A complete theoretical model for the tropical cyclone radial wind structure

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1. Introduction

In the absence of interaction with land or extratropical disturbances, the horizontal extent of the outer circulation is observed in nature to vary only marginally during the lifetime of a given tropical cyclone, but significant variation exists from storm to storm regardless of basin, location, and time of year (Merrill 1984; Frank 1977; Chavas and Emanuel 2010; Cheng-Shang et al. 2010; Knaff et al. 2014).

Here we present a complete physics-based TC radial wind profile that merges existing theoretical profiles given by Emanuel and Rotunno (2011) for the inner convecting region and Emanuel (2004) for the outer non-convecting region. We then compare this unified solution with observations, with the dual objectives of providing insight into the theoretical structure of a tropical cyclone and offering a new method for representing the tropical cyclone wind hazard in risk analysis.

2. Theory

a. Review of existing theory

Much of our current theoretical understanding of tropical cyclones is phrased in terms of absolute angular momentum, M, given by

$$M = rV + \frac{1}{2}fr^2 \quad (1)$$

where r is the radius from the axis of rotation, V is the azimuthal wind, and f is the Coriolis parameter. Quantifying the precise rate at which absolute angular momentum is lost with decreasing radius, though, requires an accounting of the broader dynamics and thermodynamics of the system. Theory currently exists that achieves such a goal, albeit in distinct thermodynamic regimes: the inner convecting region (Emanuel and Rotunno 2011) and the outer non-convecting region (Emanuel 2004). Here we seek to mathematically merge these two solutions for the purpose of creating a complete solution for the radial distribution of absolute angular momentum, and in turn the radial profile of the rotating wind, in a tropical cyclone. We begin with a review of the theory and solutions for each region.

In the non-convecting outer region of a tropical cyclone, Emanuel (2004) provides a solution not for the absolute angular momentum itself, but for its radial gradient. This outer region model assumes convection-free clear-sky conditions, such that free tropospheric air subsides at a constant rate, w_{cool}. Mathematically, the outer wind model is a Riccati equation that lacks a known analytical solution, but it can be solved numerically using a shooting method. The outer wind model has two parameters: r_0 and $\chi = \frac{2C_d}{w_{cool}}$.

As noted earlier, r_0 is observed to vary significantly in nature. In contrast, χ ought to vary minimally across storms, as w_{cool} is tied to the free tropospheric temperature profile, which is dynamically constrained, based on weak temperature gradient considerations (Sobel and Bretherton 2000), to remain relatively constant across the tropics for a given climate state, while C_d over the open ocean is a function principally of the local wind speed, independent of environment or storm (Donelan et al. 2004).

In the convecting inner region of a tropical cyclone, Emanuel and Rotunno (2011) derive a solution for the radial distribution of angular momentum. For the sake of analytical tractability, we set the ratio of exchange coefficients of enthalpy and momentum to $\frac{C_k}{C_d} = 1$.

The solution of the inner region model represents a complete radial wind profile, which is currently used in risk analysis (Lin et al. 2012). However, as noted by Emanuel and Rotunno (2011), the physics are only valid in the convecting region of the storm, thus motivating the development of a radial profile that is physically valid in both the convecting and non-convecting regions.

b. Unified radial wind profile

Here we merge the solutions for the inner convecting and outer non-convecting regions. The merged system has three equations and six unknowns, thereby leaving three free parameters. Implicitly, at least one parameter must come from the inner region and one from the outer region. In the outer region, given the theoretical basis for χ and the corresponding expectation of stability relative to r_0, one may choose to fix χ. Doing so leaves two free parameters, which offers two possible routes.

First, the model may take as input one parameter from the inner region and one from the outer region. Conceptually, this approach fixes the solution of the outer wind model in r-V space and then finds the inner model solution that matches. Second, one may elect to input r_m and V_m. Conceptually, this approach fixes the solution of the inner region and then appends the outer wind model to its tail. In both cases, the merge point solution is unique across a wide range of parameter values.

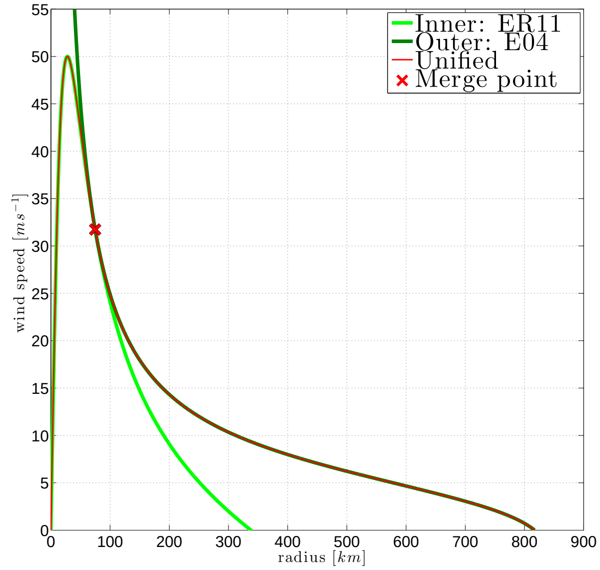

FIG. 1. Unified solution (red solid) merging numerical solution to the outer wind model and analytical inner model solution, with $V_m = 50\ ms^{-1}$.

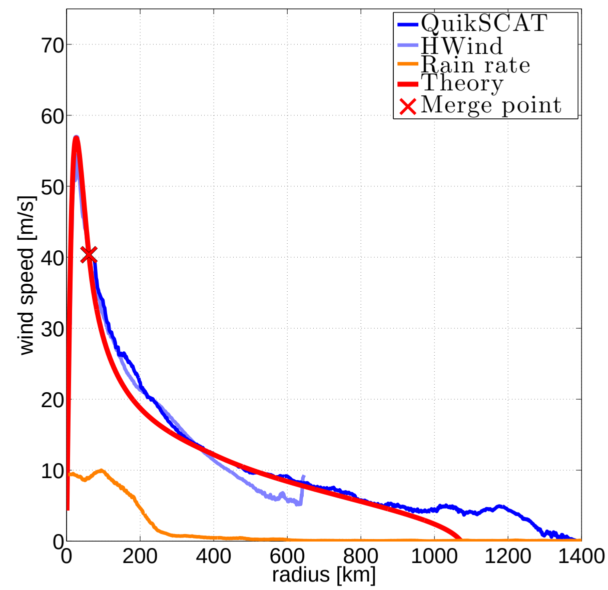

FIG. 2. Merged theoretical solution compared with observations from QuikSCAT and HWind.

3. Comparison to observations

We next seek to evaluate our merged radial wind profile against observations, particularly in the outer region, as well as examine its potential use for real world applications. We will use as a case study data from Hurricane Katrina (2005) at 1126 UTC on 28 August 2005. Our primary data source is the QuikSCAT tropical cyclone ocean near-surface ($z = 10\ m$) wind vector database (Stiles et al. (2014); `http://tropicalcyclone.jpl.nasa.gov/`). Additionally, the QuikSCAT database includes collocated data from the Hurricane Research Division HWind database (Powell et al. 1998) at 1200 UTC 28 Aug 2005 (i.e. 36 minutes later) as well as rain rate estimates using QuikSCAT microwave brightness temperature measurements (Ahmad et al. 2005). The HWind dataset combines all wind observation data to create a best-estimate wind field product in near-real-time. The center position and translation vector are determined using a piecewise cubic Hermite interpolating polynomial to interpolate the National Hurricane Center Best Track 6-hourly location and intensity data from the International Best Track archive (IBTrACS; `http://www.ncdc.noaa.gov/ibtracs/` accessed 12 Nov 2013).

Figure 2 displays the fit of the model to r_{12} from QuikSCAT and V_m from HWind, where we have allowed C_d to vary with wind speed in the outer region based on the data from Donelan et al. (2004). The model fits very well with the data over a large annulus in the outer region as well as in the inner region, where it correctly predicts the HWind estimate of r_m. The primary misfit is at intermediate radii where the non-convecting outer model is used to represent the transition region between strong convection and no convection. This result is encouraging, but more cases are needed to better assess its validity.

4. Summary and conclusions

This work develops a complete, physics-based radial wind profile for a tropical cyclone by mathematically merging existing theoretical solutions for the radial profile of absolute angular momentum at the top of the boundary layer in the inner convecting region (Emanuel and Rotunno 2011) and the outer non-convecting region (Emanuel 2004). The unified solution requires a fast ($\sim 1\ s$) numerical integration. The merged radial wind profile is then compared to wind field data from QuikSCAT and HWind, with emphasis on the former, for a case study of Hurricane Katrina (2005). The theoretical outer region solution is found to provide a very good fit to the azimuthal mean radial profile of the azimuthal wind.

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