On the Roughness Sublayer over Idealized Urban Rough Surfaces in Isothermal Conditions

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• Outline

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Introduction

• Flows over rough surfaces are characterized by the increased shear stress & enhanced momentum transfer induced by surface obstacles, such as buildings.

• The (rather homogeneous) flows in the inertial sublayer (ISL) are represented by the conventional logarithmic law of the wall (log-law).

• The inhomogeneous flows in the roughness sublayer (RSL) are often overlooked.
  • Flows are slowed down by roughness elements.
  • Transport processes are (directly) modified by individual roughness elements.
  • Log-law is no longer applicable.

• So there is a need for the functional form of mean velocity profiles that encompasses both the RSL/ISL dynamics.

• Elucidate the turbulent transport processes in the flows over rough surfaces.
Theoretical Background [1/2]

• Monin-Obukhov similarity theory (MOST)

\[ \frac{\kappa}{u_*} \frac{d\langle u \rangle}{dz} = \phi_m \left( \frac{z}{L} \right) \times \hat{\phi}_m \left( \frac{z}{L^* z_*} \right) \]

\( L = \infty \) in isothermal conditions

Dimensionless velocity gradient in isothermal conditions

\[ \frac{\kappa}{u_*} \frac{d\langle u \rangle}{dz} = \hat{\phi}_m \left( \frac{z}{z_*} \right) \]

\( \phi_m = 1 \) in isothermal conditions

Integrate from \( z_0 \) to \( z - d \)

RSL contribution

\[ \frac{\langle u \rangle_{z-d} - \langle u \rangle_{z_0}}{u_*} = \frac{1}{\kappa} \left[ \ln \left( \frac{z-d}{z_0} \right) - \int_{z_0}^{z-d} \frac{1-\hat{\phi}_m}{z} \, dz \right] \]

Logarithmic law of the wall

RSL influences vanishes asymptotically for \( z \rightarrow \infty \)

that implies

\[ \hat{\phi}_m \bigg|_{z \rightarrow \infty} = 1 \]

Hence

\[ \frac{\langle u \rangle_{z_0}}{u_*} = \int_{z_0}^{\infty} \frac{1-\hat{\phi}_m}{z} \, dz \]

Non-zero
Theoretical Background [2/2]

- **RSL velocity profile**
  \[
  \frac{\langle u \rangle}{u_*} = \frac{1}{\kappa} \left[ \ln \left( \frac{z-d}{z_0} \right) - \int_{z-d}^{\infty} \frac{1-\phi_m}{z} \, dz \right]
  \]

  \[
  \phi_m = 1 - \exp \left( -\mu \frac{z-d}{z_*} \right) \quad \mu: \text{a constant depending on RSL configuration}
  \]

- **Integral** is then simplified to an exponential integral that can be solved by series expansion. Hence,
  \[
  \frac{\langle u \rangle}{u_*} = \frac{1}{\kappa} \left[ \ln \left( \frac{z-d}{z_0} \right) - \gamma + \ln \left( \frac{\mu z-d}{z_*} \right) + \sum_{n=1}^{\infty} \frac{(-1)^n \left( \frac{\mu z-d}{z_*} \right)^n}{n \times n!} \right]
  \]

\(\gamma\): Euler constant \((= 0.5772156649)\)
Experimental Setup & Apparatus [1/4]

• Flow measurements
  • Hot-wire anemometry (HWA).
  • $\phi$ 5 $\mu$m platinum-plated tungsten wires.
  • Partly (copper plating) etched, 2-mm effective sensing length.
  • X-probe design in which the include angle is $100^\circ$.
  • HWA output signal is digitized which is then collected by 24-bit NI data acquisition modules (NI 9213) together with LabVIEW software on a digital computer.
  • Sampling frequency is 2 kHz & sampling duration is 50 sec (at each point).
  • Bruun (1971) universal HWA calibration scheme.
Experimental Setup & Apparatus [2/4]

Schematic of the test rack for surfaces with idealized roughness elements used in the wind tunnel experiments.
Idealized rough surfaces with various pitches covering different flow regimes
Reduced-scale models of idealized urban roughness used in the wind tunnel experiments.
# Rough-Surface Configuration

<table>
<thead>
<tr>
<th>Rough surfaces</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rib spacing $k$ [mm]</td>
<td>38</td>
<td>57</td>
<td>76</td>
<td>95</td>
<td>114</td>
<td>152</td>
<td>190</td>
<td>228</td>
</tr>
<tr>
<td>Rib size $h$ [mm]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pitch $k/h$</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Length of repeating unit $l$ [mm]</td>
<td>57</td>
<td>76</td>
<td>95</td>
<td>114</td>
<td>133</td>
<td>171</td>
<td>209</td>
<td>247</td>
</tr>
<tr>
<td>No. profiles in a repeating unit</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>$u_*$ [m sec$^{-1}$]</td>
<td>0.453</td>
<td>0.516</td>
<td>0.556</td>
<td>0.592</td>
<td>0.598</td>
<td>0.598</td>
<td>0.645</td>
<td>0.671</td>
</tr>
<tr>
<td>$U_\infty$ [m sec$^{-1}$]</td>
<td>8.0</td>
<td>8.4</td>
<td>8.5</td>
<td>8.5</td>
<td>8.5</td>
<td>8.4</td>
<td>9.1</td>
<td>9.0</td>
</tr>
<tr>
<td>$u_*/U_\infty$</td>
<td>0.057</td>
<td>0.062</td>
<td>0.066</td>
<td>0.069</td>
<td>0.070</td>
<td>0.071</td>
<td>0.071</td>
<td>0.074</td>
</tr>
<tr>
<td>$Re_\infty$ (= $U_\infty h/\nu$)</td>
<td>15,200</td>
<td>15,900</td>
<td>16,100</td>
<td>16,200</td>
<td>16,200</td>
<td>16,000</td>
<td>17,300</td>
<td>17,400</td>
</tr>
<tr>
<td>$Re_<em>$ (= $u_</em>h/\nu$)</td>
<td>864</td>
<td>983</td>
<td>1060</td>
<td>1,127</td>
<td>1,138</td>
<td>1,138</td>
<td>1,229</td>
<td>1,277</td>
</tr>
</tbody>
</table>
Results & Discussion
Sample Profiles: $k/h = 4$

Color symbols are the temporal averages at $x := 0$ mm (□); 9.5 mm (Δ); 28.6 mm (∇); 47.6 mm (▷); 66.7 mm (◁); 85.7 mm (◊) and 95.3 mm (○).

Filled symbols are the spatio-temporal averages.

Empty symbols are the RSL/TSL averages of the entire repeating unit.
# Flow Properties

<table>
<thead>
<tr>
<th>Rough Surfaces</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBL Thickness $\delta$</td>
<td>[mm]</td>
<td>244</td>
<td>248</td>
<td>283</td>
<td>284</td>
<td>294</td>
<td>294</td>
<td>304</td>
</tr>
<tr>
<td></td>
<td>$[h]$</td>
<td>12.84</td>
<td>13.05</td>
<td>14.89</td>
<td>14.95</td>
<td>15.47</td>
<td>15.47</td>
<td>16.00</td>
</tr>
<tr>
<td>RSL Top $z_*$</td>
<td>[mm]</td>
<td>38.00</td>
<td>44.08</td>
<td>49.97</td>
<td>53.01</td>
<td>50.92</td>
<td>45.03</td>
<td>38.00</td>
</tr>
<tr>
<td></td>
<td>$[h]$</td>
<td>2.00</td>
<td>2.32</td>
<td>2.63</td>
<td>2.79</td>
<td>2.68</td>
<td>2.37</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>$[\delta]$</td>
<td>0.16</td>
<td>0.18</td>
<td>0.18</td>
<td>0.19</td>
<td>0.17</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>ISL Thickness</td>
<td>[mm]</td>
<td>72</td>
<td>55</td>
<td>57</td>
<td>55</td>
<td>53</td>
<td>63</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>$[h]$</td>
<td>3.78</td>
<td>2.89</td>
<td>2.99</td>
<td>2.89</td>
<td>2.78</td>
<td>3.31</td>
<td>3.36</td>
</tr>
<tr>
<td></td>
<td>$[\delta]$</td>
<td>0.29</td>
<td>0.22</td>
<td>0.200</td>
<td>0.19</td>
<td>0.18</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>ISL Top</td>
<td>[mm]</td>
<td>110</td>
<td>99</td>
<td>107</td>
<td>108</td>
<td>104</td>
<td>108</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>$[h]$</td>
<td>5.78</td>
<td>5.20</td>
<td>5.62</td>
<td>5.67</td>
<td>5.46</td>
<td>5.67</td>
<td>5.36</td>
</tr>
<tr>
<td></td>
<td>$[\delta]$</td>
<td>0.45</td>
<td>0.40</td>
<td>0.38</td>
<td>0.38</td>
<td>0.35</td>
<td>0.37</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Spatio-temporally averaged velocity

\[ \langle u \rangle (\text{m sec}^{-1}) \]

\[ z - d \ (\text{m}) \]

RSL/ISL velocity profile

\[ k/h = 2 \]

\[ k/h = 3 \]

\[ k/h = 4 \]

\[ k/h = 5 \]

\[ k/h = 6 \]

\[ k/h = 8 \]

\[ k/h = 10 \]

\[ k/h = 12 \]
# RMS Error of Different Methods

<table>
<thead>
<tr>
<th>Rough Surfaces</th>
<th>Pitch $k/h$</th>
<th>Log-law</th>
<th>RSL/ISL Velocity Profile</th>
<th>Roughness Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>RSL &amp; ISL</td>
<td>ISL Only</td>
</tr>
<tr>
<td>A</td>
<td>2</td>
<td>0.9589</td>
<td>0.2256</td>
<td>0.0195</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>0.3426</td>
<td>0.1622</td>
<td>0.0340</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>0.2209</td>
<td>0.1210</td>
<td>0.0482</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
<td>0.1603</td>
<td>0.0955</td>
<td>0.0635</td>
</tr>
<tr>
<td>E</td>
<td>6</td>
<td>0.1296</td>
<td>0.0820</td>
<td>0.0622</td>
</tr>
<tr>
<td>F</td>
<td>8</td>
<td>0.1341</td>
<td>0.0943</td>
<td>0.0641</td>
</tr>
<tr>
<td>G</td>
<td>10</td>
<td>0.1020</td>
<td>0.0756</td>
<td>0.0533</td>
</tr>
<tr>
<td>H</td>
<td>12</td>
<td>0.1425</td>
<td>0.0976</td>
<td>0.0631</td>
</tr>
</tbody>
</table>
Velocity Profiles in Log Scale

\[ \frac{\langle u \rangle}{u_*} \]

\[ \frac{(z - d)}{z_0} \]

Increasing roughness

\[ \frac{\langle u \rangle}{u_*} = \frac{1}{\kappa} \left[ \ln \left( \frac{z - d}{z_0} \right) - \ln \left( \frac{\bar{z} - d}{z_{*0}} \right) - \sum_{n=1}^{n} \left( \frac{\bar{z} - d}{z_n} \right)^{\gamma} \right] \]

Enlarged Figure
Turbulent Length Scale

- Prandtl mixing-length model

\[
\langle u'' w'' \rangle = -K_m \frac{d \langle u \rangle}{dz} = -l_m^2 \left( \frac{d \langle u \rangle}{dz} \right)^2
\]

\( K_m = l_m u_* \) & \( u_* \) is the velocity scale

- Dimensionless momentum flux

\[
\frac{\langle u'' w'' \rangle}{u_*^2} = -l_m^2 \left[ \frac{d \langle u \rangle}{dz} \right]^2 = -l_m^2 \left[ \frac{1}{\kappa z} \phi_m \left( z/z_* \right) \right]^2
\]

TBL flux-gradient relationship

- Turbulent length scale

\( \approx 1 \) in the RSL/ISL

\[
l_m = \left( \frac{\langle u'' w'' \rangle}{u_*^2} \right)^{1/2} \times \frac{\kappa \times (z-d)}{1 - e^{-\mu \left( (z-d)/z_* \right)}}
\]

RSL/ISL dimensionless momentum flux is close to unity

\[
l_m \approx \begin{cases} 
\frac{\kappa \times (z-d)}{1 - e^{-\mu \left( (z-d)/z_* \right)}} & z_0 \leq z \leq z_* \\
\kappa \times (z-d) & z \to \infty 
\end{cases}
\]
Turbulent Length Scale

\[ l_m \approx \kappa \times (z - d) \]

\[ l_m \approx \frac{\kappa \times (z - d)}{1 - e^{-\mu[(z-d)/z_*]}} \]

Increasing roughness

Enlarged figure
Conclusions

• A new analytical solution to the (spatio-temporally averaged) velocity profile over hypothetical urban rough surfaces in isothermal conditions is proposed.

• An improved agreement with laboratory-scale wind tunnel measurement is demonstrated.

• It’s further developed to estimate the length scale for flows over rough surfaces in which the near- & far-field behaviors are shown.
Acknowledgment

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http://me.hku.hk/
References


On-Going Wind-Tunnel Research

Reduced-scale Hong Kong down-town 3D printing models
Profiles of dimensionless flow properties in the streamwise direction for different \( k/h \). Filled & empty symbols represent, respectively, the measured profiles at the centerline of ribs and cavities.