# BALANCED VORTEX MODEL APPLICATIONS

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### 1. INTRODUCTION

The National Hurricane Center tropical cyclone (TC) track predictions have steadily improved due to advancements in guidance. However, gains in intensity forecast skill lag behind track forecast skill. While statistical models, such as the Statistical Hurricane Intensity Prediction Scheme (SHIPS) and the Logistic Growth Equation Model (LGEM), tend to outperform dynamical models, improvements to statistical intensity forecasts have slowed in recent years (Kaplan et al. 2010). For statistical models to continue to gain skill, new methods and improvements to current techniques must be applied.

The Balanced Vortex Model (BVM) and the associated transverse circulation and geopotential tendency equations offer one possible technique for improving TC intensity guidance. Through theoretical studies (Eliassen 1951, Shapiro and Willoughby 1982, Schubert and Hack 1982, Hack and Schubert 1986, Nolan et al. 2007, Vigh and Schubert 2009, and Musgrave et al. 2012), the BVM shows that the placement of diabatic heating in relation to the inertial stability regions around the radius of maximum wind (RMW) account for TC behavior and development. As a first step towards understanding how the BVM would predict TC intensity for real storms in the Atlantic and Eastern Pacific Ocean basins, forecast fields from the Hurricane Weather Research and Forecasting model (HWRF) are applied to the BVM equations (Gopalakrishnan et al. 2011).

### 2. BALANCED VORTEX MODEL

The BVM assumes inviscid, axisymmetric, quasistatic, gradient-balanced motions of a stratified atmosphere on an *f*-plane. The BVM used here has a logpressure coordinate system,  $z = H \ln(p_0/p)$ , where  $H = RT_0/g$  is the constant scale height and  $p_0$  and  $T_0$  are constant reference values.  $H \approx 8.79$  km when  $p_0$  and  $T_0$  are chosen to be 1000 hPa and 300 K. The governing equations for the BVM are

$$\left(f + \frac{v}{r}\right)v = \frac{\partial\phi}{\partial r},\tag{1}$$

$$\frac{\partial v}{\partial t} + u \left[ f + \frac{\partial (rv)}{r\partial r} \right] + w \frac{\partial v}{\partial z} = 0, \tag{2}$$

$$\frac{\partial \phi}{\partial z} = \frac{g}{T_0}T,\tag{3}$$

$$\frac{\partial(ru)}{r\partial r} + \frac{\partial(\rho w)}{\rho\partial z} = 0, \qquad (4)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \left( \frac{\partial T}{\partial z} + \frac{RT}{c_p H} \right) = \frac{Q}{c_p},$$
(5)

where u and v are the radial and tangential components of the velocity field [m s<sup>-1</sup>], w is the log-pressure vertical velocity [m s<sup>-1</sup>],  $\phi$  is the geopotential [m<sup>2</sup> s<sup>-2</sup>], f is the Coriolis parameter [s<sup>-1</sup>],  $c_p$  is the specific heat capacity at constant pressure [J kg<sup>-1</sup> K<sup>-1</sup>],  $\rho(z) = \rho_0 \exp{(-z/H)}$  is the pseudodensity [kg m<sup>-3</sup>],  $\rho_0 = p_0/(RT_0) \approx 1.16$  kg m<sup>-3</sup> is the constant density, and Q is the diabatic heating.

Following the procedure presented by Eliassen 1951, Vigh and Schubert 2009, and Musgrave et al. 2012, the geopotential tendency equation is

$$\frac{\partial}{r\partial r} \left( r \frac{A}{D^2} \frac{\partial \phi_t}{\partial z} + r \frac{B}{D^2} \frac{\partial \phi_t}{\partial z} \right) \\
+ \left( \frac{\partial}{\partial z} - \frac{1}{H} \right) \left( \frac{B}{D^2} \frac{\partial \phi_t}{\partial r} + \frac{C}{D^2} \frac{\partial \phi_t}{\partial z} \right) \\
= \frac{g}{c_p T_0} \left[ \frac{\partial}{r\partial r} \left( r \frac{B}{D^2} Q \right) + \left( \frac{\partial}{\partial z} - \frac{1}{H} \right) \left( \frac{C}{D^2} Q \right) \right], \quad (6)$$

where  $\phi_t = \partial \phi / \partial t$ , *A* is the static stability, *B* is the baroclinity, *C* is the inertial stability,  $D = AC - B^2$ . When D > 0, (6) satisfies the condition for ellipticity. The resulting partial differential equation can be further reduced to produce the ordinary differential equation for the temperature tendency

$$\hat{T}_t - \frac{d}{rdr} \left( \ell^2 r \frac{d\hat{T}_t}{dr} \right) = \frac{\hat{Q}}{c_p},\tag{7}$$

where the Rossby length is

$$\ell(r) = \left[\frac{A}{C(r)} \left(\frac{\pi^2}{z_T^2} + \frac{1}{4H^2}\right)^{-1}\right]^{\frac{1}{2}}.$$
 (8)

From (7), a conceptual model is built. If the diabatic heating  $\hat{Q}/c_p$  is located outside of the RMW in the low inertial stability region, the TC will not undergo large intensity changes. However, if diabatic heating is located within the high inertial stability region inside the RMW, a localized temperature tendency develops allowing for an increase in TC intensity.

### 3. HWRF DATA AND THE BVM

To assess the conceptual model presented in the previous section, 2011 forecast fields from HWRF will be used with the BVM equations. To make a prediction, the BVM requires an azimuthally averaged profile of

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tangential velocity, relative vorticity, and diabatic heating to generate the tangential velocity tendency for a given period. In this study, the BVM will be applied to the typical flight level (700 hPa) of hurricane reconnaissance aircraft in preparation for use with real data. To "initialize" the BVM, profiles are generated from each HWRF forecast time represented in the National Hurricane Center A-Deck (an Automated Tropical Cyclone Forecasting (ATCF) formatted product that provides track and intensity information from model guidance) (Sampson and Schrader 2000).

The HWRF data is filtered to only include storms in which diabatic heating is present at 700 hPa and where land is further than 11.1 km (0.1 degrees) from the center of the TC. Also, only dates after 3 August 2011 are used to avoid potential inconsistencies with the mid-season changes to HWRF. The sample includes 15 Atlantic named storms (05L to 19L) and 10 Eastern Pacific named storms (06E to 13E) with 4800 and 3200 BVM forecasts respectively using the procedure described below.

This study only shows 6 hr BVM predictions using the 6 hr to 120 hr forecast fields from HWRF. The first 6 hr of the HWRF run is not included to avoid the spin-up period. The general procedure for making BVM predictions is as follows. The 6 hr HWRF forecast field is used to generate profiles of tangential velocity, relative vorticity, and diabatic heating. These variables are treated as the BVM "initial conditions." Then a 6 hr prediction is made and the BVM result is compared to the 12 hr HWRF forecast fields. The procedure is then repeated using the 12 hr HWRF forecast fields as the "initial conditions." This continues until the end of the HWRF forecast period (120 hr). As an example from 2011, initial conditions are created from the 54 hr forecast of HWRF on 1200 UTC 30 August for Katia. The 6 hr BVM prediction is then verified using the 60 hr HWRF forecast field from the same model run.

# 4. RESULTS

To show the effects of diabatic heating on the BVM tangential wind tendency from HWRF data, two relatively ideal HWRF cases are selected. The first has the peak diabatic heating located within the high inertial stability region of the TC. The second contains a peak diabatic heating outside the high inertial stability region. After showing the two relatively ideal cases, the skill of determining TC intensity change through the BVM using HWRF data is assessed for the 2011 Atlantic and Eastern Pacific Hurricane Season.

### 4.1 Inside the High Interial Stability Region

In the first example of a prediction from the BVM, a case is selected where the diabatic heating is within the high inertial stability region inside the radius of maximum tangential velocity. The "initial" fields are taken from the Irene 78 hr forecast on 1800 UTC August 2011 with the tangential velocity verification from the 84 hr forecast (Fig. 1a). The BVM intensified the storm and responded to the

secondary peak in the diabatic heating which is outside the radius of maximum tangential velocity. Also, HWRF widened the storm slightly but has the same maximum intensity. While this slight variation exists, the BVM is in rather close agreement with HWRF.

# Azimuthally Averaged BVM and HWRF Profiles , v(r), $\zeta(r)$ , and $\hat{Q}/c_{p}$



FIG. 1. Azimuthally averaged profiles of tangential velocity v(r) (blue), relative vorticity  $\zeta(r)$  (red), diabatic heating  $\hat{Q}/c_p$  (gray) taken from HWRF and used as initial conditions in the BVM. Profiles of tangential velocity v(r) for the 6 hr BVM prediction (green solid line) and verification from HWRF (green dashed line). (a) HWRF profiles and BVM prediction from the Irene 78 hour forecast on 1800 UTC August 2011 with the peak in diabatic heating located within the high inertial stability region inside the RMW at 700 hPa. (b) HWRF profiles and BVM prediction from Katia 24 hr forecast on 0000 UTC 01 September 2011 with the peak in diabatic heating located within the low inertial stability region outside the RMW at 700 hPa.



FIG. 2. Prediction comparison between the BVM and HWRF for the Atlantic (top row) and the Eastern Pacific (bottom row) for a 6 hr prediction made using HWRF forecasts starting on 1200 UTC 3 August 2011 to the end of the season. In each plot, HWRF is on the abscissa and the BVM is on the ordinate. The one-to-one correlation is the black diagonal line. Plots are shaded relative to normalized values for number of cases to give a sense of the general trend for each basin.. Blues indicated a low number of cases and reds indicated a high number of cases. (a) & (e) Comparisons of the maximum azimuthally averaged tangential velocity. (b) & (f) Comparisons of the change in maximum azimuthally averaged tangential velocity. (c) & (g) Comparisons of the radius of maximum azimuthally averaged tangential velocity. (d) & (h) Comparisons of the change in radius of maximum azimuthally averaged tangential velocity.

### 4.2 Outside the High Interial Stability Region

For another prediction from the BVM, a case is selected where the diabatic heating is within the low inertial stability region outside or near the radius of maximum tangential velocity. The "initial" fields are taken from the Katia 24 hr forecast on 0000 UTC 01 September 2011 with the tangential velocity verification from the 30 hr forecast (Fig. 1b). In this case, the BVM increases the peak velocity slightly as well as wind profile further from the radius of maximum intensity. In this case, the BVM is not able to match the intensity change shown by HWRF.

# 4.3 2011 Hurricane Season

To evaluate the BVM predictions for the Atlantic and Eastern Pacific 2011 Hurricane Season, general trends in predicted maximum intensity, intensity change, radius of maximum intensity, and change in radius of maximum intensity are assessed. Fig. 2a shows the predicted tangential velocity of the HWRF versus the BVM. In the figure, the BVM appears to underestimate mid-intensity TCs (30-50 m s<sup>-1</sup>) while overpredicting TCs greater than 55-60 m s<sup>-1</sup>. This underprediction can be seen in the upper right quadrant of Fig. 2b. The BVM falls short of the one-to-one correlation but is still intensifying the storms. Fig. 2b also highlights an expected result. Since

the BVM does not allow for decay, no change in intensity for the BVM is expected. Fig. 3a also demonstrates that the BVM does not change intensity for cases where HWRF dissipates the storm. For intensifying storms, the BVM and HWRF distribution are rather similar especially for larger intensity changes. In Fig. 2c and 2d, both the BVM and HWRF show little variation in the radius of maximum tangential velocity for the 6 hr period.

In the Eastern Pacific, the same trend of overpredicting strong TCs can been seen in Fig. 2e. However, mid-intensity TCs do not appear underpredicted. Fig. 2f shows that HWRF is actually dissipating most of the storms in the Eastern Pacific which means that the BVM is maintaining the storms initial tangential velocity profile. Fig. 3b also shows less variation in intensity for the BVM. The changing in radius of maximum intensity has more variability than in the Atlantic (Figs. 2g and 2h). For the Eastern Pacific, the BVM does not vary the radius much as a result of the tangential velocity tendency being zero.

### 5. DISCUSSION

Musgrave et al. (2012) show that the BVM is highly sensitive to the placement of the diabatic heating in relation to the inertial stability and the RMW wind. With HWRF in the 2011 season having a resolution of 8 km with grid output of 11 km (0.1 degrees), the placement of the diabatic heating in the BVM "initial" fields may not be conducive to TC development. In addition, Fig. 2 shows that the BVM begins overpredicting intensity change when HWRF reaches a maximum attainable velocity around 55-60 m s<sup>-1</sup> while underpredicting mid-intensity storms.



FIG. 3. Normalized histograms of the change in intensity for the Atlantic (a) and Eastern Pacific (b) 2011 Hurricane Season. The change in intensity is binned by 2 m s<sup>-1</sup> from -10 to 10 m s<sup>-1</sup>. HWRF is in red and the BVM is in blue.

There are two explanations for the differences between the BVM and HWRF forecasts. First, HWRF could be allowing diabatic heating to enter the high inertial stability region during the mature phase of TC development. Second, the assumptions made in the BVM do not allow for friction and diffusion to keep the model in alignment with the HWRF forecasts. The placement of the diabatic heating is also an issue for weaker TCs. In HWRF, weaker TCs tend to have the diabatic heating located near the radius of maximum tangential velocity but still within the low inertial stability region (not shown here).

Another issue related to the model and the diabatic heating is in how the BVM is run using the fields from HWRF. For this study, the BVM extends the instantaneous diabatic heating profile at 700 hPa for the entire 6 hr period. While this assumption would work well for steady development, the results would indicate that using an instantaneous profile does not adequately depict the storm.

The comparison of the BVM to HWRF reveals key features and issues related to how the BVM would react to observations. As shown in the results section, the BVM predicted tangential velocity is highly sensitive to the amount and position of the diabatic heating. When shifting to running the BVM on real data, attention will need to be given to the location and amount of diabatic heating in relation to the inertial stability region.

Acknowledgments. The author wishes to thank Wayne Schubert, Mark DeMaria, Scott Fulton, Kate Musgrave, and Vijay Tallapragada for their advice, helpful comments, and technical support. This research was supported by the National Oceanographic Partnership Program through ONR contract N000014-10-1-0145.

### REFERENCES

- Eliassen, A., 1951: Slow thermally or frictionally controlled meridional circulation in a circular vortex. Astrophys. Norv., 5, 19-60.
- Gopalakrishnan, S., Q. Liu, T. Marchok, D. Sheinin, N. Surgi, M. Tong, V. Tallapragada, R. Tu-Yablonsky, and X. Zhang, 2011. leva. R. Weather Research and Forecast-Hurricane ing (HWRF) Model: 2011 Scientific Documentation. Development Testbed Center. <http://www.emc.ncep.noaa.gov/HWRF/HWRF ScientificDocumentation2011.pdf>.
- Hack, J. J., and W. H. Schubert, 1986: Nonlinear response of atmospheric vortices to heating by organized cumulus convection. *J. Atmos. Sci.*, 43, 1559-1573.
- Kaplan, J., M. DeMaria, J. A. Knaff, 2010: A revised tropical cyclone rapid intensification index for the atlantic and eastern north pacific basins. *Wea. Forecasting*, 25, 220241.
- Musgrave, K., R. K. Taft, J. L. Vigh, B. D. McNoldy, and W. H. Schubert, 2012: Time evolution of the intensity and size of tropical cyclones. J. Adv. Model. Earth Syst., submitted.
- Nolan, D. S., Y. Moon, D. P. Stern, 2007: Tropical cyclone intensification from asymmetric convection: energetics and efficiency. J. Atmos. Sci., 64, 3377-3405.
- Sampson, C. R., A. J. Schrader, 2000: The Automated Tropical Cyclone Forecasting System (Version 3.2). *Bull. Amer. Meteor. Soc.*, **81**, 12311240.
- Schubert, W. H. and J. J. Hack, 1982: Inertial stability and tropical cyclone development. J. Atmos. Sci., 39, 1687-1697.
- Shapiro, L. J., H. E. Willoughby, 1982: The response of balanced hurricanes to local sources of heat and momentum. J. Atmos. Sci., 39, 378-394.
- Vigh, J. L., W. H. Schubert, 2009: Rapid development of the tropical cyclone warm core. J. Atmos. Sci., 66, 3335-3350.