# 4A.6
# LINEAR & NONLINEAR MOTION OF A BAROTROPIC VORTEX

Israel Gonzalez* III and Hugh E. Willoughby
Florida International University, Miami, Florida

It is generally accepted that the beta gyres (Fig, 1a) have a key role in observed tropical cyclone propagation (as distinct from advection). In the 1980s and 90s, beta gyre dynamics was the focus of intense modeling and observational efforts. Willoughby (1992, 1994) developed a series of semispectral, shallow-water, barotropic linear and nonlinear models to simulate vortex motion on a beta plane. In the linear model, which simulated only wavenumber one asymmetries, the vortex accelerated toward the NW without limit (Fig. 1b), ostensibly through the resonant growth of a free linear mode.

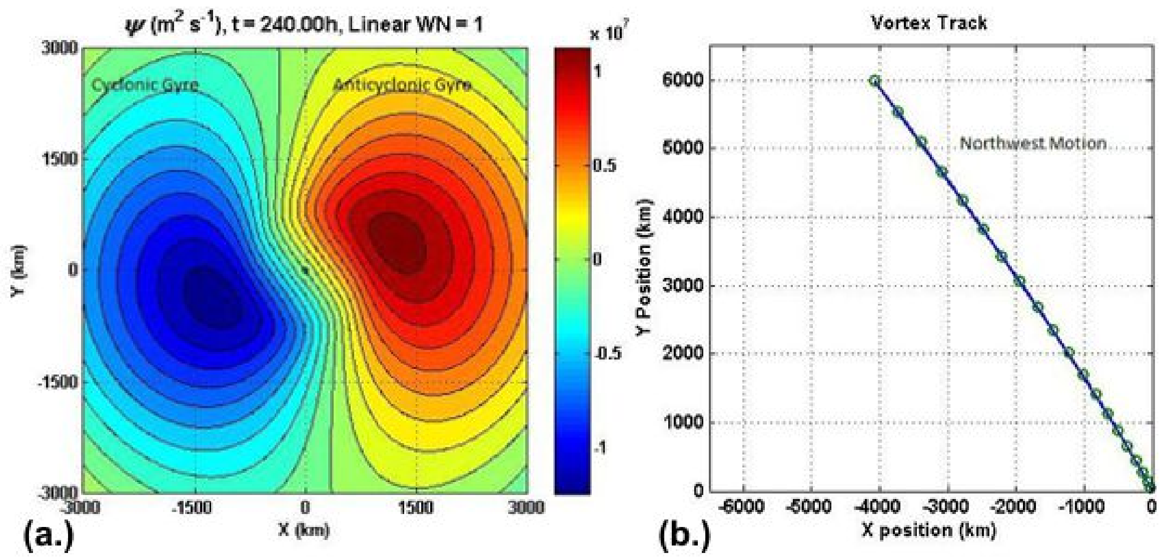


Fig. 1. (a) Linear wavenumber-1, beta-gyre streamfunction after simulated 240h in a still environment of a β plane. (b) The linear vortex track 0 through 240h, showing uniform northwestward acceleration.

This asymmetry was forced through advection of the gradient of planetary vorticity by the swirling flow of the maintained axially symmetric mean vortex. Closure for the vortex translation involved identifying the apparent asymmetry that arises when the axis of vortex rotation and the origin of the vortex do not coincide. The vortex was then repositioned to minimize this “alpha-gyre” asymmetry. We argue that the alpha gyres reflect the invariance of the dynamics under different choices of coordinates and that their growth is the consistent representation of vortex motion in this formulation.

In the analogous nonlinear model, wave-wave interaction limited the westward and poleward motion to reasonable speeds of 1-2 m $s^{-1}$, consistent with the observed beta drift.

* *Corresponding Author Address*: Israel Gonzalez, III, Dept. of Earth and Environment, Florida International University, Miami, FL 33199. Email: igonz008@fiu.edu

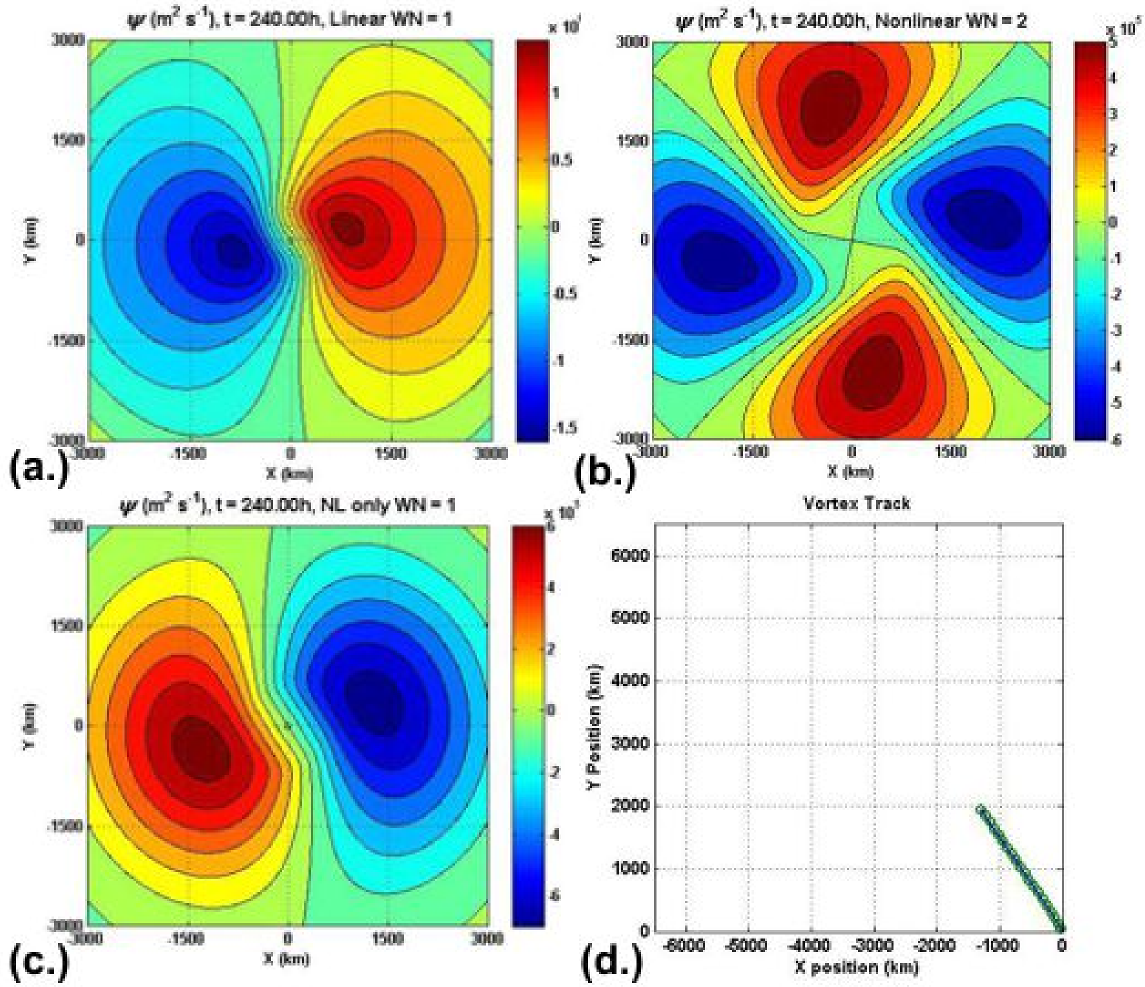


Fig. 2. (a.) Linearly forced part of the wavenumber-1, beta-gyre streamfunction after 240 simulated hours in a still environment on a β plane. (b.) Nonlinearly forced wavenumber-2 asymmetry. (c.) Nonlinearly forced wavenumber-1 anti-beta gyres. (d.) Vortex track for 0-240 simulated hours illustrating how the nonlinearly forced wavenumber-1 asymmetry limits poleward and westward acceleration.

Subsequent work by Montgomery et al. (1997) was unable to replicate the linear result. Here, we revisit the problem in a Barotropic Non-Divergent (BND) context to resolve the question of the free mode's existence and role and to clarify the nonlinear dynamics.

Our linear BND model is time-dependent and uses a beta-like forcing that yields an increasing magnitude and a phase reversal in the neighborhood of a low (period of ~100 days) cyclonic frequency. It was able to correctly reproduce the beta gyres (Fig.1a) as WN1 streamfunction dipoles of opposite polarity with a northeast-southwest orientation such that the counter-flow between them produces a uniform south-southeasterly current (i.e., ventilation flow) across the vortex center. This causes the simulated storm to accelerate to the NW linearly. The theory behind this behavior is that the beta gyres appear to be free waves resonantly excited by the beta effect. Thus, the results are consistent with Willoughby's and inconsistent with Montgomery's Asymmetric Balance Model, where it produced finite linear drift speeds.

The nonlinear version of the BND model accounts for wave-wave interaction such that the beta effect forces linear beta gyres (Fig 2a); linear wavenumber-1 interacts with itself to force wavenumber-2 nonlinearly (Fig 2b); and nonlinear wavenumber-2 interacts with linear wavenumber-1 to force nonlinear wavenumber-1 (Fig 2c). The most important result from the complete solution is the nonlinearly-forced wavenumber-1 streamfunction gyres that have opposite phase to the linear gyres. The ventilation flow from these asymmetries counteracts the uniform current of the linear

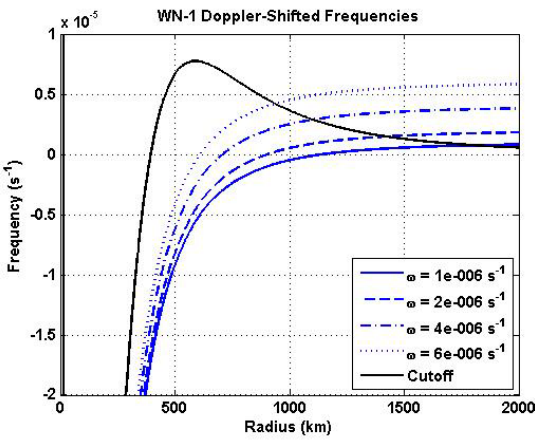


Fig.3. Doppler shifted vortex Rossby wave frequencies in the outer waveguide. Here VRWs are confined to a narrow passband between zero frequency and the cutoff frequency for downstream propagating VRWs in the reversed peripheral vorticity gradient.

gyres to reduce the overall vortex speed. Therefore, these “anti-beta gyres” provide a mechanism for limiting vortex motion in the nonlinear framework.

The mean vortex is bounded at some finite radius where the circulation approaches zero. As a result, the Circulation Theorem requires a reversal of sign for the relative vorticity and radial vorticity gradient (Fig. 4a). Thus, a bounded vortex has two Vortex Rossby-Wave (VRW) waveguides. In the inner waveguide, the negative radial gradient of axially symmetric relative vorticity (Fig. 4b) supports upstream propagating VRWs whose frequencies lie in a passband between the VRW cutoff frequency (actually the frequency of one-dimensional VRWs in the BND system) and zero frequency. Inward propagating waves are Doppler shifted to the cutoff frequency at the inner radius of the annular waveguide, where they are reflected. Outward propagating waves are Doppler shifted to zero frequency at the outer radius of the waveguide, where they are absorbed at the VRW critical radius as their radial wavenumber becomes locally infinite (Cotto 2012).

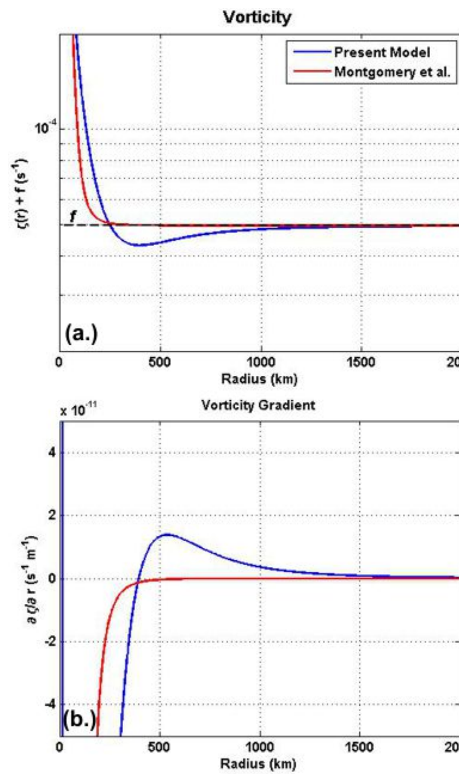


Fig. 4. (a) Radial profiles of absolute vorticity used in the present model and Montgomery et al. (1997), illustrating the reversal of vorticity gradient that supports VRWs in the outer waveguide, and (b.) Corresponding radial vorticity gradients.

In the outer waveguide, the situation is reversed. Here the absolute vorticity increases from a value somewhat less than the Coriolis frequency to the Coriolis frequency at large radius. The beta gyres are downstream propagating VRWs that propagate on the positive peripheral mean vorticity gradient. They are confined in a narrow frequency band between zero and the 1-dimensional VRW cutoff frequency. In the outer waveguide the critical surface with zero frequency lies nearer the center and the turning point farther away. The beta gyres appear to be a continuous spectrum of free waves confined to a very narrow range of cyclonic frequencies. The waveguide is also "leaky" because, despite very slow radial group velocity, energy that propagates to the critical radius is absorbed. If the linear calculation is run for several days on a beta plane and then restarted on an *f* plane the vortex track curves cyclonically and decelerates, consistent with the foregoing interpretation.

Montgomery et al. (1997) were unable to reproduce Willoughby's earlier results because their completely cyclonic mean vortex had no outer waveguide. Nonetheless, upstream propagating, low frequency VRWs were probably present in a narrow radial interval where their mean vorticity profile approached *f*. Although these waves may explain the unrealistically fast propagation (6 m $s^{-1}$) of their vortex, they were confined to extremely narrow intervals of both radius and frequency so that they were forced more weakly than waves in a bounded vortex would be. In the limit of large radius, the vortex of Montgomery et al. approached a Rankine vortex, so that it had constant circulation at large radius and infinite total kinetic energy and relative angular momentum.

**References**

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