DEVELOPING A SIMPLE NUMERICAL MODEL IN MOIST AIR FOR STUDYING CLOUD FORMATION PROCESSES IN THE TROPICS

Chantawan Noisri*
Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi, Bangkok, Thailand

Robert Harold Buchanan Exell
The Joint Graduate School of Energy and Environment, Center for Energy Technology and Environment, King Mongkut's University of Technology Thonburi, Bangkok, Thailand

1. INTRODUCTION

Condensation of water above the Earth's surface creates clouds. Normally, clouds develop in any air mass that becomes saturated by way of atmospheric mechanisms that cause the temperature of an air mass to be cooled to its dew point.

Numerical weather prediction (NWP) models are techniques used to predict the future state of the weather by solving a set of equations which govern the behavior of the atmosphere. A simplified numerical model for studying the behavior of air in the troposphere in a tropical climate is described in this paper. It shows students how NWP models are constructed without involving the complicated transformations in actual operational models.

A simple numerical model in dry air with horizontal grid steps of 1 km has shown by experiments that a time step of 0.3 seconds is too large for the model without the deep convection approximation (Noisri and Sukawat, 2011). But with the deep convection approximation one can use a time step of 0.4 seconds (Noisri and Exell, 2011) and get reasonable results in numerical experiments on the vertical movement of air over a heated surface representing a city heat island.

2. MODEL DESCRIPTION

In our model there is one horizontal dimension, the vertical dimension, and the time dimension. The variables are located on a staggered grid with stretched grid spacing in the vertical dimension and constant grid spacing in the horizontal dimension.

**Corresponding author address:* Chantawan Noisri, Department of Mathematics Faculty of Science, King Mongkut's University of Technology Thonburi, Bangkok, Thailand; e-mail: tsu_004@hotmail.com.

Symbol	Description
x	Horizontal distance
z	Vertical height
t	Time
g	Acceleration of gravity
R	Gas constant for air
c_v	Specific heat of air
ρ	Air density
u	Horizontal velocity
w	Vertical velocity
T	Temperature
z_0	Roughness length of the surface
q_s	Surface heating rate per unit area
i, k	Horizontal and vertical cell indices
ρ_s	Saturated vapor density
ρ_v	Water vapor density
m_c	Condensed cloud water per unit volume
L	Latent heat of condensation of water

Table1. List of symbols

The molecular viscosity terms are omitted; the body forces are friction at the Earth's surface in the horizontal momentum equation and gravity in the vertical momentum equation; heating and cooling of the air occur at the Earth's surface; kinetic energy and potential energy in the temperature equation are omitted; the Coriolis force is omitted; no distinction is made between liquid water and ice; the effects of the moisture on the thermodynamic properties of the air are neglected; rain is not included; and the deep convection approximation is used.

The deep convection approximation (Pielke, 2002) is

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=-\frac{w}{\rho^0}\frac{\partial \rho^0}{\partial z}, \quad (1)$$

where $\rho^0(z)$ is calculated from a steady background temperature profile $T^0(z)$ and the assumption of hydrostatic equilibrium in the undisturbed atmosphere.

The steady background temperature profile is an approximation to the annual mean upper air temperatures at Bangkok represented by the formula

$$T^0(z)=302-0.00675z, \quad (2)$$

where T^0 is in kelvins.

2.1 *Governing Equations*

The model equations listed below are derived from the fundamental system of partial differential equations of computational of fluid dynamics (Anderson, 1995). The temperature and moisture equations used depend on whether or not the air is saturated. A simple equation for the saturation vapor density of water as a function of temperature is obtained by integrating the Clausius Clapeyron equation (Rogers and Yau,1989) assuming that the latent heat of condensation of water vapor is constant and water vapor is an ideal gas.

The density equation

The density equation without the deep convection approximation is

$$\frac{D\rho}{Dt}=-\rho\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right), \quad (3)$$

and the density equation with the deep convection approximation is

$$\frac{D\rho}{Dt}=-\rho\frac{w}{T^0}\left(\frac{\partial T^0}{\partial z}+\frac{g}{R}\right). \quad (4)$$

The vertical velocity equation

$$\frac{Dw}{Dt}=-\frac{RT}{\rho}\frac{\partial \rho}{\partial z}-R\frac{\partial T}{\partial z}-g \quad (5)$$

The wind equation

$$\frac{Du}{Dt}=-\frac{RT}{\rho}\frac{\partial \rho}{\partial x}-R\frac{\partial T}{\partial x} -\frac{0.16u|u|}{\left[\ln\left(0.5\Delta z/z_0\right)\right]^2\Delta z} \quad (6)$$

The last term is horizontal friction, which is applied only in the layer of air of thickness Δz at the Earth's surface where the roughness length is z_0.

Unsaturated air

The temperature equation

$$\frac{DT}{Dt}=-\frac{RT}{c_v}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+\frac{q_s}{c_v\rho\Delta z} \quad (7)$$

The surface heating term is applied only in the layer of air at the Earth's surface.

The water vapor equation

$$\frac{D\rho_v}{Dt}=-\rho_v\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right) \quad (8)$$

The condensed cloud water equation

$$m_c=0 \quad (9)$$

Saturated air

The temperature equation

$$\frac{DT}{Dt}=\frac{W}{1+EQ} \quad (10)$$

The water vapor equation

$$\frac{D\rho_v}{Dt}=\frac{EW}{1+EQ} \quad (11)$$

The condensed cloud water equation

$$\frac{Dm_c}{Dt}=-\left(\rho_v+m_c\right)\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)-\frac{EW}{1+EQ} \quad (12)$$

where $W=-\frac{RT}{c_v}\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+\frac{q_s}{c_v\rho\Delta z}$,

$$E=\frac{A\left(B-T\right)}{R_vT^3}e^{-B/T},$$

$$Q = \frac{L}{c_v \rho}, \; L = 2.50 \times 10^6 \; Jkg^{-1}.$$

2.2 *Finite difference and experiments*

2.2.1 *Finite difference set up*

The domain of the model is divided into a 25×100 array of cells. The horizontal resolution is one kilometer. A vertical coordinate s is used in accordance with the transformation $z = 75s + 25s^2$ in order to give thin layers at the Earth's surface and thick layers at the top of the troposphere (Exell, 2009).

The model variables are evaluated at points on an Arakawa-C grid. The horizontal velocity is on the left side of the cell, the vertical velocity is on the bottom of the cell, and other variables are in the center of each cell, as shown in Fig1.

The leapfrog method is used to calculate the model variables at next time step. The Euler method is used for the first time step. First and second order finite difference approximations are used in the modeling of space derivatives. In the row of cells at the Earth's surface one-sided second order difference approximations to derivatives with respect to z are used.

The initial values of the model variables in each cell are functions of the height of the cell above the Earth's surface, but are constant along the horizontal rows of cells. The temperature equation is given by

$$T_k^0 = 302.211 - 0.3375k - 0.16875k^2, \quad (13)$$

where $k = 1, 2, ..., 25$.

It is assumed the initial values of the velocity, surface heating and amounts of cloud water are zero everywhere. The initial values of the temperature and density satisfy the hydrostatic equation, and the initial values of vapor density are calculated on the assumption that the dew point depression below the initial air temperature is a constant at all heights. The model variables are fixed at the boundary.

2.2.2 *Numerical Experiments*

Two different experiments were done to study the effects of a heated surface in the middle of the domain:

Case1.

Without the deep convection approximation.

- relaxation at the boundary
- smoothing in time

Case2.

With the deep convection approximation.

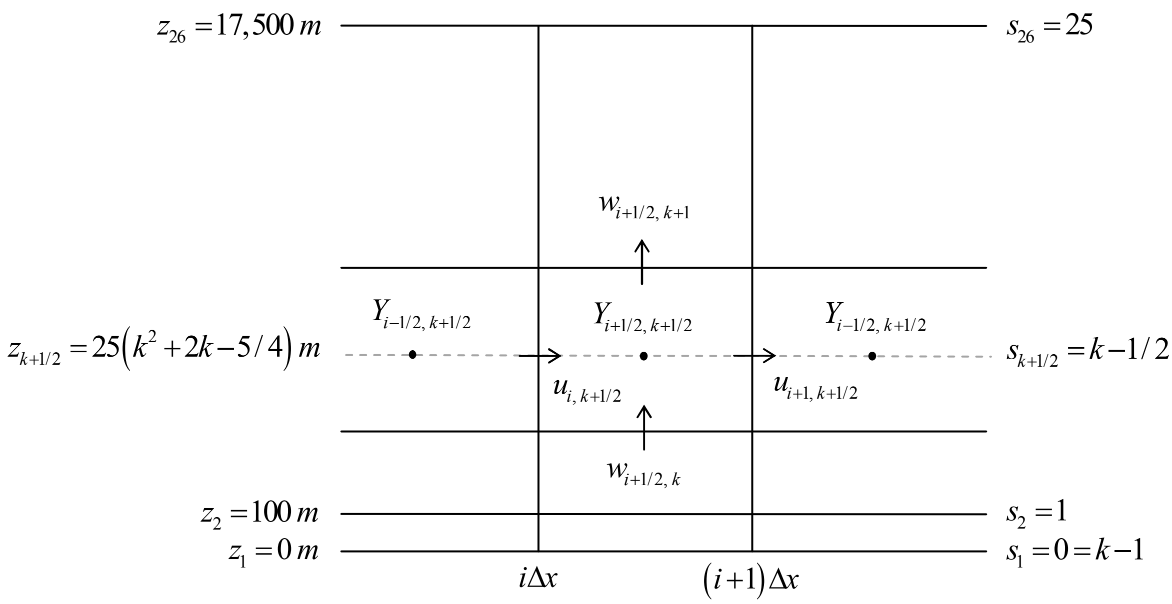

Fig1. The model variables are evaluated on an Arakawa-C grid with the stretched grid. The horizontal velocity is on the left side of the cell, the vertical velocity is on the bottom of the cell, and other variables are in the center of each cell.

3. RESULTS AND DISCUSSION

The object of these experiments will be to find the largest temporal resolution that gives stable results for model times of the order an hour and to determine the errors in the results.

The model	Time step	Total time
without deep convection approximation	0.2 s	more than an hour
	0.3 s	less than a minute
with deep convection approximation	0.4 s	more than an hour
	0.5 s	less than a minute

Table2. The temporal resolution and the total time that give stable results for the model.

Preliminary results have shown that a time step of 0.3 seconds is too large for the model without the deep convection approximation and using the deep convection approximation allows double the size of time step. In addition, relaxation at the boundary and smoothing in time by a Robert-Asselin filter have little effect on the model without the deep convection approximation.

However, previous results have shown that this very simple model can give reasonable results in numerical experiments on vertical movement processes in the tropics.

4. FUTURE WORK

Experiments with the model:

- A heated area in the middle of a cooled area.
- A cooled area in the middle of a heated area.
- Random heating and cooling at the surface in space and time across the domain.
- A rough area in the middle of a smooth surface.
- A smooth area in the middle of a rough surface.
- Random variation of surface roughness across the domain.
- Cloud formation or the absence of cloud in air with high and low humidity.
- The effect of variable surface properties on the formation of clouds.
- The effect of vertical wind shear on cloud forms.

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