THRESHOLD MODELS FOR RAINFALL AND CONVECTION: DETERMINISTIC VERSUS STOCHASTIC TRIGGERS.

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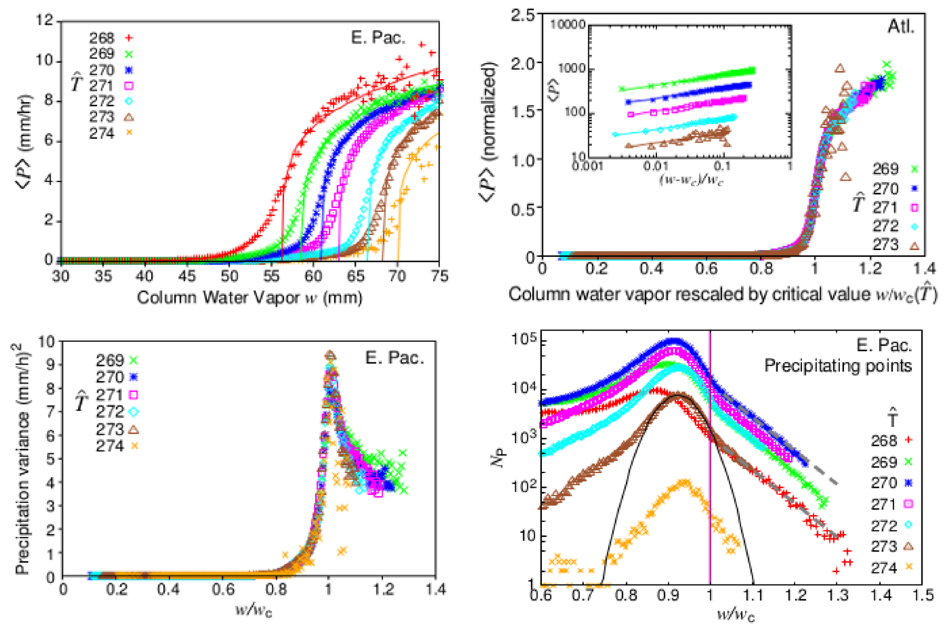
WATER VAPOR'S ROLE IN CONVECTION

Water vapor plays a major role in the onset of convection in tropical regions. Recent observations have shown that water vapor has a critical value, depending on temperature, that signals a transition to convection [1]. Our study aims to answer the question: **what are the underlying mechanisms of the transition to convection?** Specifically, we aim to develop a simple mathematical model of water vapor that resembles the observational statistics of critical phenomena.



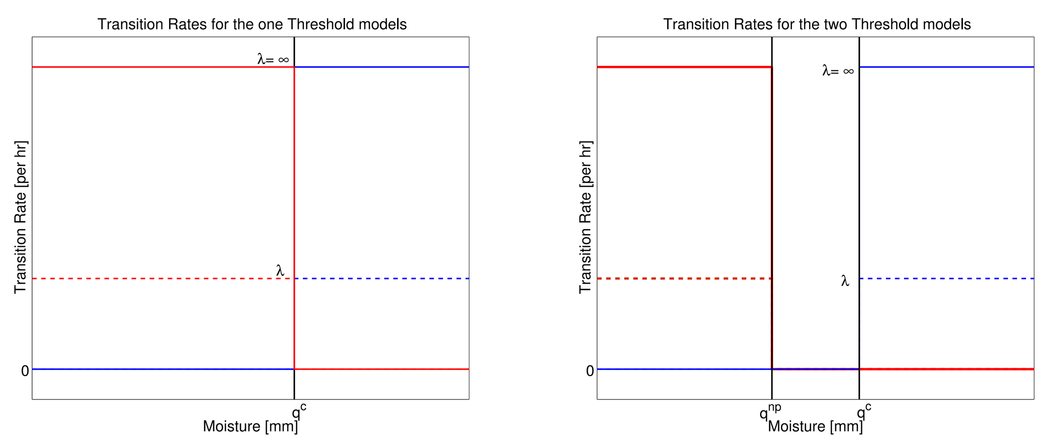
Observational statistics of water vapor from [1]. The plots are of: (a) the mean, (b) variance of precipitation, (c) the normalized mean precipitation and the (d) frequency of precipitating points with respect to the water vapor value.

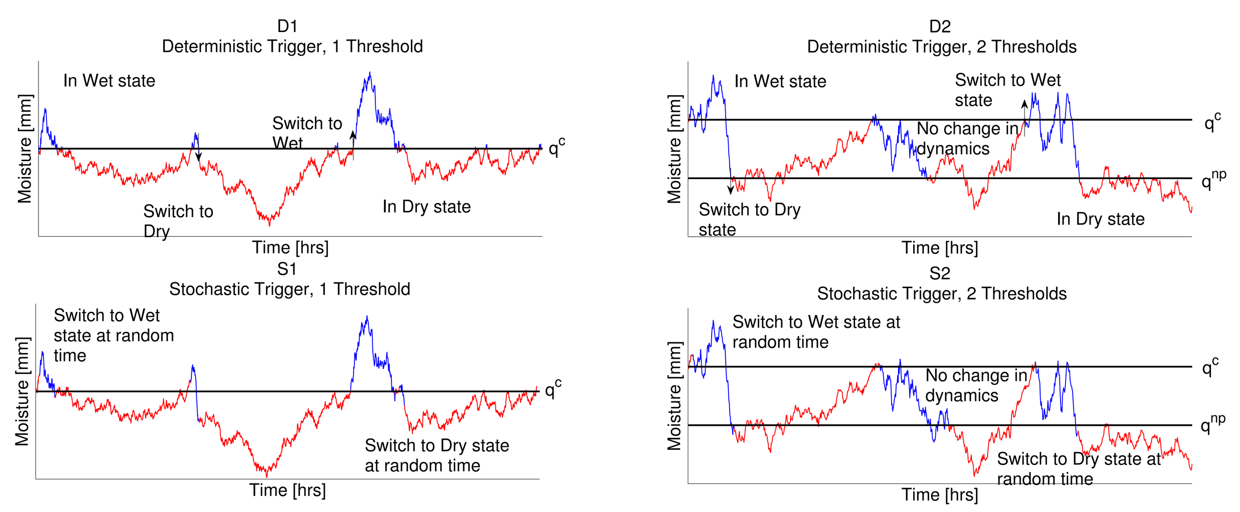
MODELS

To model column water vapor q_t as a function of time t, we use a simple Stochastic Differential Equation (SDE). The SDE changes dynamics depending on the state of $\sigma_t \in \{0,1\}$.

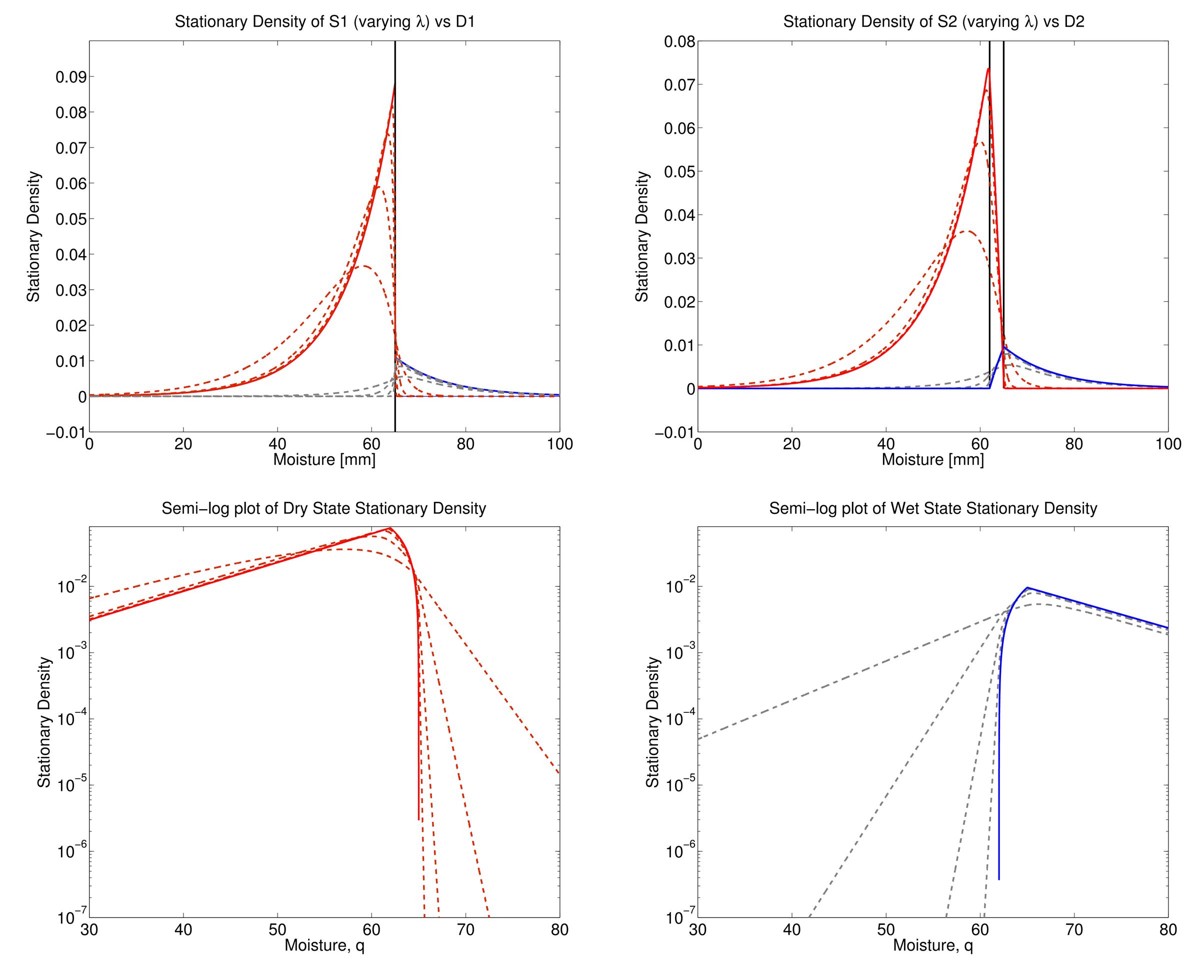
$$dq_t = \begin{cases} m dt + D_0 dW_t, & \sigma_t = 0 \\ -r dt + D_1 dW_t, & \sigma_t = 1 \end{cases} \qquad D_1 > D_0 > 0$$

To define σ_t, we define the transition rates in the plots below with one threshold (left) and two thresholds (right).





STATISTICS OF THE MODELS: FREQUENCY



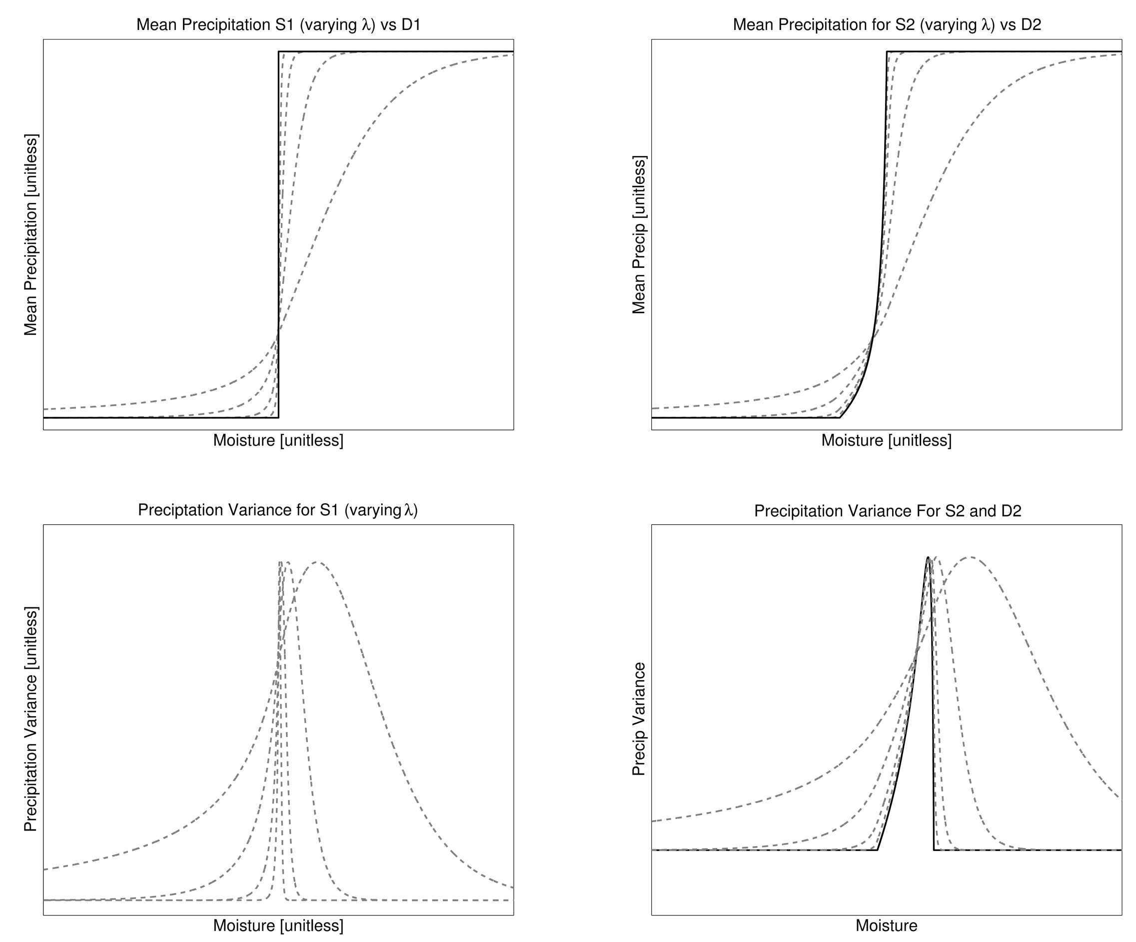
The water vapor pdfs are plotted above with D1 on the left and D2 on the right in solid red (dry state) and blue (wet state). The corresponding stochastic triggers are plotted (S1 on left, S2 on right) with various increasing values of the rate $\lambda^{-1} = 4, .4, .04, .004$ hours. The **cloud fraction** is defined as the fraction of time, on average, that the cloud is raining.

$$E[\sigma = 1] = \frac{r}{m+r}$$

The cloud fraction does not change when the type of trigger or the number of thresholds is changed.

STATISTICS: PRECIP. MEAN AND VARIANCE

The simplicity of the models allows exact formulas for all of the statistics.



Further Reading: Stechmann, Neelin: A Stochastic Model for the Transition to Strong Convection, J. Atmos. Sci. (2011)

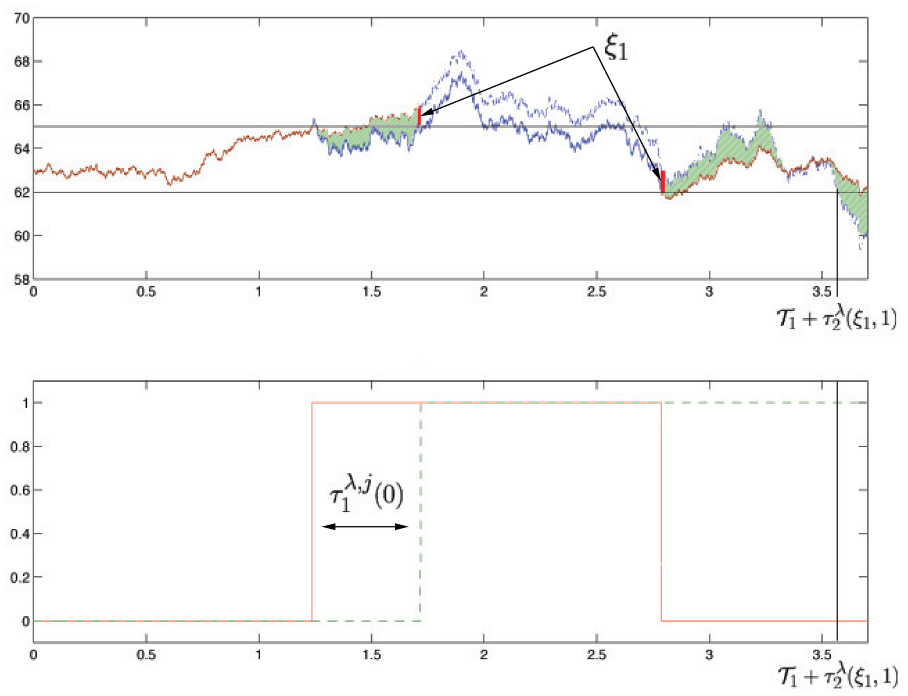
Stechmann, Neelin: First-Passage Time Prototypes for Precipitation Statistics, J. Atmos. Sci. (2014)

CONVERGENCE OF MODELS

The S2 model captures most statistics well, but the exact formulas are hard to analyze. The D2 model has simple formulas and the statistics resemble observations. For large transition rates λ, S2 is approximated by D2.

Theorem: Let $(q_t^\lambda, \sigma_t^\lambda)$ be the solution of SDE (S2) with initial conditions (q_0, σ_0) constant for every λ and let (q_t, σ_t) be the solution to SDE (D2) with the same initial condition (q_0, σ_0). Then

$$\lim_{\lambda \to \infty} E\left[\left(\sup_{0 \le t \le T} |q_t^\lambda - q_t|\right)^2\right] = 0.$$



The main idea: Two types of Error. 1) ξ from delay in jumping time, and 2) ζ accruing "catch up" error, can be made small for $\lambda \gg 1$.

CONCLUSIONS

- The onset of convection is well modeled by a simple SDE. The SDE must have **either a deterministic trigger with two thresholds or a stochastic trigger** to capture the relevant statistics.
- The stochastic trigger captures important statistics better than the deterministic trigger.
- The deterministic trigger model is preferable to work with because formulas are easier to solve exactly. The D2 model still captures the observational statistics.
- The D2 model is a good approximation to the stochastic trigger model because the stochastic model converges to the deterministic trigger model for large transition rates λ.

References:

[1] J.D. Neelin, O. Peters, K. Hales: The transition to strong convection. J. Atmos. Sci., 66, 2367-2384, doi:10.1175/2009JAS2962.1 (2009).

Acknowledgements:

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