# Beyond the Textbook EBL: Mean and Turbulence in Unsteady Ekman Boundary Layers

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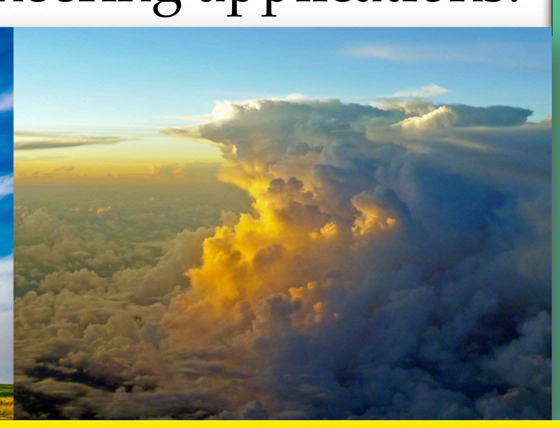
## Motivations

- Geophysical wall-bounded flows are dynamical systems that are **almost always changing** with time. Unsteady geostrophic forcing in the atmosphere or ocean can strongly influence the mean wind and higher order turbulence statistics.
- It is important to understand when and if turbulence can be considered in **quasi-equilibrium**, and what are the implications of unsteadiness and disequilibrium on flow characteristics and on the classic equilibrium-based models.
- The present study focuses on the **unsteady Ekman boundary layer** (EBL) where pressure gradient forces, Coriolis forces, and turbulent friction forces interact but are not necessarily in equilibrium.

The knowledge obtained from studying these questions help us understand the underlying fundamental physical dynamics of the unsteady boundary layers, and develop better turbulence closures for weather/climate models and engineering applications.



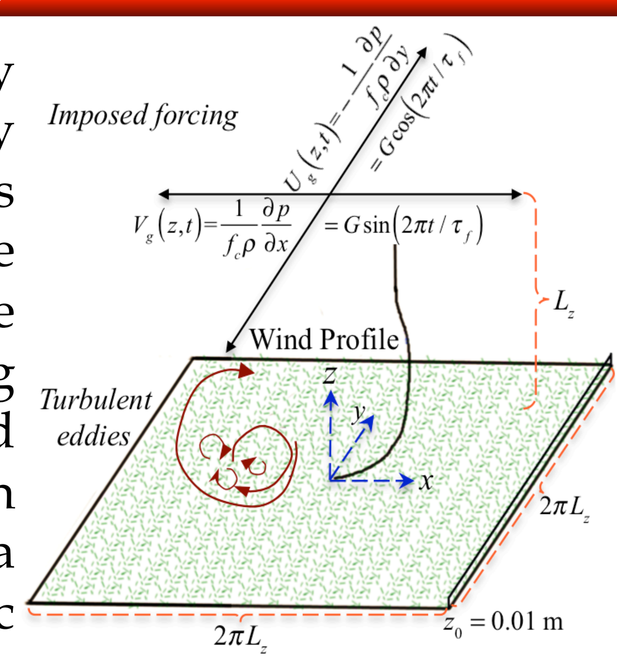
## Objectives

The aim of this research is to answer some of the key questions regarding the dynamics of the unsteady EBLs such as:

1) How do the mean and turbulence respond to different forcing time scales, relative to the mean and turbulence time scales?
2) When will the quasi-equilibrium assumption fail, and what are the implications for the mean velocity and turbulence behaviour?
3) Are existing theories, such as the log-law, still valid in some unsteady regimes?

## Large Eddy Simulation

LES technique explicitly calculates the large-eddy field and parameterizes eddies smaller than the grid/filter size. We use the geostrophic forcing to drive the flow and will represent the mean pressure gradient as a horizontal geostrophic wind.




➢ Governing equations in the LES:

1. **Continuity** Equation : $\frac{\partial \tilde{u}_i}{\partial x_i} = 0$

2. **Navier-Stokes** Momentum Equations :

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j\left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i}\right) = -\frac{1}{\rho}\frac{\partial \tilde{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + f_c(U_g - \tilde{u}_1)\delta_{i2} - f_c(V_g - \tilde{u}_2)\delta_{i1}$$

A **scale-dependent Lagrangian dynamic model** for SGS modeling, Bou-Zeid *et al.* [1], is being used.

## Resolved TKE Budget

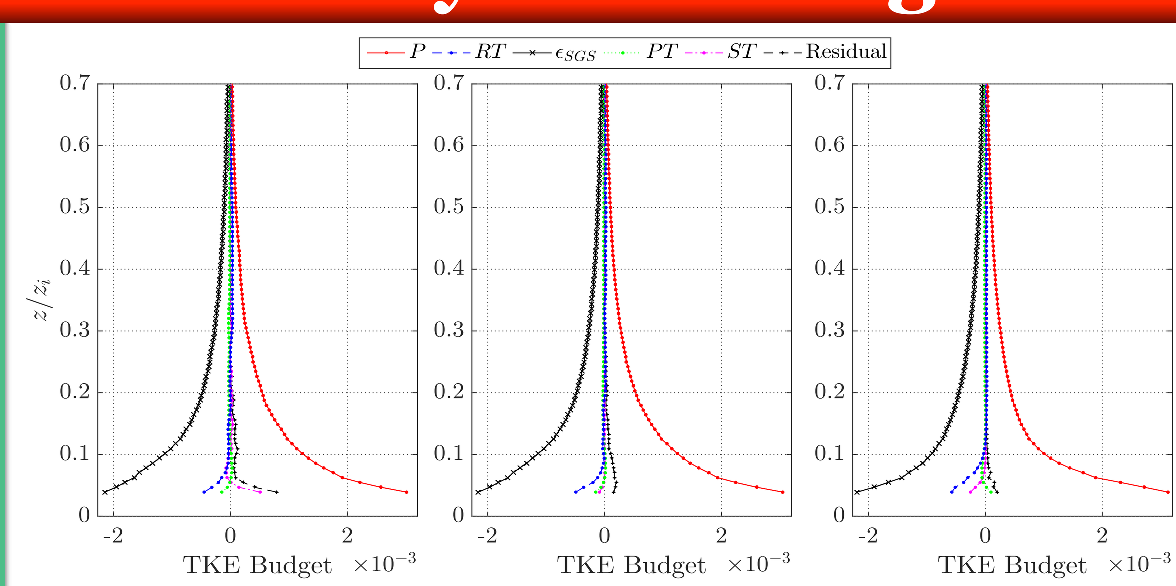
- We assume no subsidence $w=0$ and periodic statistical homogeneity in both horizontal directions. We apply the Reynolds decomposition to N-S and get:

$$\frac{\partial \bar{e}}{\partial t} = -\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z} - \frac{\partial \overline{w'p'}}{\partial z} - \frac{\partial \overline{w'e}}{\partial z} - \frac{\partial \overline{u_i'\tau_{i3}'}}{\partial w} + \overline{\tau_{ij}'S_{ij}'}$$

*P* (production) Pressure *T* Resolved *T* SGS *T* SGS *D*

*T* means transport, and *D* means dissipation.

## Steady TKE Budget

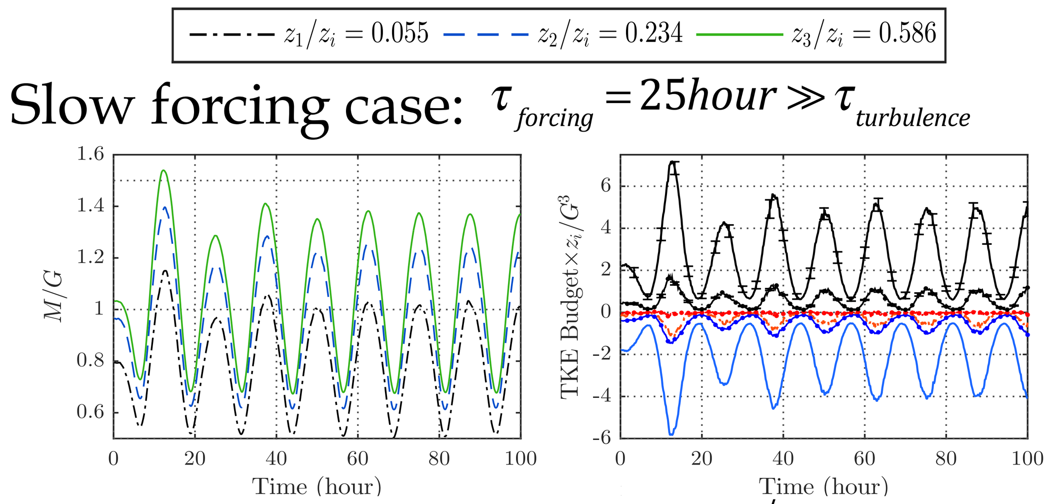



TKE budget profiles (normalized using $G$ and $z_i$) with spatial averaging (left). Steady case with both spatial and temporal averaging using LASD (middle), and using Smagorinsky SGS model (right).

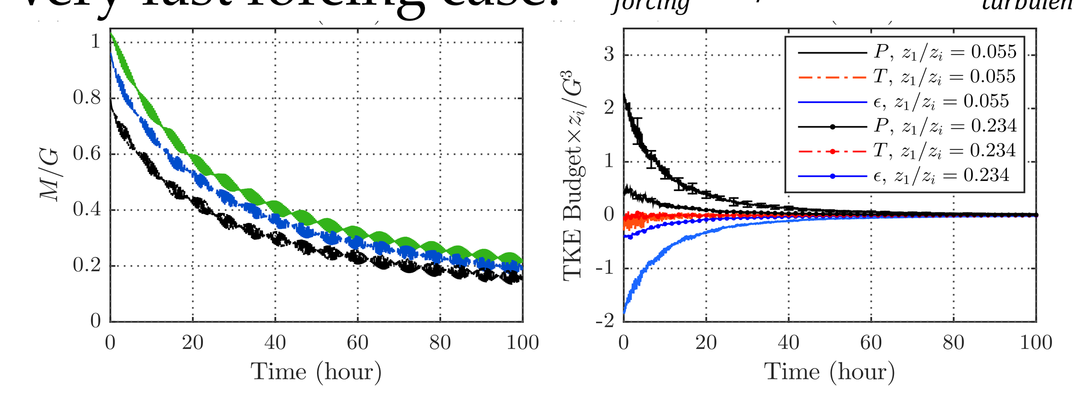
- Timescales:
  1) Inertial timescale: $\tau_{EBL} = 2\pi/f_c \approx 12.5$ hours (see Momen &Bou-Zeid 2016-a)
  2) Turbulence timescale: $\tau_t = z_i/u_* <= 2$ hours
  3) Forcing timescale: $\tau_f$

## Unsteady TKE Budget

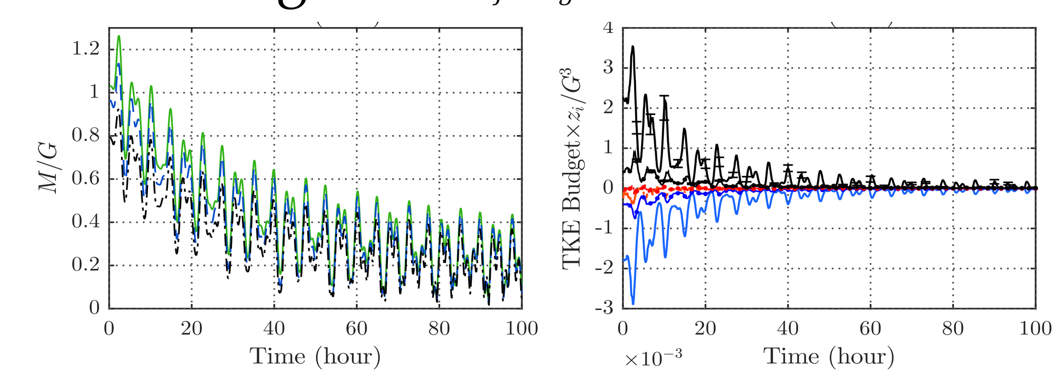
1) Slow forcing case: $\tau_{forcing} = 25 hour \gg \tau_{turbulence}$




2) Very fast forcing case: $\tau_{forcing} = 1/3$ hour $\ll \tau_{turbulence}$



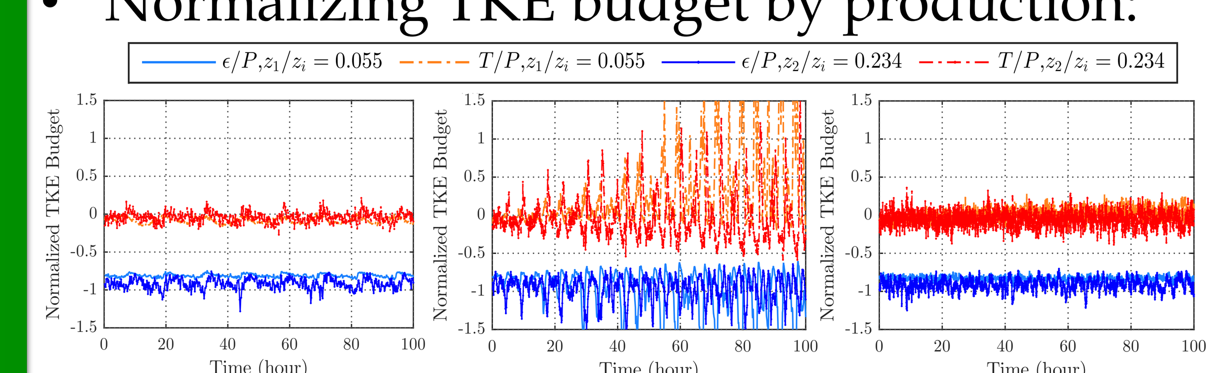

3) Fast forcing case: $\tau_{forcing} = 3.125$ hours $\approx \tau_{turbulence}$




See Momen & Bou-Zied (2016-b) for other cases.

## Turbulence equilibrium

- Normalizing TKE budget by production:




$\tau_{forcing} = 25$ hours $\gg \tau_{turbulence}$ $\tau_{forcing} = 3.125$ hours $\approx \tau_{turbulence}$ $\tau_{forcing} = 1/3$ hour $\ll \tau_{turbulence}$

## Validity of the log-law

- Log-law departure parameters :

$$\begin{cases} \gamma \equiv \dfrac{\sqrt{\mathrm{var}\left(M^+ - M^+_{\text{log fit}}\right)}}{\overline{M^+}} \\ \gamma_d \equiv \dfrac{\gamma}{\gamma_0}, \kappa_d \equiv \dfrac{\kappa_{FittedLog}}{\kappa_0} \end{cases}$$

where $M^+ = M/u_*$; $\gamma_d$ is a normalized curvature parameter that indicates the departure from a log profile compared to the steady-state simulation for which $\gamma_0 = 0.3$.

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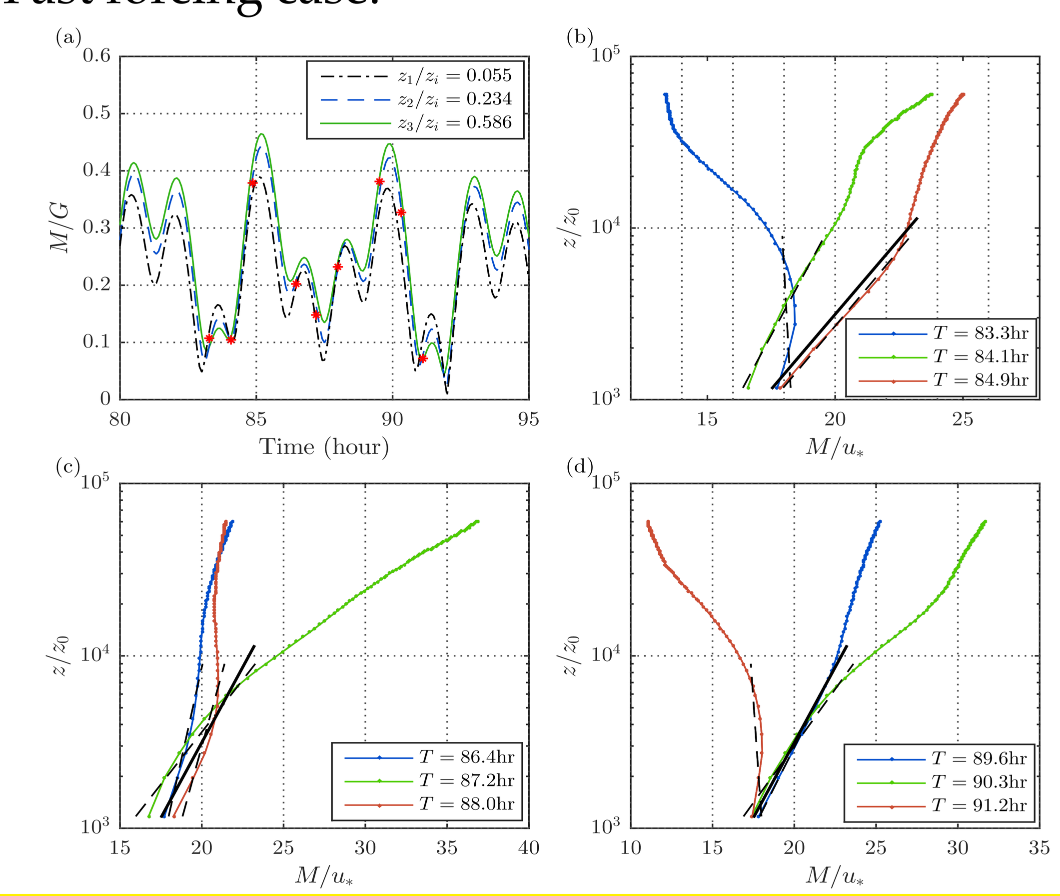
## The log-law validity

➢ Log-law departures in three forcing cases:

| Case | Times (hour) | $\gamma_d$ (min, average, max) | $\kappa_d$ (min, average, max) |
|---|---|---|---|
| **Slow** | 81.75, 88.00, 94.25 | (1.3, 1.3, 1.4) | (0.96, 0.97, 0.99) |
| **Very fast** | 92.42, 92.50, 92.55, 95.53, 95.63, 95.68, 98.65, 98.75, 98.80 | (1.3, 2.1, 2.3) | (0.89, 1.1, 1.25) |
| **Fast** | 83.3, 84.08, 84.87, 86.43, 87.21, 87.99, 89.55, 90.33, 91.15 | (1.0, 4.2, 7.3) | (–15.8, –1.5, 2.5) |

- Fast forcing case:




## Conclusions and Future Work

- When the forcing is slow ($\tau_f >> \tau_t$), or fast ($\tau_f << \tau_t$) turbulence is in **quasi-equilibrium** condition and the shape of the normalized TKE budget profile does not change. However, only when the forcing time scale is on the order of the turbulence time scale ($\tau_f \sim \tau_t$) this quasi-equilibrium breaks down.
- For $\tau_{forcing} \sim \tau_{turbulence}$ , our results showed **unusual wind profiles** and very strong departures from the log-law, both instantaneously and on the average.
- High-order or **non-equilibrium closures** should be used (e.g. in URANS) to account for this out of equilibrium conditions.

## References

1. Bou-Zeid E, Meneveau C, Parlange M (2005) *A scale-dependent Lagrangian dynamic model for LES of turbulent flows*. Phys Fluids
2. Momen M, Bou-Zeid E (2016-a) *Large Eddy Simulations and Damped-Oscillator Models of the Unsteady EBL*. J Atmos Sci.
3. Momen M, Bou-Zeid E (2016-b) *Mean and Turbulence Dynamics in Unsteady Ekman Boundary Layers*. JFM (in review)