Possible Attractor-Like Flow in a Tornadic Supercell

Will Frost, Doug Dokken, Kurt Scholz, and Misha Shvartsman, Department of Mathematics, University of St. Thomas, St. Paul, MN, USA, Pavel Bělík, Department of Mathematics, Augsburg College, Minneapolis, MN, USA,

American Meteorological Society Meeting, 28th Conference on Severe Local Storms, November 7th, 2016, Portland, OR

Introduction

In a recent papers Y. Sasaki develops a model for tornado genesis and maintenance. Using the calculus of variations, he derives the Euler equations for the fluids, subject to constraints on entropy and mass. An important role is played by an interpretation of the Lagrange multipliers associated to the constraints. In his paper he suggests that the flow in the neighborhood of the pre-tornado and tornado is like an attractor.

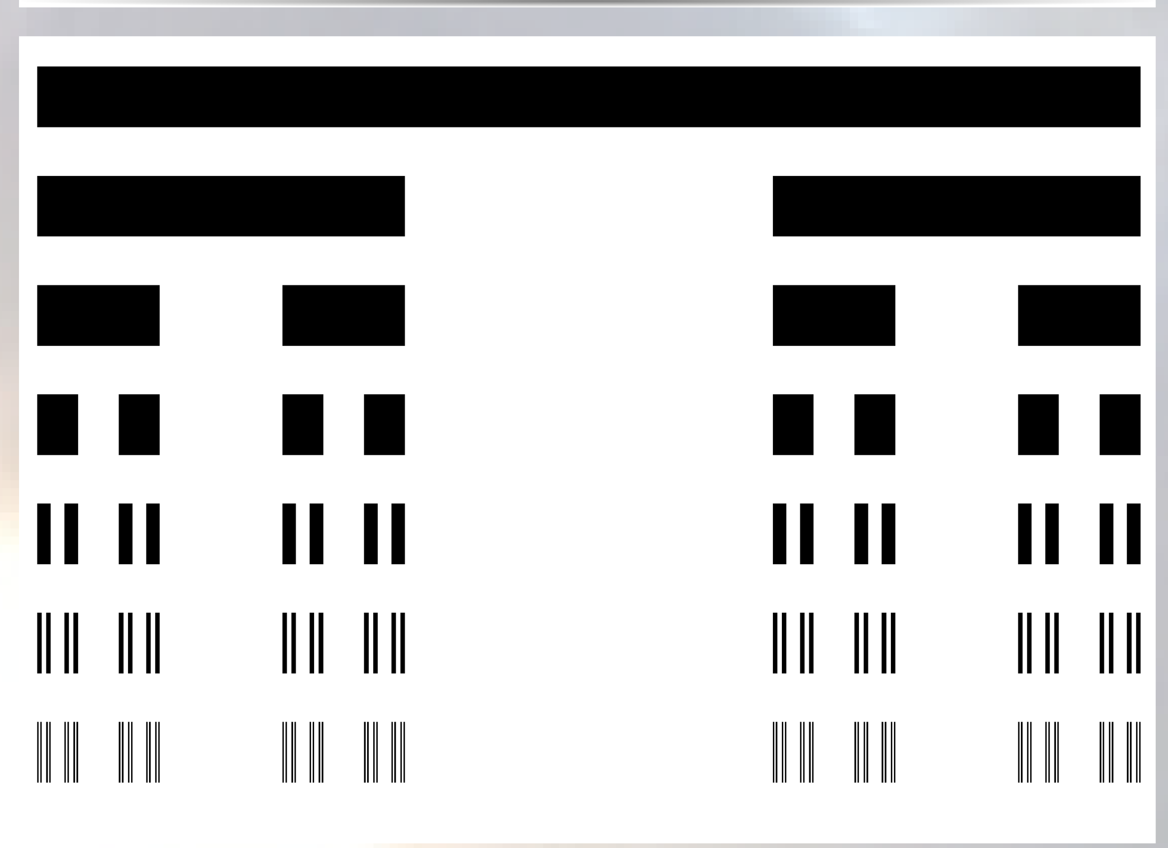
What is an Attractor?

An attractor is a set that a dynamical system or a physical system evolves toward.

Cantor middle thirds set

Begin with the unit interval [0,1]. The first step: Remove the open middle third of the interval, (1/3, 2/3). The second step: From the remaining two intervals, remove the open middle thirds of the remaining intervals. Continue,..... At the k^{th} step: From the remaining intervals, remove the open middle thirds. Repeat this process indefinitely. The set of points that remain is called a Cantor set.

Cantor Set construction



Example: Quadratic Map $f_\mu(x)$

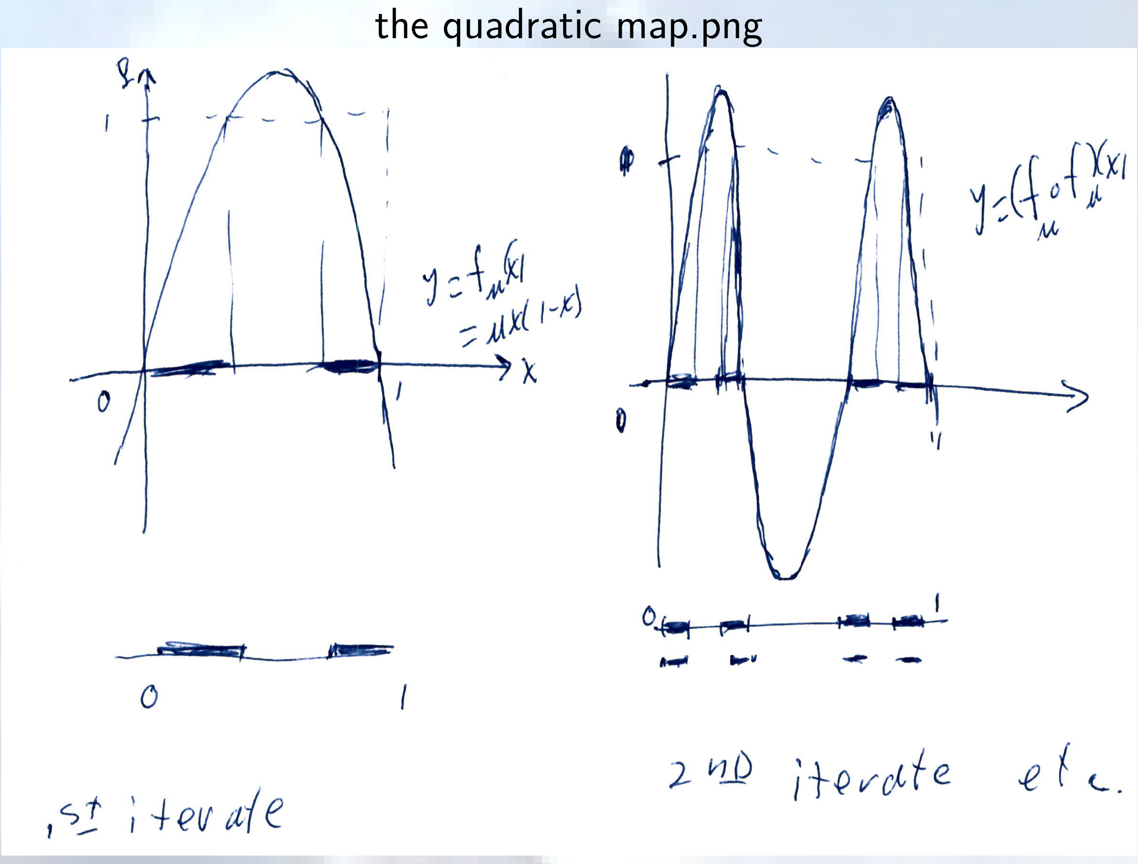
The quadratic family is defined by $f_\mu(x) = \mu x(1-x)$, where μ is a constant greater than 0. The graphs of the curves in this family are parabolas opening downward with x-intercepts at $x = 0$ and $x = 1$. For $\mu > 4$, iteration of the map $f_\mu(x)$ on the set $[0, 1]$ leads to a Cantor set. The first iteration of the map removes middle portion of the interval $[0, 1]$. The second iteration removes middle portions of the remaining two intervals. Continuing to iterate the map on $[0, 1]$ leads to a Cantor set.

Acknowledgements

Doug Dokken, Kurt Scholz, Misha Shvartsman, and Pavel Bělík were supported by the NSF CSUMS grant DMS-0802959.

Interating the quadratic map

the quadratic map.png



Example: The Smale Williams Attractor

Consider a torus (donut shaped object). This can be identified with a circle S^1 fattened by a disk D cross-section. Mathematically, this can be thought of as a 3-dimensional object with $S^1 \times D$. The Smale-Williams Attractor is formed by repeatedly iterating the map $f(t, z) = (2t, \frac{1}{4}z + \frac{1}{2}e^{(}it))$ on the set $S^1 \times D$, where $t \in S^1$(the unit circle) and $z \in D$, and the first component of $f(t, z)$ is taken $mod\ 2\pi$(i.e. the remainder after subtracting off a multiple of 2π. The first iteration stretches the torus in the direction of the axis (first component) and then contracts the cross-section and rotates it depending on the position along the axis. The image shows the nested image of the successive iterations to the left of the image of a cross section. No mass is lost in the iterations of the Smale-Williams attractor, the construction uses compression of volume.

Smale-Williams Attractor

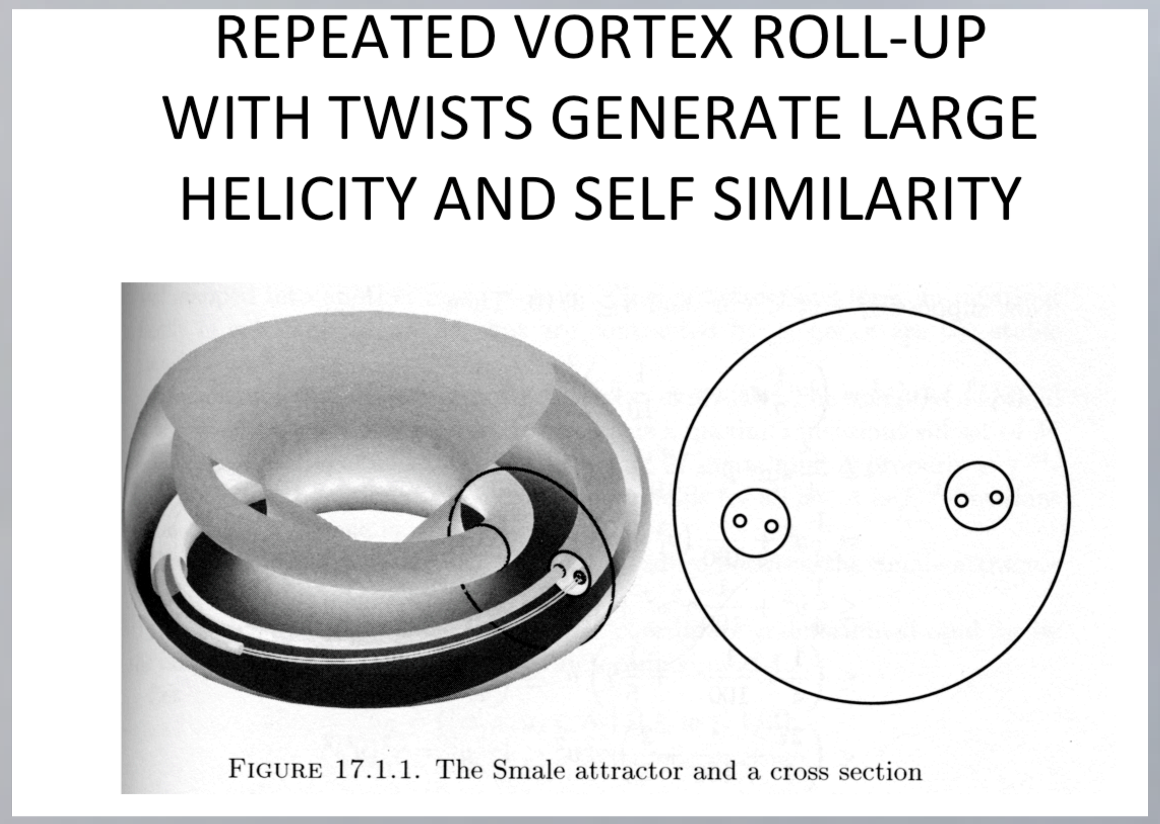
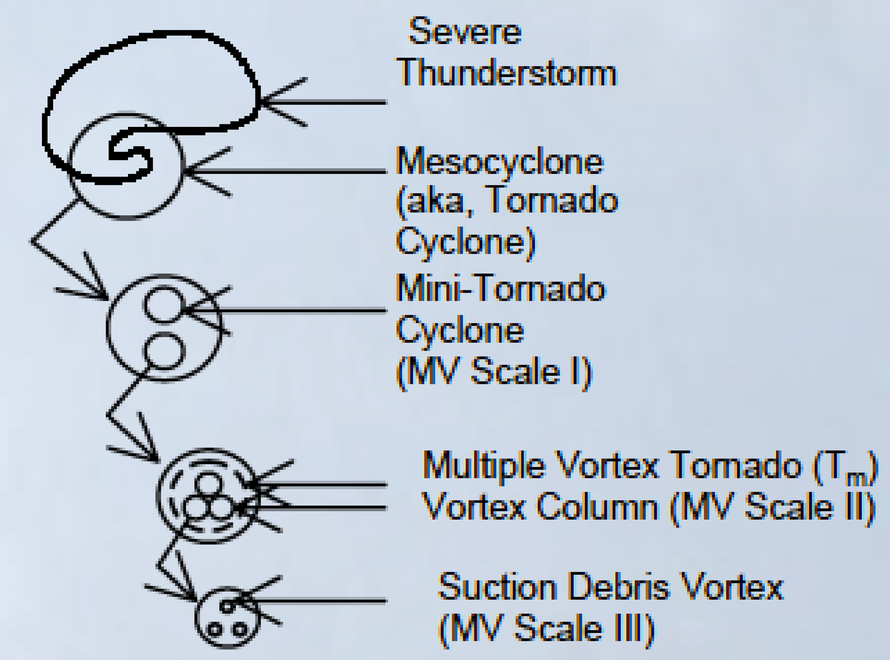


FIGURE 17.1.1. The Smale attractor and a cross section



Fractal hierarchy of multiple vortices, *BAMS*, **58**, 900–908

Fujita's Recycling Hypothesis

Some of the rear-flank downdraft is recycled into the updraft. Surface moisture is captured by the updraft, as the parcels are lifted they cool and latent heat is released, sustaining the updraft, as the moisture condenses it forms cloud water. Subsequent cooling of cloud water leads to formation of hail and grapple, and further latent heat release, sustaining the updraft further up in the storm. Mass and volume in the updraft overshoot their equilibrium at the top of the storm flowing out the top of the storm(anvil) and fall back into the anvil or out other parts of the upper level of the storm. Some of this moisture to falls out the backside of the storm, mixing of this moisture falling into dry warm air from the west (at upper levels), southwest (at lower levels) leads to evaporative cooling of water species (hail, grappel, snow, and super cooled water droplets) formed in the updraft. This leads to the development of the RFD (rear flank downdraft). Mass and volume of the downdraft spread out at the surface. If the downdraft air is buoyant enough some of it recycles into the updraft.

Fujita Recycling Hypothesis

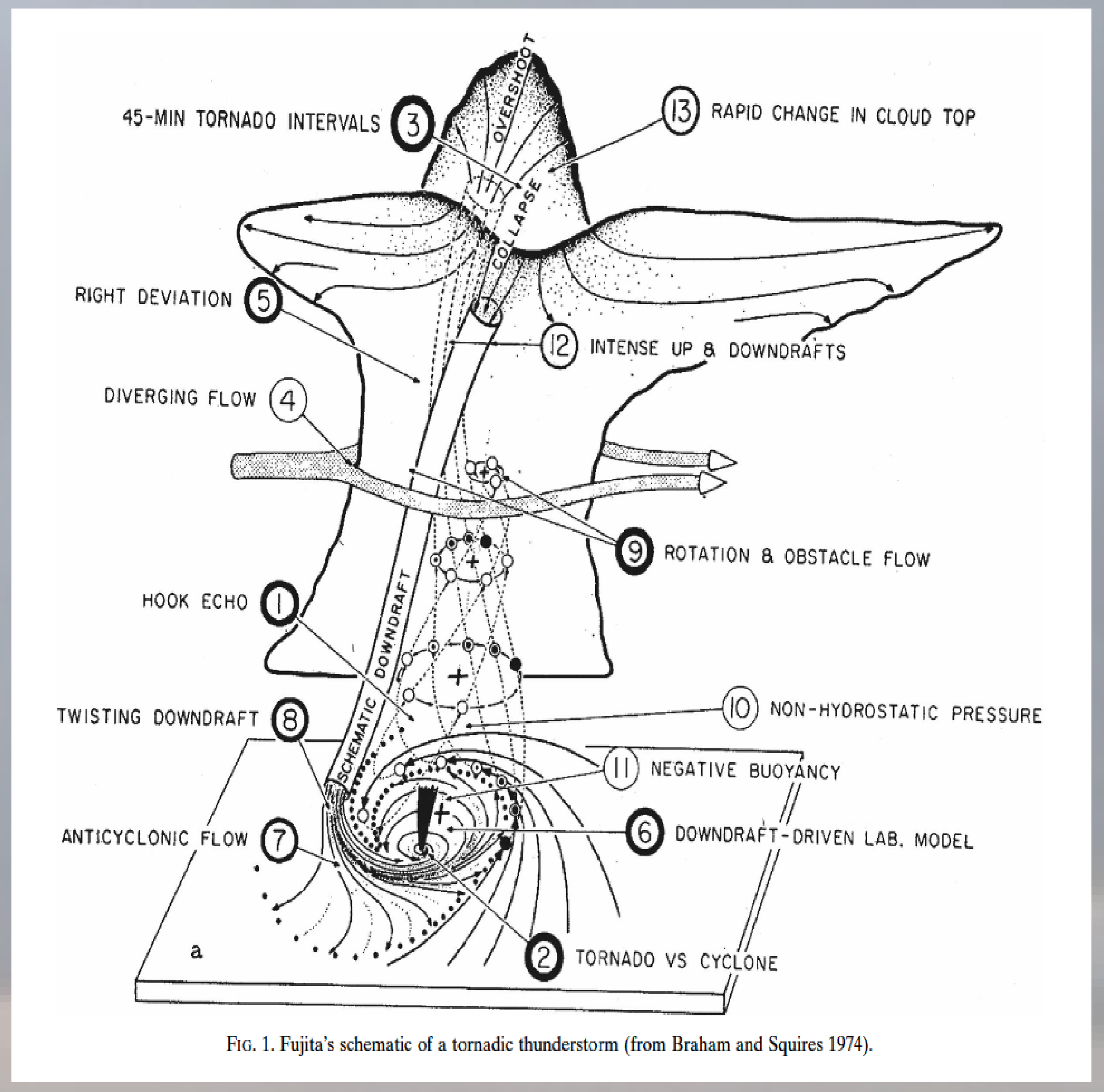


FIG. 1. Fujita's schematic of a tornadic thunderstorm (from Braham and Squires 1974).

Iteration of the Recycling Hypothesis

Freeze time: look at the stream lines. If one iterates the recycling hypothesis flow along the stream lines in the updraft and downdraft (assuming enough mixing in the updraft and downdraft and the RFD air is buoyant enough) then one sees similar structures to the nested loops in the Smale-Williams attractor forming. If the flow is helical at the surface (i.e. streamlines and vortex lines are collinear) then nested vortices would form at the surface. Thus the flow would have cross section leading to the hierarchy of vortices. It is important to notice that the iteration of the recycling hypothesis leads to loss of mass and volume with each loop and hence this process differs from the Smale-Williams atttractor, where contraction, leads to the nesting of the tubes. The iteration of the recycling hypothesis is similar to the interation of the quadratic map f_μ $\mu > 4$, on the unit interval, in that the mass of the line is lost with each iteration, leading to a Cantor set.

References

[1] Katok, A. Introduction to the Modern Theory of Dynamical Systems , Cambridge Univ. Press, (1995).

[2] Davies-Jones, R., Can a Descending Rain Curtan in a Supercell Tornadogenesis Barotropically, J. Atmos. Sci., **43**, (1986).

[3] Braham, R. R., Jr., and P. Squires, Cloud physics -1974. Bull. Amer. Meteor. Soc., **55**, 543-586.