). This shear is applied over the depth domain because v is a passive scalar and its contours serve as vortex lines for (ξ,0,ζ). The component of environmental vorticity parallel to the gust front, η_0, is 0.02 s^{-1} in the lowest 1000 m (Fig. 3e). The northward vorticity component (i.e., westerly vertical shear; Fig. 3a) is included to offset the strong southward vorticity generated solenoidally by the density current and, hence, to maintain the density current's almost vertical leading edge in accord with the RKW discovery. There is no zonal wind shear above 1 km.

After ten minutes, the head of the density current is still over 3 km deep with its leading edge staying steep (Fig. 3). Just ahead of this almost vertical wall, warm air is rising rapidly with vertical velocities in excess of 20 m s^{-1} located as low as 1 km above ground (Fig. 3c). The peak vertical vorticity is 0.02 s^{-1} and is located well aloft at 3 km (Fig. 3f). Despite the large environmental horizontal vorticity in the lowest 1 km, the maximum vertical vorticity at 25 m (the lowest scalar level) in the warm air ahead of the density current is only 1.25×10^{-3} s^{-1}. The vertical vorticity is small there even though vortex lines are being tilted very abruptly near the surface by the nearly vertical density current head (Fig. 3i). The reasons for this rather surprising result are contained in the pressure field (Fig. 3g) and in the ξ field (Fig. 3d). A stagnation high is present at the surface at the leading edge of the density current (Fig. 3g). Thus, warm parcels encounter an adverse pressure gradient and decelerate as they approach within about 2 km of the gust front. Consequently, they are compressed in the east-west direction (and stretched vertically to conserve mass). Owing to the east-west compression, the westward vorticities of these parcels are greatly reduced before the parcels encounter large gradients of vertical velocity. The magnitude of ξ in the lowest 100 m decreases from 0.02 s^{-1} to 0.002 s^{-1} (Fig. 3d) by the time the streamlines (Fig. 3h) turn upward at the density current's leading edge. The vertical stretching of parcels implies that air rises gradually at first about 2 km ahead of the gust front. This is evident in the streamlines (Fig. 3h) and vortex lines (which coincide with v contours; Fig. 3b).

5. Summary

Based on the following line of reasoning, we conclude that tilting of horizontal vortex tubes by a gust front does not cause a tornado. For two-dimensional ($\partial/\partial y = 0$), three-directional, inviscid, isentropic flow in a nonrotating atmosphere, the velocity component in the y direction, v, is a conservative passive scalar. It serves as a "streamfunction" for the $(\xi, 0, \zeta)$ vector field so the vortex lines lie in surfaces of constant v. If the environment is horizontally homogeneous and thus devoid of vertical vorticity and Z is the original height of a parcel, then the vortex lines lie in constant-Z surfaces, which coincide with the constant-v surfaces and the isentropic surfaces. Vorticity components ξ and ζ are related by $\zeta = \xi(\partial z/\partial x)_{Z \text{ or } v \text{ or } \theta}$. This suggests that abrupt upturning of vortex lines by a density current or topography could produce appreciable vertical vorticity next to the ground. However, a circulation argument shows that, an updraft by itself cannot produce at very low levels the large differences in horizontal velocity associated with significant rotation. Differential downward transport of y momentum (or angular momentum in an axisymmetric flow) is required.

If the flow is also *steady*, v and hence $dv/d\psi$ are constant along a streamline. The x and z components of vorticity and wind satisfy a Beltrami relationship, namely $\alpha(\xi, 0, \zeta) = (u, 0, w)dv/d\psi$ so the streamlines coincide with the projections of the vortex lines onto the x-z plane. Vorticity components ξ and ζ are related by $\zeta = \xi(\partial z/\partial x)_\psi$. Along a

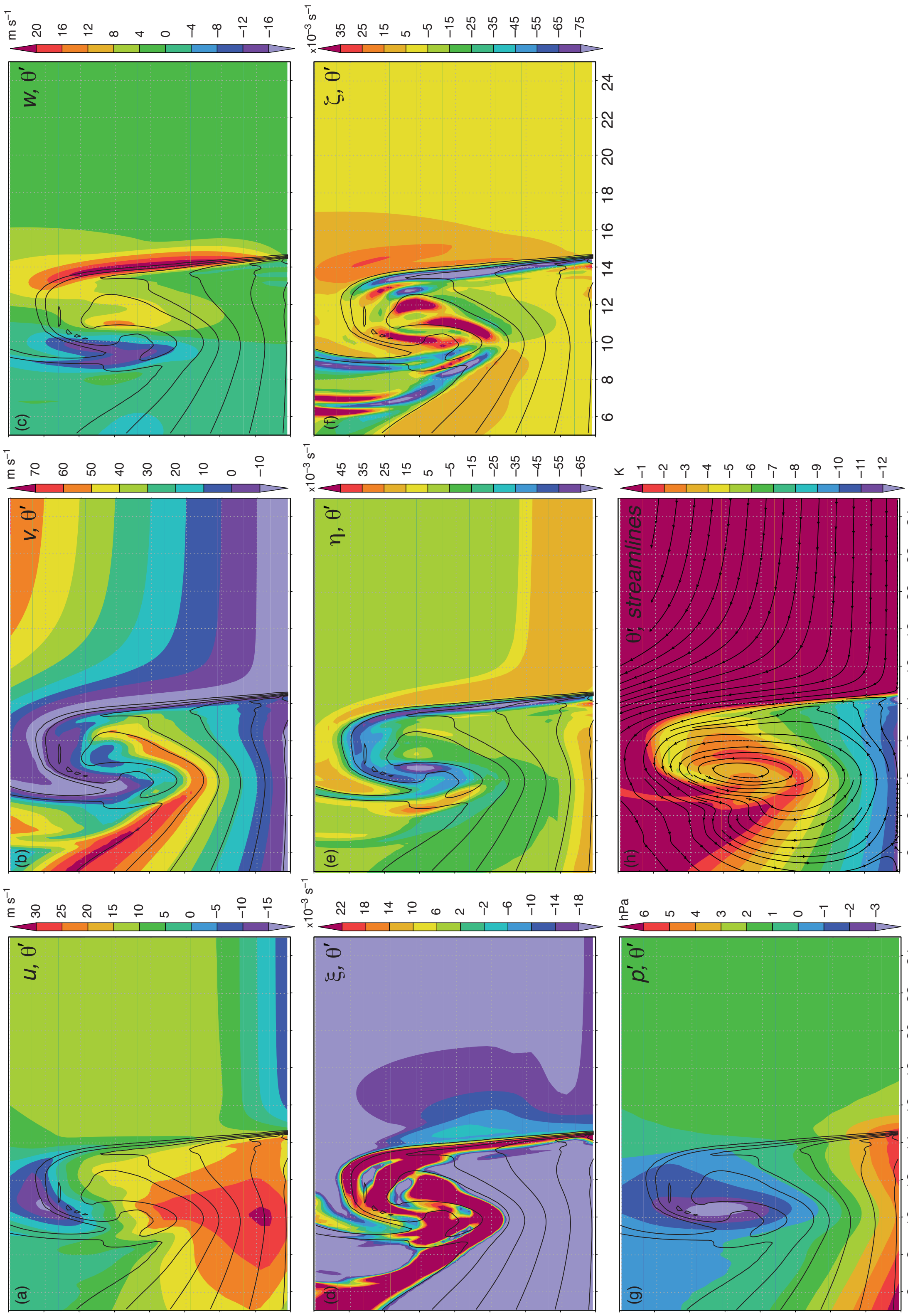

FIG. 3. Vertical cross-sections 10 min after a cold block is released into a neutrally stratified environment having strong westerly and southerly low-level shear. In (a)–(g), isopleths of potential temperature perturbation are shown every 2 K (starting at 1 K) in black, overlaid on (color shading) u, v, w, ξ, η, ζ, and p', respectively. Streamlines (black) are overlaid on the field of potential temperature perturbation (color shading) in (h). Axis labels indicate distances in kilometers.

streamline with $dv/d\psi > 0$, the maximum values of $\alpha\zeta$ and w are collocated, and $(\xi, 0, \zeta)$ vanishes at a stagnation point. The Bernoulli function B is constant along a streamline, so $B = c_p\theta_0\pi + gz + (u^2 + w^2)/2 = c_p\theta_0\pi_0 + gz_0 + u_0^2/2$. At the stagnation point at the front of the density current, the pressure is high (dubbed a "stagnation high"). Along a streamline, low dynamic pressure is collocated with high values of $\alpha^2(\xi^2 + \zeta^2)$. Air parcels approaching the stagnation high decelerate in strong adverse pressure gradient and are compressed horizontally. Along a streamline $\alpha\xi$ is proportional to u. So, near the surface, the horizontal vorticity available for upward tilting is greatly reduced before it is tilted. Consequently, uplifting of horizontal vortex lines by a density current does not lead to appreciable vertical vorticity just off the ground. A time-dependent numerical simulation verifies this finding and generalizes it to unsteady flow.

Therefore, one cannot argue that because there is large amount of horizontal vorticity in a surface-based layer in the environment, abrupt tilting of it at an "obstacle" (such as a gust front or topographical barrier) will produces similar strength vertical vorticity very close to the surface. Linear thinking (i.e., assuming that horizontal vorticity is unmodified from environmental values) is misleading in this case because the abrupt tilting is unavoidably associated locally with a stagnation flow that greatly compresses the horizontal vortex tubes prior to tilting.

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