The Centre for Australian Weather and Climate Research
A partnership between CSIRO and the Bureau of Meteorology

A new approach to Potential Vorticity tendency

Kevin Tory, Jeff Kepert, (CAWCR)
Jason Sippel, (NASA), Mai Nguyen (Monash University)
The Centre for Australian Weather and Climate Research – A partnership between the Bureau of Meteorology and CSIRO, Melbourne, Australia
Correspondence: K.tory@bom.gov.aul

Potential Vorticity (PV) is a very useful and widely used meteorological quantity that is conserved in adiabatic frictionless flow. An understanding of how and why PV changes can give great insight into atmospheric dynamics. Mathematically PV is the dot product of two vectors: one representing the wind field (absolute vorticity) and one the mass field (potential temperature gradient divided by density):

$$PV = \vec{\eta} \bullet \left(\vec{\nabla}\theta / \rho \right)$$

1. A/D cancellation problem

In convective systems there can often be large opposition between the PV advection and diabatic heating terms of the traditional PV tendency equation, with the total tendency some small residual value. An example is given in Fig. 1 (right): This is *the adiabatic/diabatic (A/D) cancellation problem* (Tory et al. 2012).

adv = PV advection
$diab$ = diabatic heating source term
$fric$ = Friction source term

$$\frac{\partial}{\partial t} PV = -\vec{u} \bullet \vec{\nabla} PV_{adv} + \frac{1}{\rho} \vec{\eta} \bullet \vec{\nabla} \dot{\theta}_{diab} + \frac{1}{\rho} \left(\vec{\nabla}\theta \bullet \vec{\nabla} \times \vec{F} \right)_{fric}$$

The A/D cancellation is illustrated schematically in Fig. 2 (below). Diabatic heating depresses the central isentrope in panel b, which weakens PV in the upper layers and strengthens PV in the lower layers by changing the vertical gradient of potential temperature. The heating-induced upward flow reverses the isentrope movement, which contributes to an opposing PV tendency from advection. The A/D cancellation of Fig.1 is described by the upper layer of Fig. 2.

Dynamically, the PV change is due to vorticity convergence and divergence in the lower and upper layers respectively, coupled with the small changes in mass field required to balance this changed wind field (Fig. 2d). Such physical insight is not apparent in the traditional PV tendency formulation.

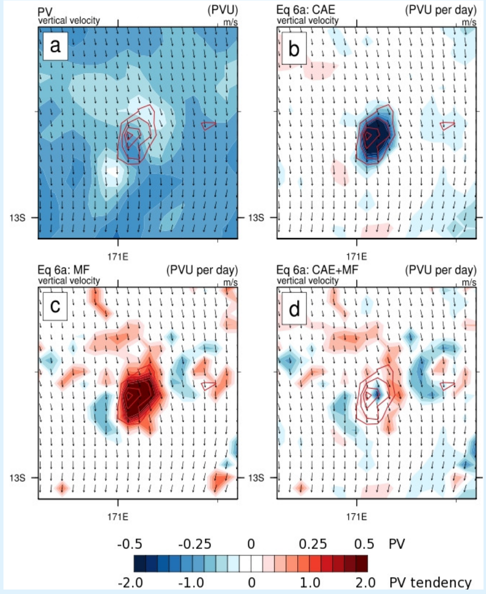

Figure 1: *A small convective burst from an ACCESS simulation of TC Yasi (Z=3380 m). Shaded quantities: (a) PV, (b) PV advection, (c) diabatic Heating term, and (d) the sum of (b) and (c). Vertical velocity is contoured in red (0.5, 0.75, 1.0,1.25 m/s).*

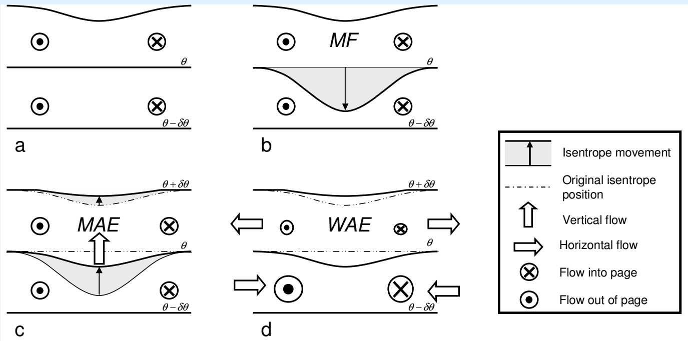

Figure 2: *Schematic representation of the A/D cancellation problem, depicting a vertical cross-section through a cyclonic circulation (northern hemisphere). Thick solid lines represent isentropes of increasing magnitude with height. (a) Initially vorticity is constant in both layers. (b) Diabatic heating is applied which induces a downward distortion of the central isentrope. (c) The atmosphere responds with ascent in the heated region, and (d) the associated inflow in the lower layer and outflow in the upper layer converges (enhances) and diverges (weakens) vorticity respectively.*

2. Horizontal PV conservation

Haynes and McIntyre (1987, HM87) revolutionised PV thinking by demonstrating the conservation of a PV quantity on isentropic surfaces. The conservation of the quantity, termed PV substance, included fluids impacted by diabatic heating and friction.

In isentropic coordinates PV substance reduces to the *vertical component* of absolute vorticity and the HM87 formulation reduces to a simple vorticity tendency equation,

$$\frac{\partial}{\partial t}\eta = -\vec{\nabla} \bullet \left[(u,v,0)\eta + \dot{\theta}(\omega_1,\omega_2,0) + (-G,F,0) \right] \quad \text{where} \quad \vec{\eta} = \vec{\nabla} \times \vec{u} + f\vec{k} = (\omega_1,\omega_2,\eta)$$

There is no problem with A/D cancellation in this formulation and the dynamical insight gained from the horizontal PV substance conservation is profound.

The terms are illustrated schematically in Fig. 3. First is the isentrope-parallel advective flux (Fig. 3d), and second is the non-advective-tilting flux (curved line Fig. 3c), which essentially combines the familiar vorticity tendencies of vertical advection and tilting (Fig. 3b). The third term is the frictional flux.

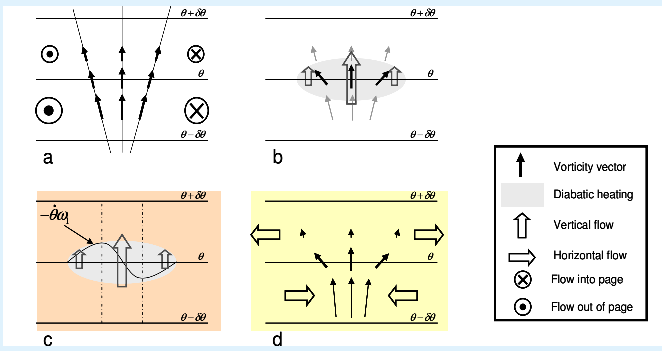

Figure 3: *Schematic representation of PVS tendency, depicting a vertical cross-section through a cyclonic circulation (northern hemisphere) in isentropic coordinates. Horizontal lines represent isentropes. (a) Initial state with vorticity decreasing with height (indicated by the vorticity vectors on vortex lines). Diabatic heating is applied (grey shade), which in isentropic coordinates is analogous to vertical flow (open arrows) that (b) advects and tilts vorticity vectors (black arrows) and (c) introduces a non-advective-tilting flux (curved line), which is convergent (+η tendency) between the dashed lines. (d) The associated inflow and outflow enhances and weakens the vorticity respectively.*

3. Alternative PV tendency formulation

The HM87 formulation is subject to the following limitations:
(i) it can be difficult to implement in real data,
(ii) it offers no direct insight into the evolving mass field, and
(iii) PV substance is not PV.

To address these limitations we constructed an alternative, true PV tendency equation in geometric coordinates valid for fully compressible non-hydrostatic fluids (see below). It offers HM87-type insight from an isentrope-parallel advective flux term and a non-advective tilting flux term, includes mass-change tendency terms, and contains no A/D cancellation.

References

Haynes, P. H., and M. E. McIntyre, 1987: On the evolution of vorticity and potential vorticity in the presence of diabatic heating and frictional or other forces. *J. Atmos. Sci.*, **44**, 828–841.

Tory, K.J., J. D. Kepert, J. A. Sippel and C. M. Nguyen, 2012: On the use of potential vorticity tendency equations for diagnosing atmospheric dynamics in numerical models.*J. Atmos. Sci.*, **69**, 942—960.

$$\frac{\partial PV}{\partial t} = -\frac{1}{\rho}\left(\vec{\nabla}\theta \bullet \left(\frac{\partial}{\partial y}(v\omega_1) + \frac{\partial}{\partial z}(w\omega_1), \frac{\partial}{\partial x}(u\omega_2) + \frac{\partial}{\partial z}(w\omega_2), \frac{\partial}{\partial x}(u\eta) + \frac{\partial}{\partial y}(v\eta) \right) \right) + \frac{1}{\rho}\left(\vec{\nabla}\theta \bullet \left(\frac{\partial}{\partial y}(u\omega_2) + \frac{\partial}{\partial z}(u\eta), \frac{\partial}{\partial x}(v\omega_1) + \frac{\partial}{\partial z}(v\eta), \frac{\partial}{\partial x}(w\omega_1) + \frac{\partial}{\partial y}(w\omega_2) \right) \right)$$

$$+ \frac{1}{\rho}\left(\vec{\nabla}\theta \bullet \vec{\nabla} \times \vec{F} \right) + \frac{Q}{\rho}\vec{\nabla} \bullet (\rho\vec{u}) + \frac{1}{\rho}\left(\vec{\eta} \bullet \vec{\nabla}\frac{\partial \theta}{\partial t} \right)$$