22nd Symposium on Boundary Layers and Turbulence
June 20 to 24, 2016; Salt Lake City, UT

On the Roughness Sublayer over Idealized Urban Rough Surfaces in Isothermal Conditions

Chun-Ho LIU* & Yat-Kiu HO

Department of Mechanical Engineering, The University of Hong Kong, Hong Kong

Paper No. 4A.2; Session 4A: Boundary Layer Processes IV

3:45 pm to 4:00 pm; June 20, 2016 (Monday)

The Canyons, Sheraton Salt Lake City Hotel

***Corresponding Authors**: Chun-Ho LIU; Department of Mechanical Engineering, 7/F Haking Wong Building, The University of Hong Kong, Pokfulam Road, HONG KONG; *Tel*: +852 2859 7901; *Fax*: +852 2858 5415; liuchunho@graduate.hku.hk

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Introduction

- Flows over rough surfaces are characterized by the increased shear stress & enhanced momentum transfer induced by surface obstacles, such as buildings.
- The (rather homogeneous) flows in the inertial sublayer (ISL) are represented by the conventional logarithmic law of the wall (log-law).
- The inhomogeneous flows in the roughness sublayer (RSL) are often overlooked.
  - Flows are slowed down by roughness elements.
  - Transport processes are (directly) modified by individual roughness elements.
  - Log-law is no longer applicable.
- So there is a need for the functional form of mean velocity profiles that encompasses both the RSL/ISL dynamics.
- Elucidate the turbulent transport processes in the flows over rough surfaces.

Theoretical Background [1/2]

- Monin-Obukhov similarity theory (MOST)

ϕ_m = 1 in isothermal conditions

$$\frac{\kappa z}{u_*} \times \frac{\mathrm{d}\langle \overline{u} \rangle}{\mathrm{d}z} = \phi_m\left(\frac{z}{L}\right) \times \hat{\phi}_m\left(\frac{z}{L}, \frac{z}{z_*}\right)$$

L = ∞ in isothermal conditions

L: Obukhov length scale
z: wall-normal distance
z_*: RSL thickness
u_*: friction velocity
κ: von Kármán constant (= 0.41)
ϕ_m: ISL stability function
$\hat{\phi}_m$: RSL profile function
$\overline{\bullet}$: temporal average
<•>: spatial average

$$\frac{\kappa z}{u_*} \times \frac{\mathrm{d}\langle \overline{u} \rangle}{\mathrm{d}z} = \hat{\phi}_m\left(\frac{z}{z_*}\right)$$

Dimensionless velocity gradient in isothermal conditions

Integrate from z_0 to z - d

RSL contribution

$$\frac{\langle \overline{u} \rangle\big|_{z-d} - \langle \overline{u} \rangle\big|_{z_0}}{u_*} = \frac{1}{\kappa}\left[\ln\left(\frac{z-d}{z_0}\right) - \int_{z_0}^{z-d} \frac{1-\hat{\phi}_m}{z}\,\mathrm{d}z\right]$$

Logarithmic law of the wall

RSL influences vanishes asymptotically for $z \rightarrow \infty$ that implies

$$\hat{\phi}_m\big|_{z\to\infty} = 1$$

Hence

Non-zero

$$\frac{\langle \overline{u} \rangle\big|_{z_0}}{u_*} = \int_{z_0}^{\infty} \frac{1-\hat{\phi}_m}{z}\,\mathrm{d}z$$

Theoretical Background [2/2]

- RSL velocity profile

RSL contribution

$$\frac{\left.\langle\overline{u}\rangle\right|_{z-d}}{u_*} = \frac{1}{\kappa}\left[\ln\left(\frac{z-d}{z_0}\right) - \int_{z-d}^{\infty} \frac{1-\widehat{\phi}_m}{z}\,\mathrm{d}z\right]$$

- Raupach (1992); Harman and Finnigan (2007) & De Ridder (2010)

$$\widehat{\phi}_m = 1 - \exp\left(-\mu\frac{z-d}{z_*}\right)$$

μ: a constant depending on RSL configuration

- Integral is then simplified to an exponential integral that can be solved by series expansion. Hence,

RSL/ISL velocity profile

$$\frac{\left.\langle\overline{u}\rangle\right|_{z-d}}{u_*} = \frac{1}{\kappa}\left\{\ln\left(\frac{z-d}{z_0}\right) - \left[\gamma + \ln\left(\mu\frac{z-d}{z_*}\right) + \sum_{n=1}^{\infty}\frac{(-1)^n\left(\mu\frac{z-d}{z_*}\right)^n}{n\times n!}\right]\right\}$$

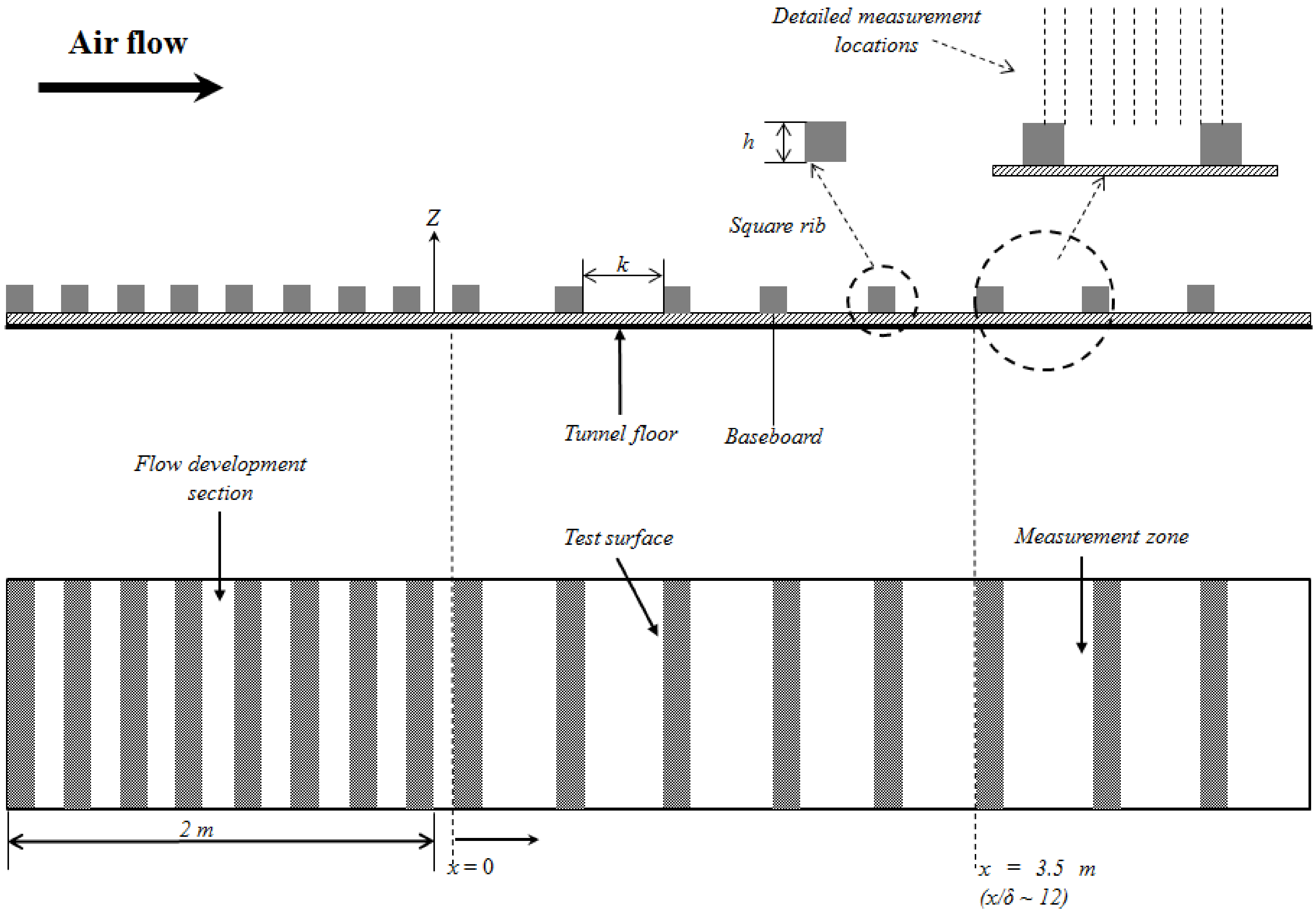
γ: Euler constant (= 0.5772156649)

RSL contribution

Experimental Setup & Apparatus [1/4]

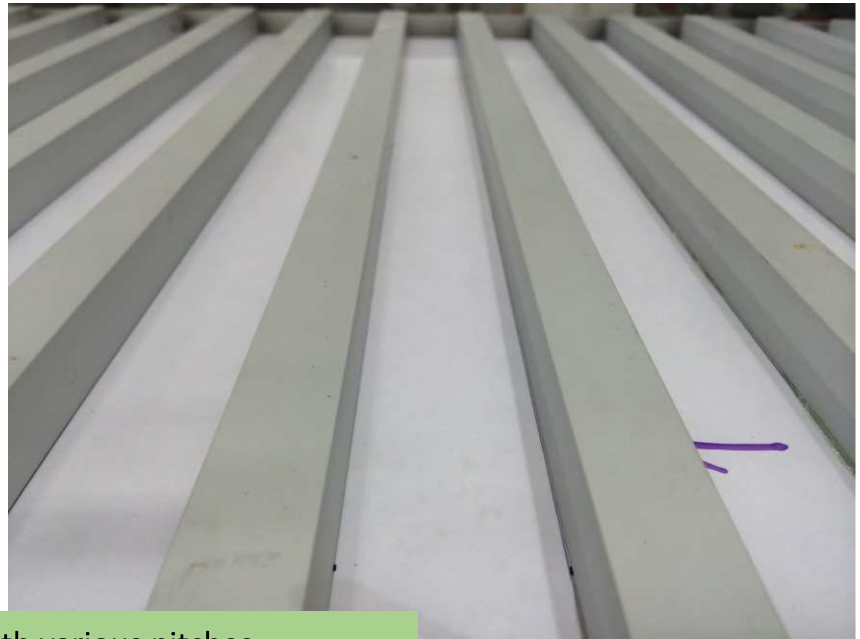
- Flow measurements
  - Hot-wire anemometry (HWA).
  - ϕ 5 μm platinum-plated tungsten wires.
  - Partly (copper plating) etched, 2-mm effective sensing length.
  - X-probe design in which the include angle is 100^o.
  - HWA output signal is digitized which is then collected by 24-bit NI data acquisition modules (NI 9213) together with LabVIEW software on a digital computer.
  - Sampling frequency is 2 kHz & sampling duration is 50 sec (at each point).
  - Bruun (1971) universal HWA calibration scheme.

Experimental Setup & Apparatus [2/4]

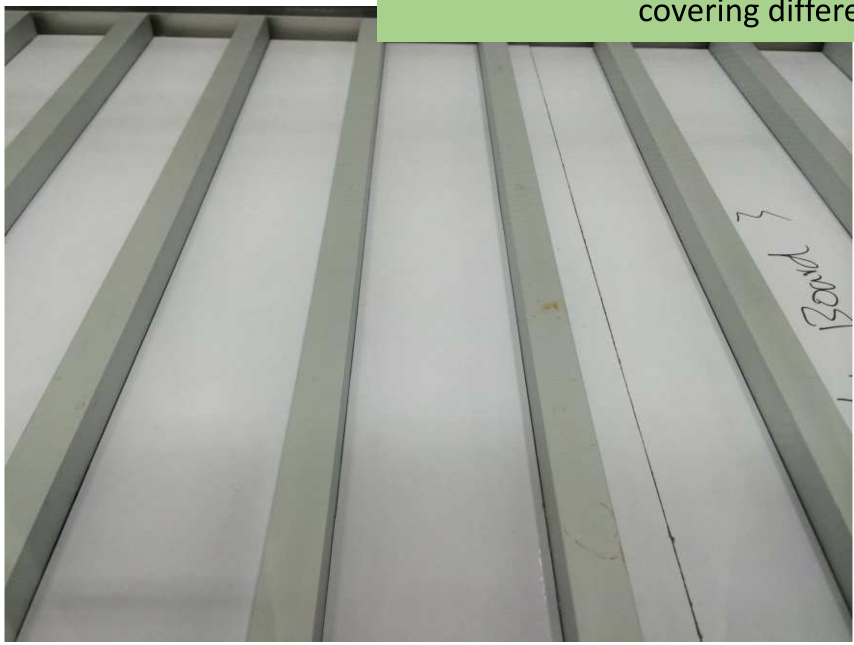


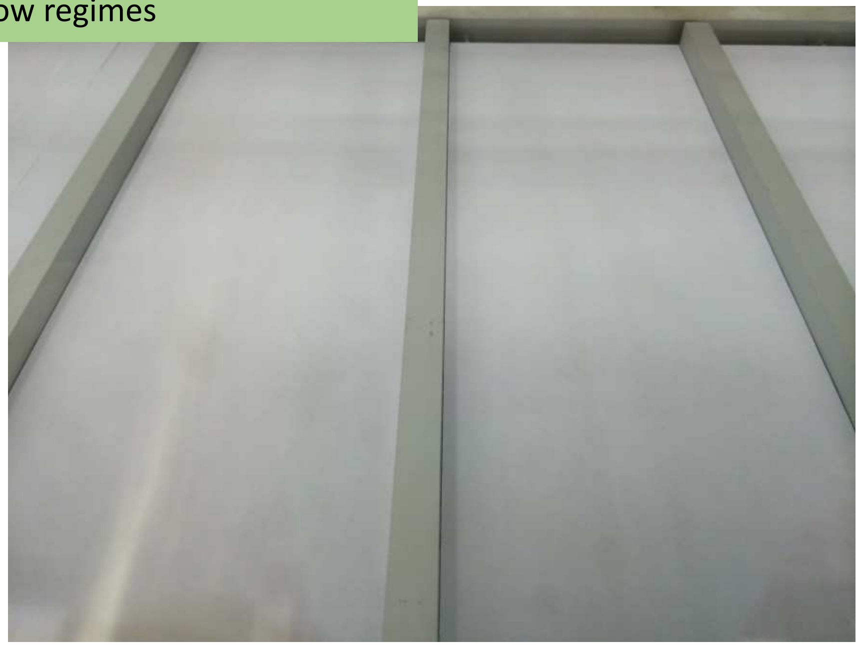
Schematic of the test rack for surfaces with idealized roughness elements used in the wind tunnel experiments.

Experimental Setup & Apparatus [3/4]



Idealized rough surfaces with various pitches
covering different flow regimes



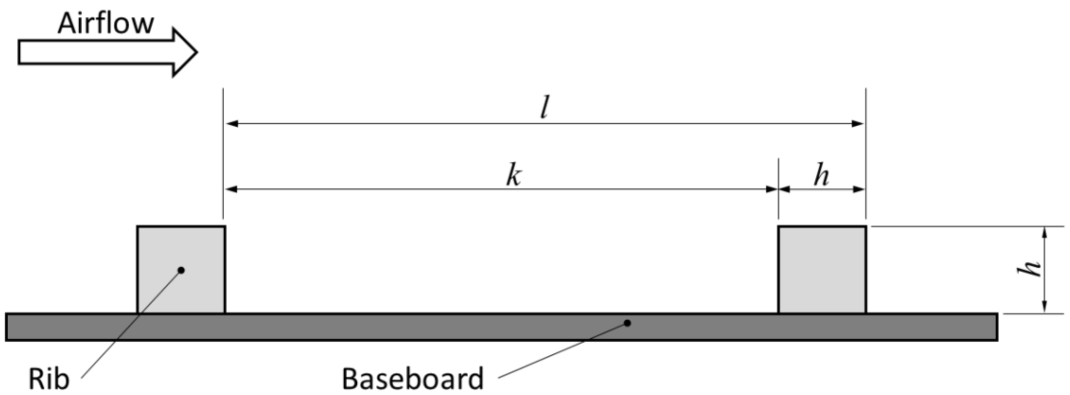


Experimental Setup & Apparatus [4/4]

Reduced-scale models of idealized urban roughness used in the wind tunnel experiments.

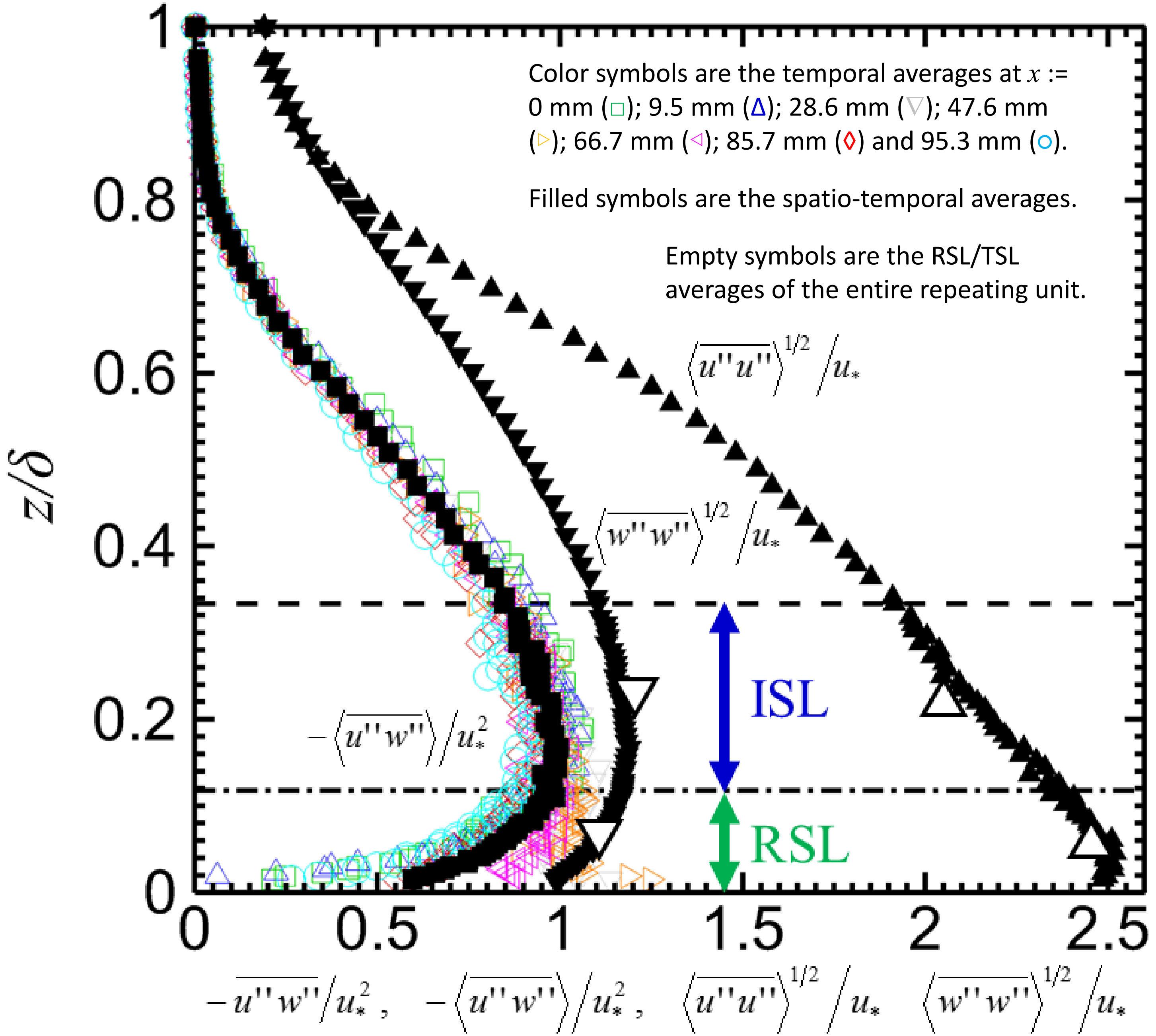
Rough-Surface Configuration

Rough surfaces	A	B	C	D	E	F	G	H
Rib spacing k [mm]	38	57	76	95	114	152	190	228
Rib size h [mm]	19							
Pitch k/h	2	3	4	5	6	8	10	12
Length of repeating unit l [mm]	57	76	95	114	133	171	209	247
No. profiles in a repeating unit	7					9		
u_* [m sec^{-1}]	0.453	0.516	0.556	0.592	0.598	0.598	0.645	0.671
U_∞ [m sec^{-1}]	8.0	8.4	8.5	8.5	8.5	8.4	9.1	9.0
u_*/U_∞	0.057	0.062	0.066	0.069	0.070	0.071	0.071	0.074
Re_∞ (= $U_\infty h/\nu$)	15,200	15,900	16,100	16,200	16,200	16,000	17,300	17,400
Re_* (= $u_* h/\nu$)	864	983	1060	1,127	1,138	1,138	1,229	1,277



Results & Discussion

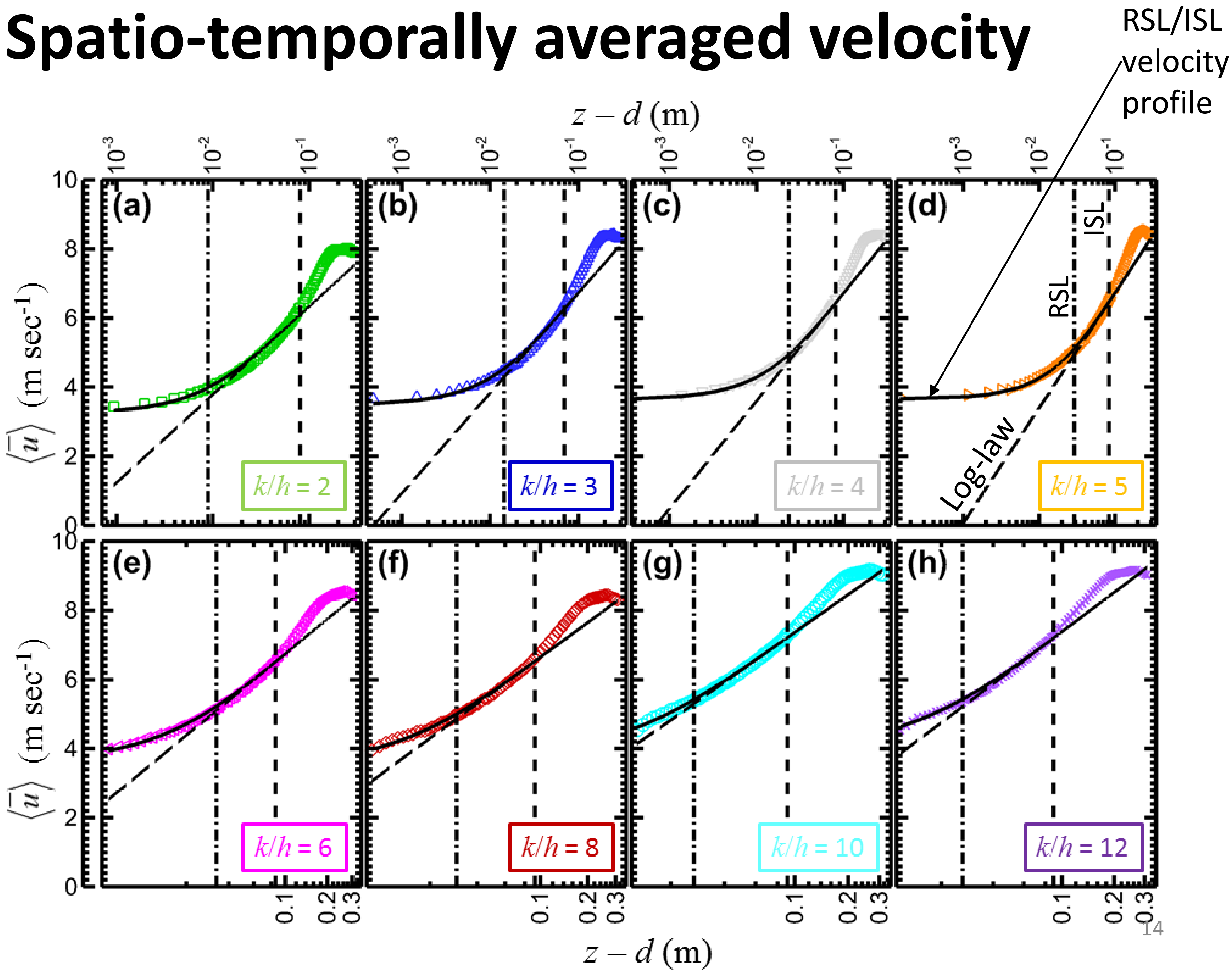
Sample Profiles: k/h = 4



Flow Properties

Rough Surfaces		A	B	C	D	E	F	G	H
TBL Thickness δ	**[mm]**	244	248	283	284	294	294	304	304
	[h]	12.84	13.05	14.89	14.95	15.47	15.47	16.00	16.00
RSL Top z_*	**[mm]**	38.00	44.08	49.97	53.01	50.92	45.03	38.00	38.95
	[h]	2.00	2.32	2.63	2.79	2.68	2.37	2.00	2.05
	[δ]	0.16	0.18	0.18	0.19	0.17	0.15	0.13	0.13
ISL Thickness	**[mm]**	72	55	57	55	53	63	64	63
	[h]	3.78	2.89	2.99	2.89	2.78	3.31	3.36	3.31
	[δ]	0.29	0.22	0.200	0.19	0.18	0.21	0.21	0.21
ISL Top	**[mm]**	110	99	107	108	104	108	102	102
	[h]	5.78	5.20	5.62	5.67	5.46	5.67	5.36	5.36
	[δ]	0.45	0.40	0.38	0.38	0.35	0.37	0.33	0.33

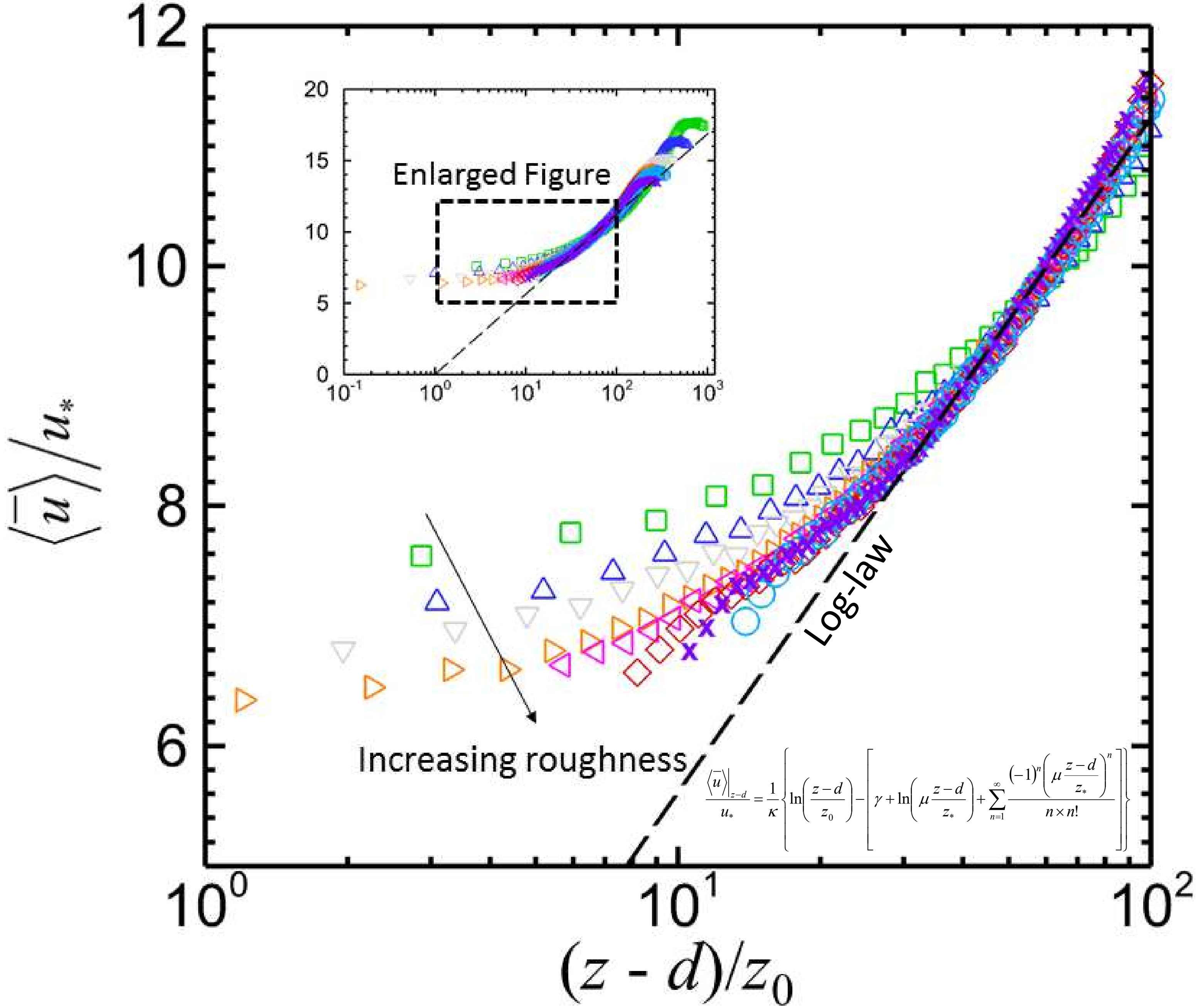
Spatio-temporally averaged velocity



RMS Error of Different Methods

Model		RMS Error with Wind Tunnel Data		Roughness Parameters			
				z_0/h		d/h	
Rough Surfaces	Pitch k/h	Log-law	RSL/ISL Velocity Profile	RSL & ISL	ISL Only	RSL & ISL	ISL Only
A	2	0.9589	0.2256	0.0195	0.0172	1.2818	1.5323
B	3	0.3426	0.1622	0.0340	0.0252	0.9576	1.5565
C	4	0.2209	0.1210	0.0482	0.0371	0.7312	0.4038
D	5	0.1603	0.0955	0.0635	0.0501	0.5561	1.2582
E	6	0.1296	0.0820	0.0622	0.0519	0.4128	0.9702
F	8	0.1341	0.0943	0.0641	0.0554	0.3576	0.8126
G	10	0.1020	0.0756	0.0533	0.0490	0.3267	0.5862
H	12	0.1425	0.0976	0.0631	0.0577	0.2796	0.6559

Velocity Profiles in Log Scale



Turbulent Length Scale

- Prandtl mixing-length model

$$\langle \overline{u''w''} \rangle = -K_m \frac{\mathrm{d}\langle \overline{u} \rangle}{\mathrm{d}z} = -l_m^2 \times \left(\frac{\mathrm{d}\langle \overline{u} \rangle}{\mathrm{d}z} \right)^2$$

$K_m = l_m u_*$ & u_* is the velocity scale

- Dimensionless momentum flux

$$\frac{\langle \overline{u''w''} \rangle}{u_*^2} = -l_m^2 \times \left[\frac{\mathrm{d}\left(\langle \overline{u} \rangle / u_* \right)}{\mathrm{d}z} \right]^2 = -l_m^2 \times \left[\frac{1}{\kappa z} \widehat{\phi}_m \left(z/z_* \right) \right]^2$$

TBL flux-gradient relationship

- Turbulent length scale

≈ 1 in the RSL/ISL

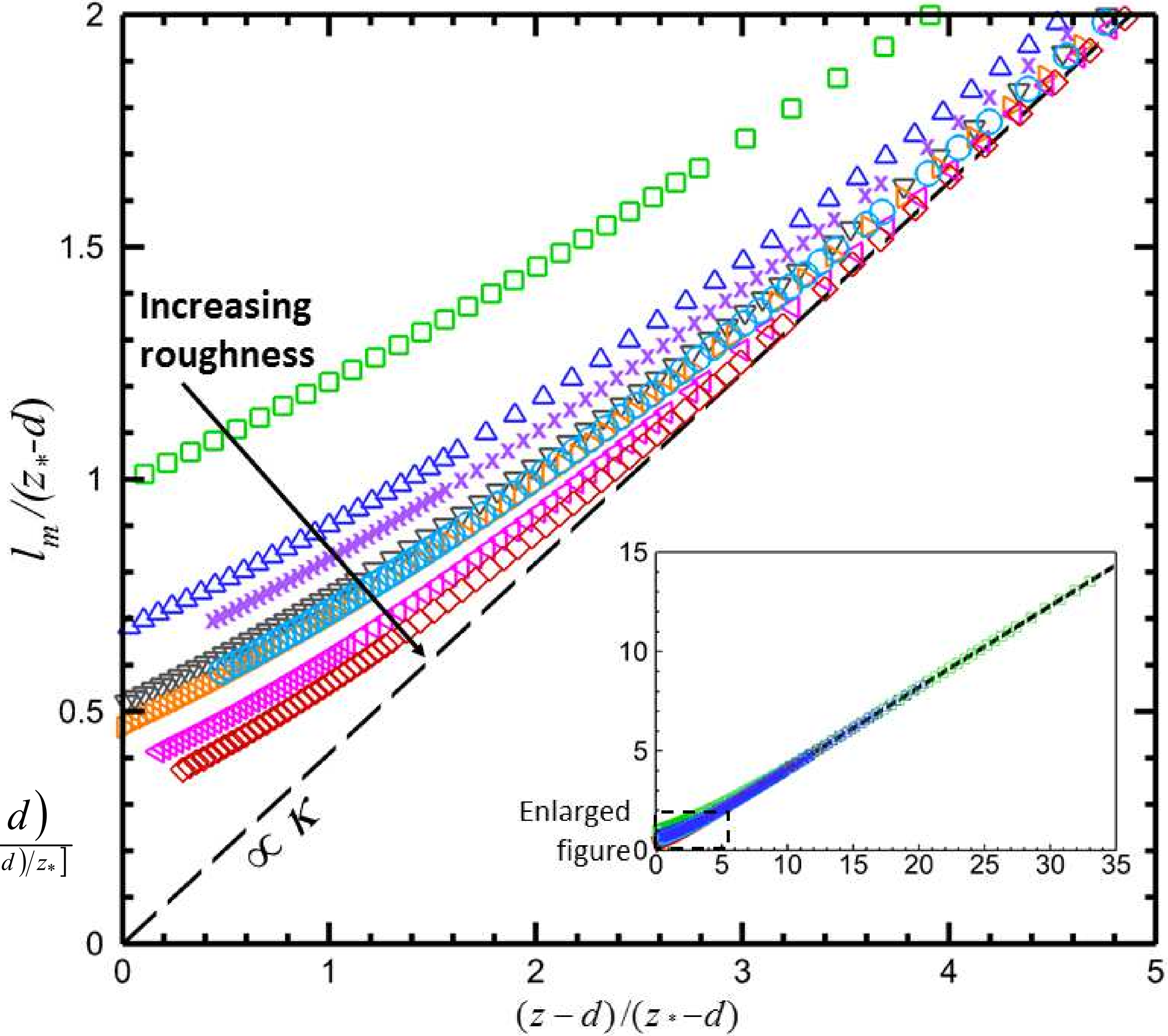
$$l_m = -\left(\frac{\langle \overline{u''w''} \rangle}{u_*^2} \right)^{1/2} \times \frac{\kappa \times (z-d)}{1 - e^{-\mu[(z-d)/z_*]}}$$

RSL/ISL dimensionless momentum flux is close to unity

$$l_m \approx \begin{cases} \dfrac{\kappa \times (z-d)}{1 - e^{-\mu[(z-d)/z_*]}} & z_0 \le z \le z_* \\[2ex] \kappa \times (z-d) & z \to \infty \end{cases}$$

Turbulent Length Scale

$$l_m \approx \kappa \times (z-d)$$



$$l_m \approx \frac{\kappa \times (z-d)}{1-e^{-\mu[(z-d)/z_*]}}$$

Conclusions

- A new analytical solution to the (spatio-temporally averaged) velocity profile over hypothetical urban rough surfaces in isothermal conditions is proposed.
- An improved agreement with laboratory-scale wind tunnel measurement is demonstrated.
- It's further developed to estimate the length scale for flows over rough surfaces in which the near- & far-field behaviors are shown.

Acknowledgment

- The 2nd author wishes to thank the Hong Kong Research Grants Council (RGC) for financially supporting his study through the Hong Kong PhD Fellowship (HKPF) scheme. This project is partly supported by the General Research Fund (GRF) of RGC HKU 714913E. Technical support from Mr. Vincent K.W. Lo is appreciated.

http://me.hku.hk/

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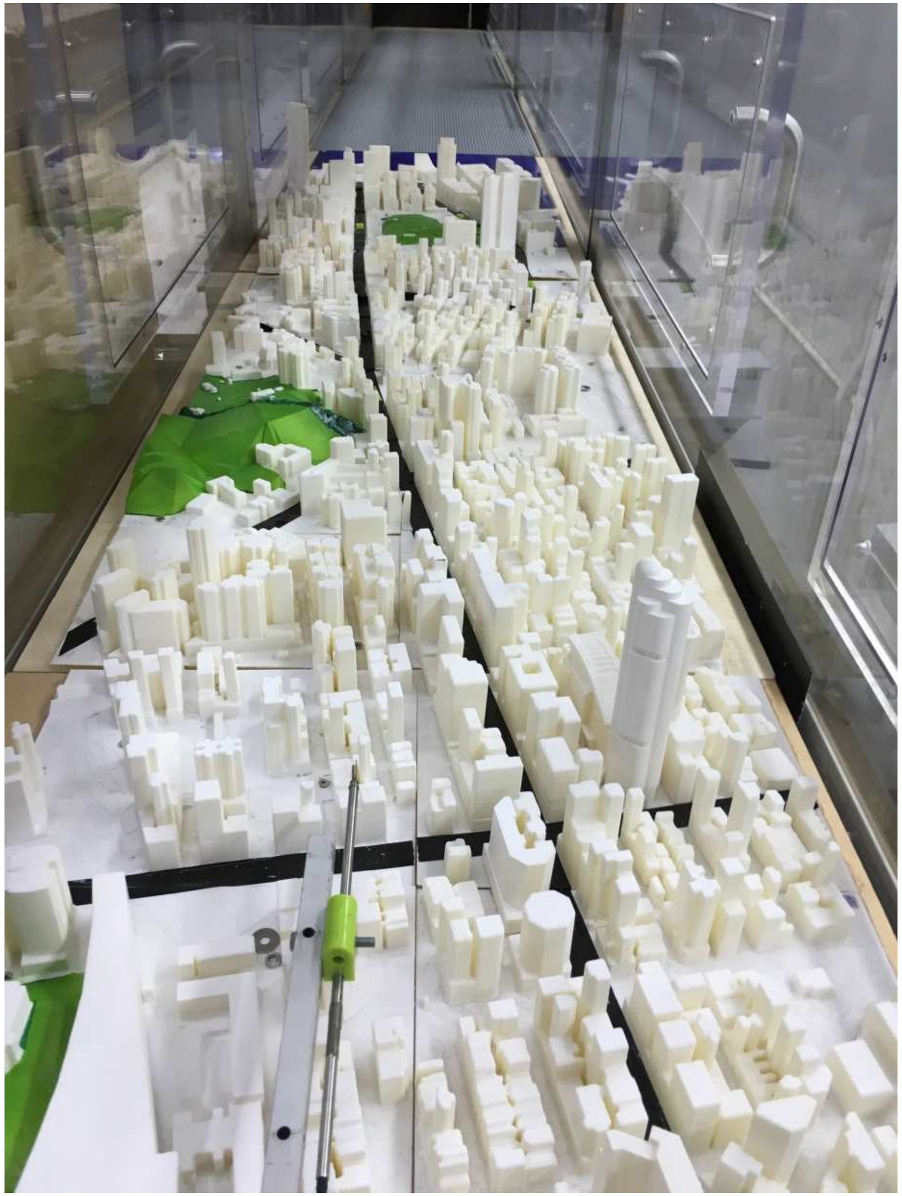
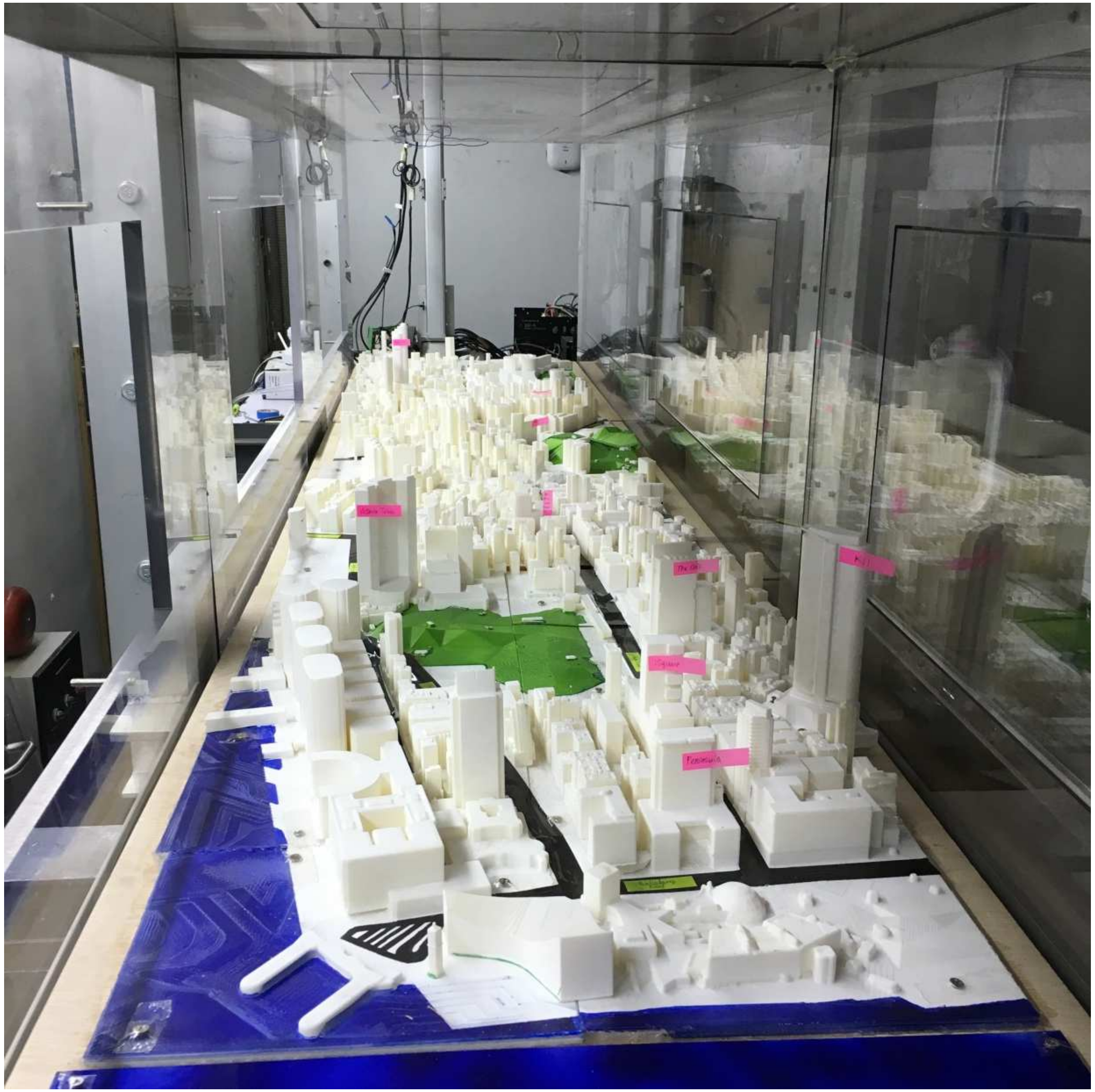
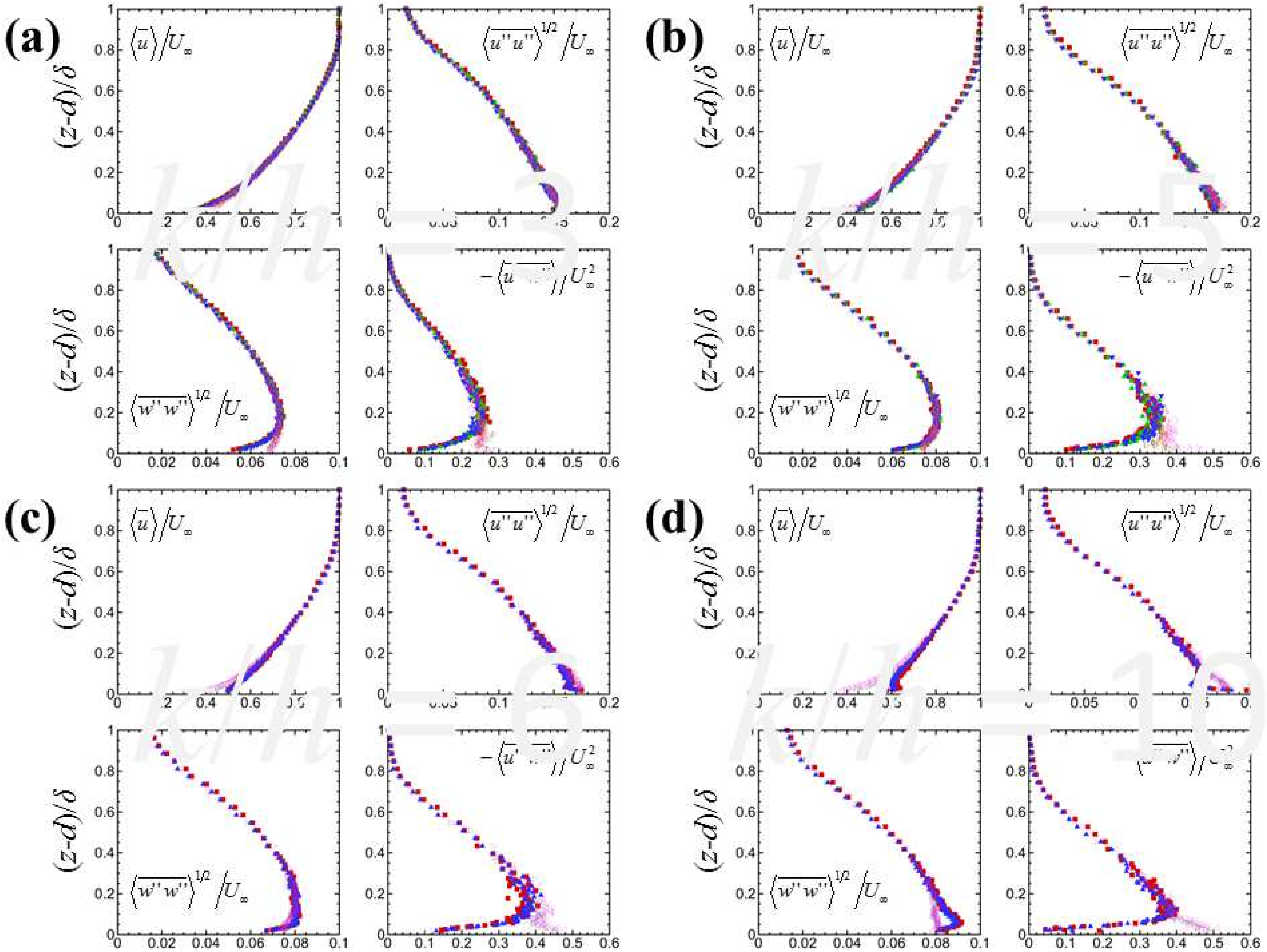
On-Going Wind-Tunnel Research





Reduced-scale Hong Kong down-town 3D printing models



Profiles of dimensionless flow properties in the streamwise direction for different k/h. Filled & empty symbols represent, respectively, the measured profiles at the centerline of ribs and cavities.