Large-Eddy Simulation of the Wake Flow Inside and Downwind of a Large Finite-Size Wind Farm

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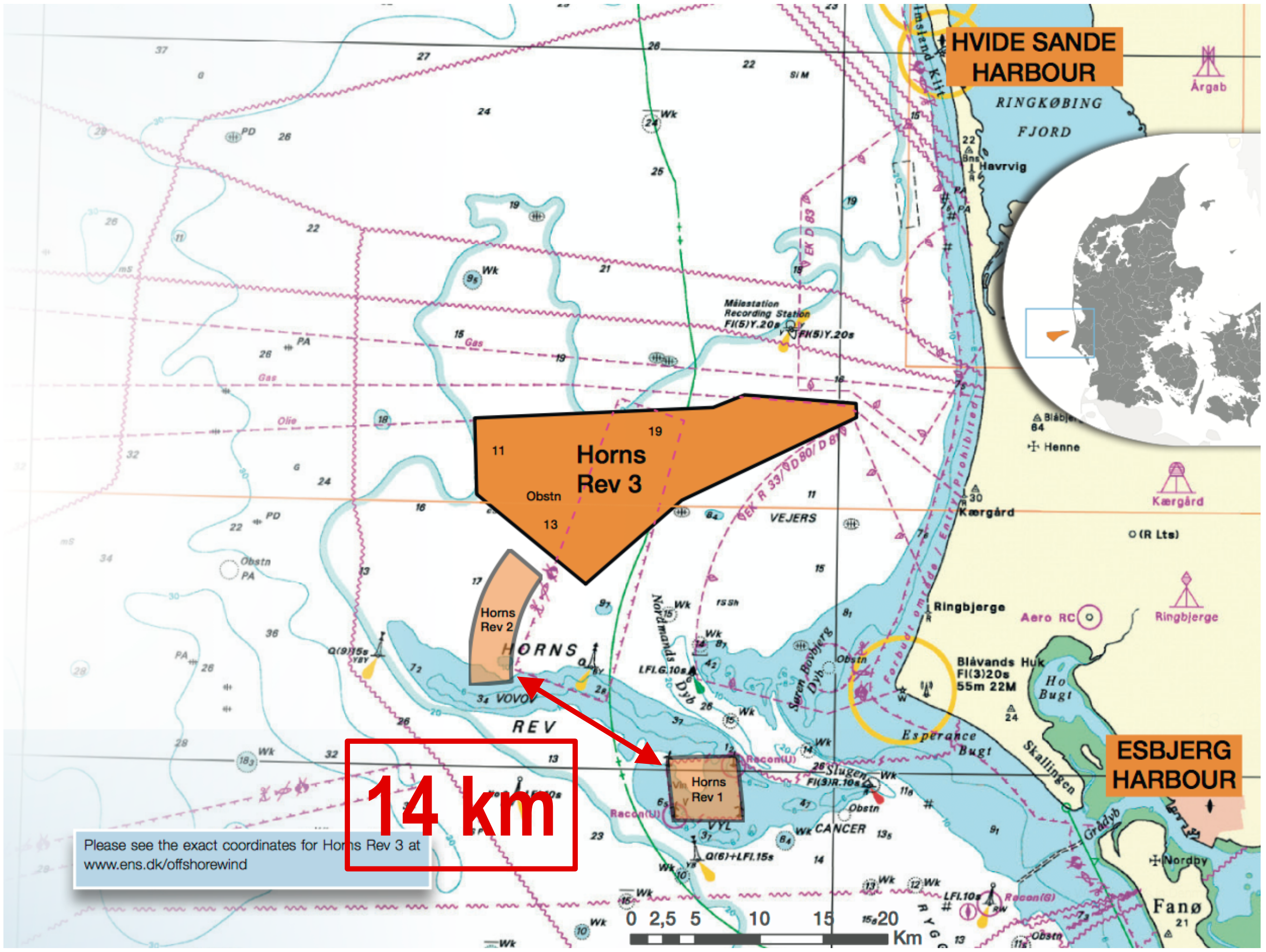
Photo credit: https://wall.alphacoders.com/big.php?i=351853

Introduction

Large Finite-Size Wind Farms

- Large wind farms are built closer and closer to each other
  e.g. Horns Rev 2 is placed only 14 km from Horns Rev 1 in the North Sea
- Offshore ABLs are often considered as conventionally neutral boundary layers (CNBL)



Source: http://www.ens.dk/sites/ens.dk/files/dokumenter/publikationer/downloads/new_offshore_wind_tenders_in_denmark_final.pdf

Objectives

To study:

- the adjustment of the ABL above and behind a large wind farm
- the structure of the wake flow inside and behind the wind farm
- the validity and limitations of the infinite wind farm approximation

METHODOLOGY - LARGE-EDDY SIMULATIONS (LES)

In-house WIRE-LES code solving filtered governing equations [Abkar & Porté-Agel (2013)]

Filtered Navier-Stokes Equations

$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right) = -\frac{\partial \tilde{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}^d}{\partial x_j} + \delta_{i3} g \frac{\tilde{\theta} - \langle \tilde{\theta} \rangle}{\theta_0} + f_c \epsilon_{ij3} \tilde{u}_j + F_i$$

Filtered Continuity Equation

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0$$

Filtered Scalar Transport Equation

$$\frac{\partial \tilde{\theta}}{\partial t} + \tilde{u}_j \frac{\partial \tilde{\theta}}{\partial x_j} = -\frac{\partial q_j}{\partial x_j}$$

Scale-dependent Lagrangian dynamic eddy-viscosity model to parameterize subgrid-scale (SGS) stress and heat flux [Stoll & Porté-Agel (2006)]

SGS Stress

$$\tau_{ij}^d = \tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij} = -2 \tilde{\Delta}^2 C_s^2 |\tilde{S}| \tilde{S}_{ij}$$

SGS Flux

$$q_j = -\tilde{\Delta}^2 C_s^2 Pr_{sgs}^{-1} |\tilde{S}| \frac{\partial \tilde{\theta}}{\partial x_j}$$

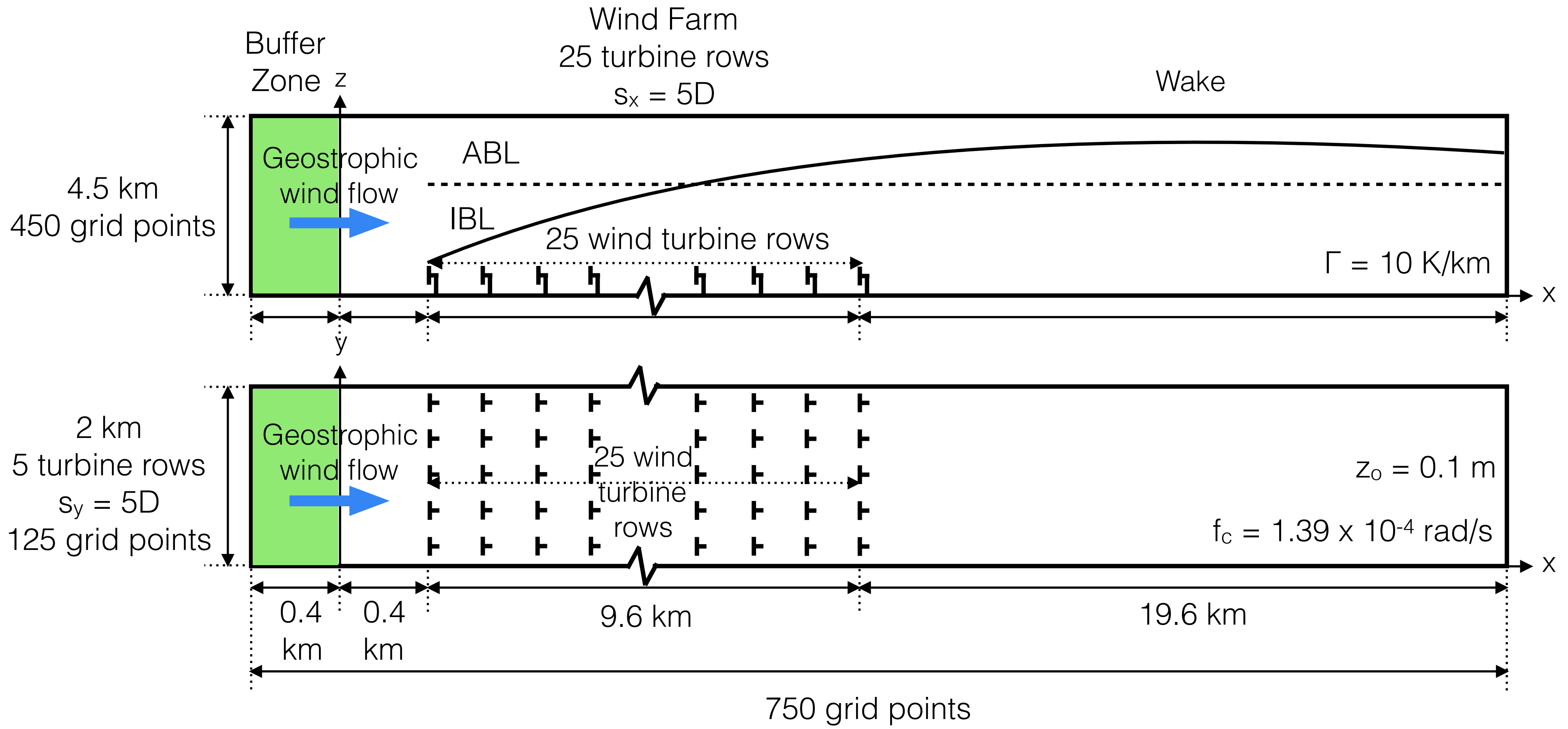
Actuator-disk model with rotation for wind-turbine parameterization [Wu & Porté-Agel (2011)]:

$$\mathbf{f}_{disk} = \frac{d\mathbf{F}}{dA} = \frac{1}{2} \rho V_{rel}^2 \frac{Bc}{2\pi r} (C_L \mathbf{e_L} + C_D \mathbf{e_D})$$

Model of wind turbines simulated: Vestas V80-2MW, with a 80 m diameter

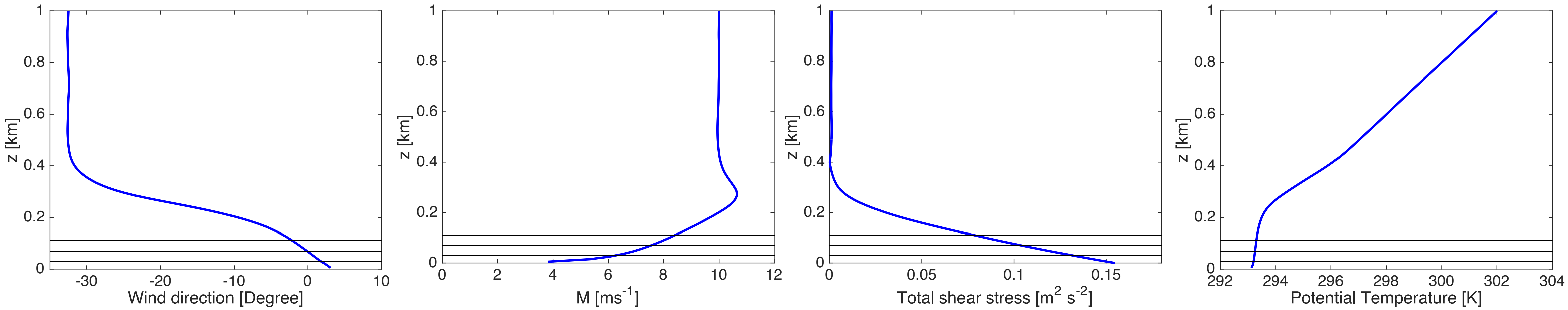
METHODOLOGY - LARGE-EDDY SIMULATIONS (LES)

Large Finite-Size Wind Farm Setup

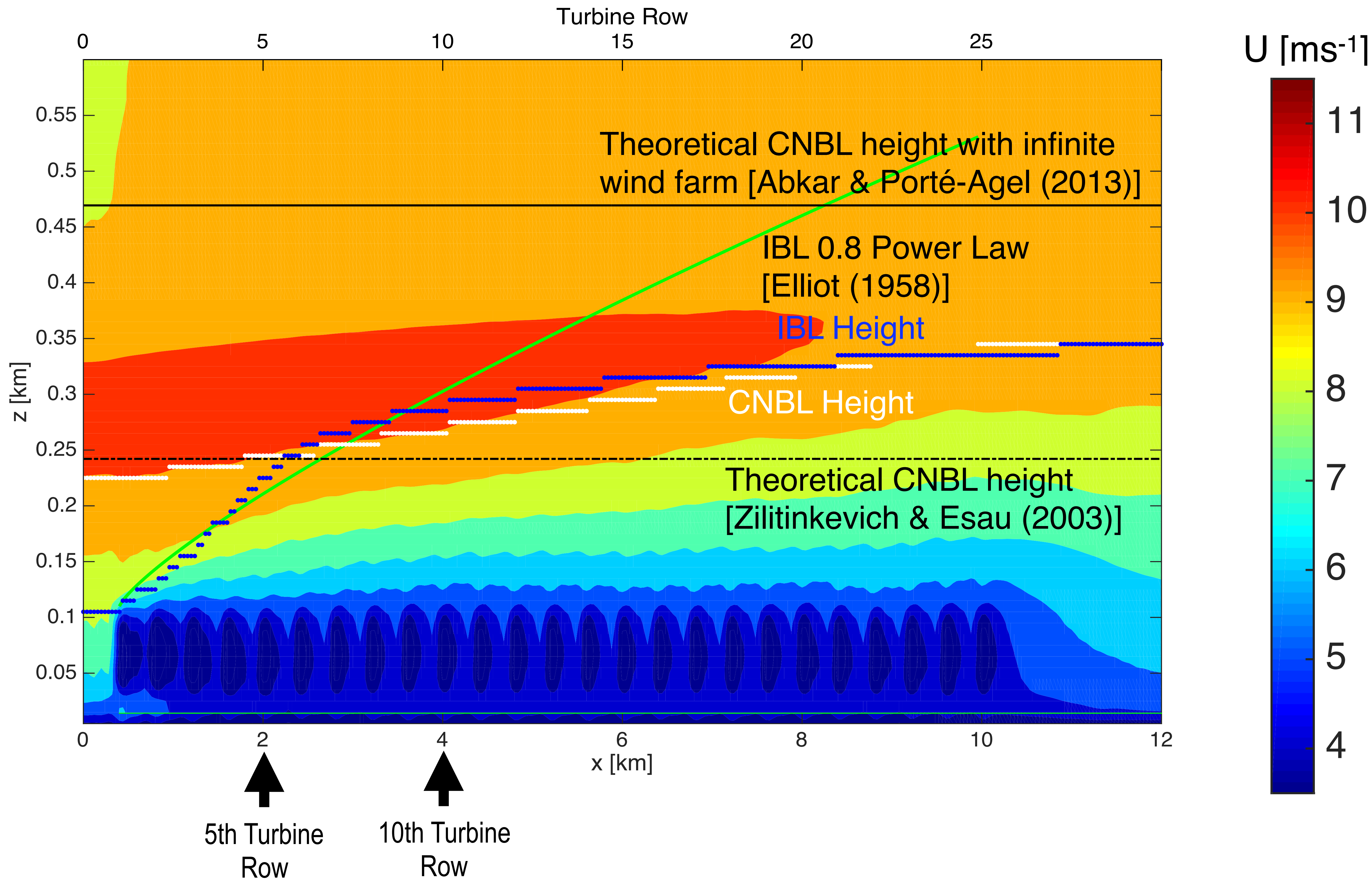


D = Wind turbine diameter

Inflow Conditions (from Precursor Simulation)



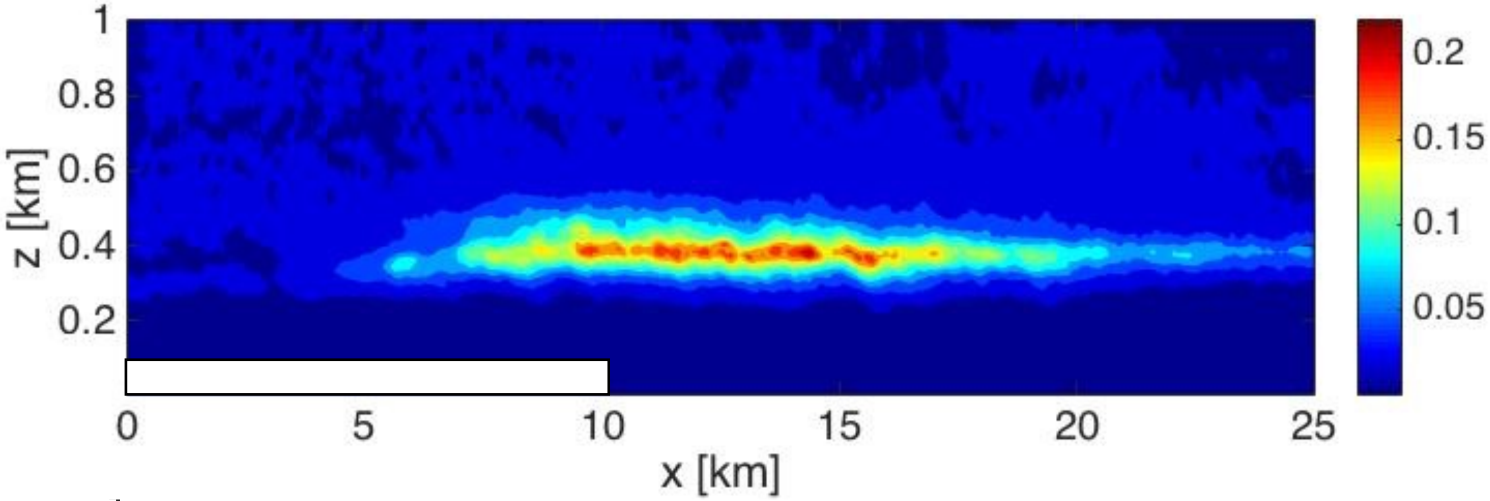
Boundary Layer Growth



Boundary Layer Growth

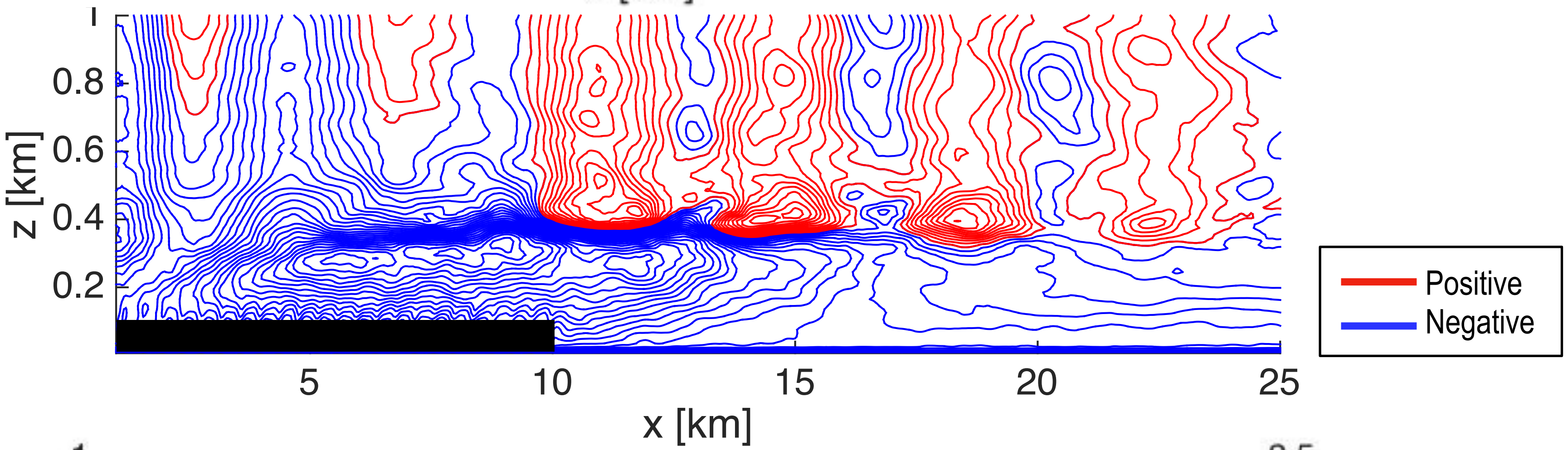
Temperature Variance <θ'θ'>

↑ <θ'θ'>



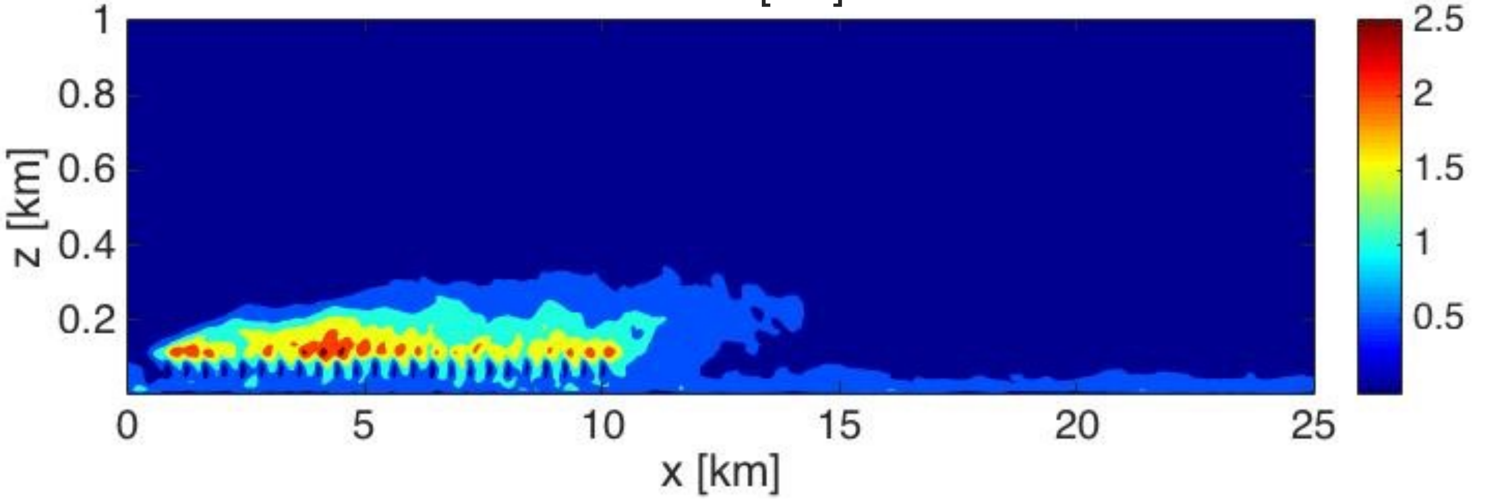
Temperature Flux <w'θ'>

↓magnitude of negative <w'θ'>



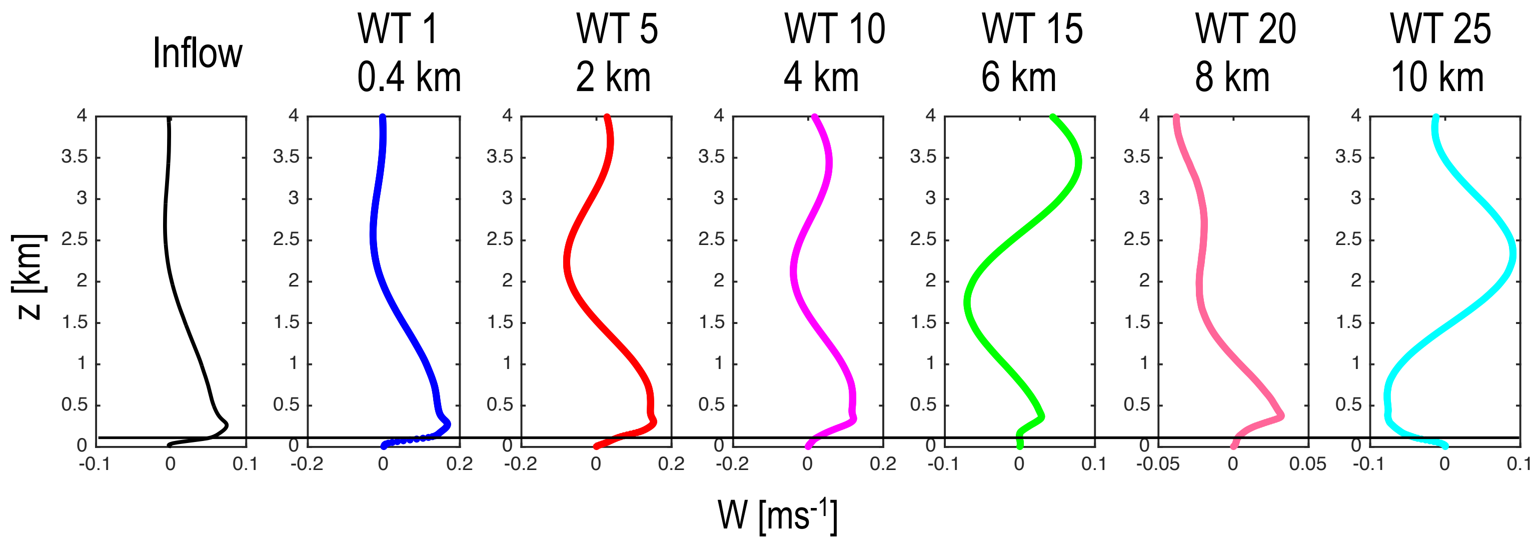
Turbulence Kinetic Energy

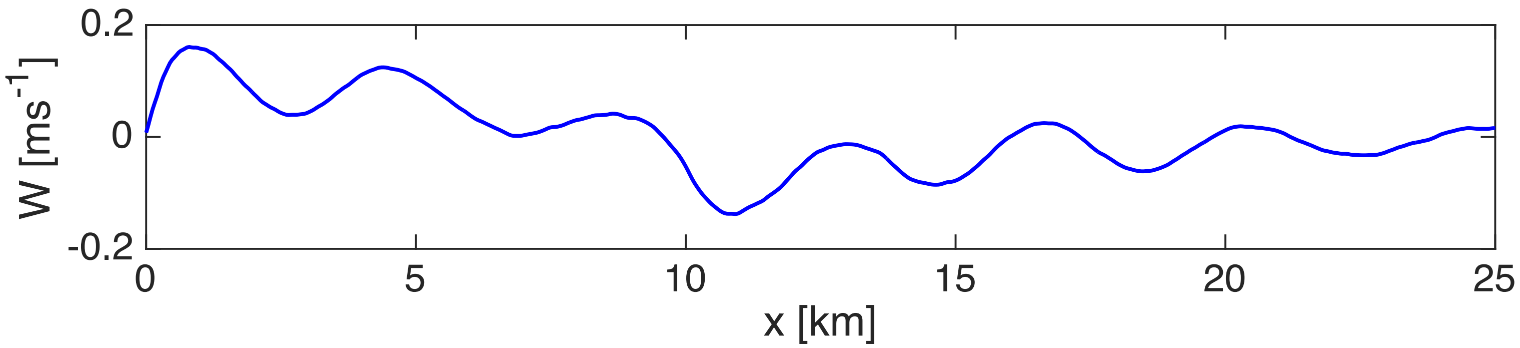
↑ TKE



Gravity Wave Generation

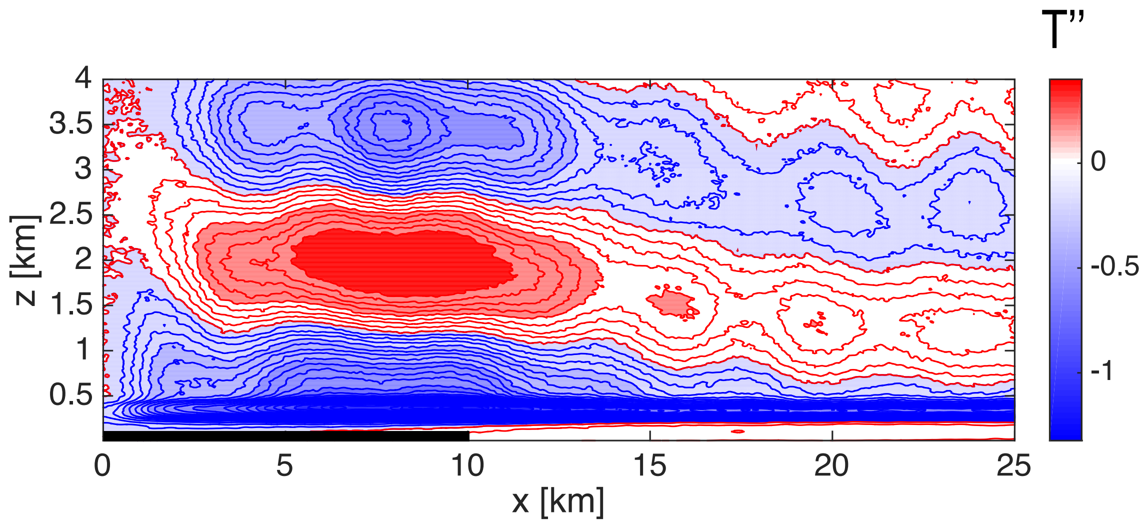
Velocity Profile at Different Locations





At z = 0.4 km

Temperature Perturbation from Inflow



Horizontal Wave Length

~4000m

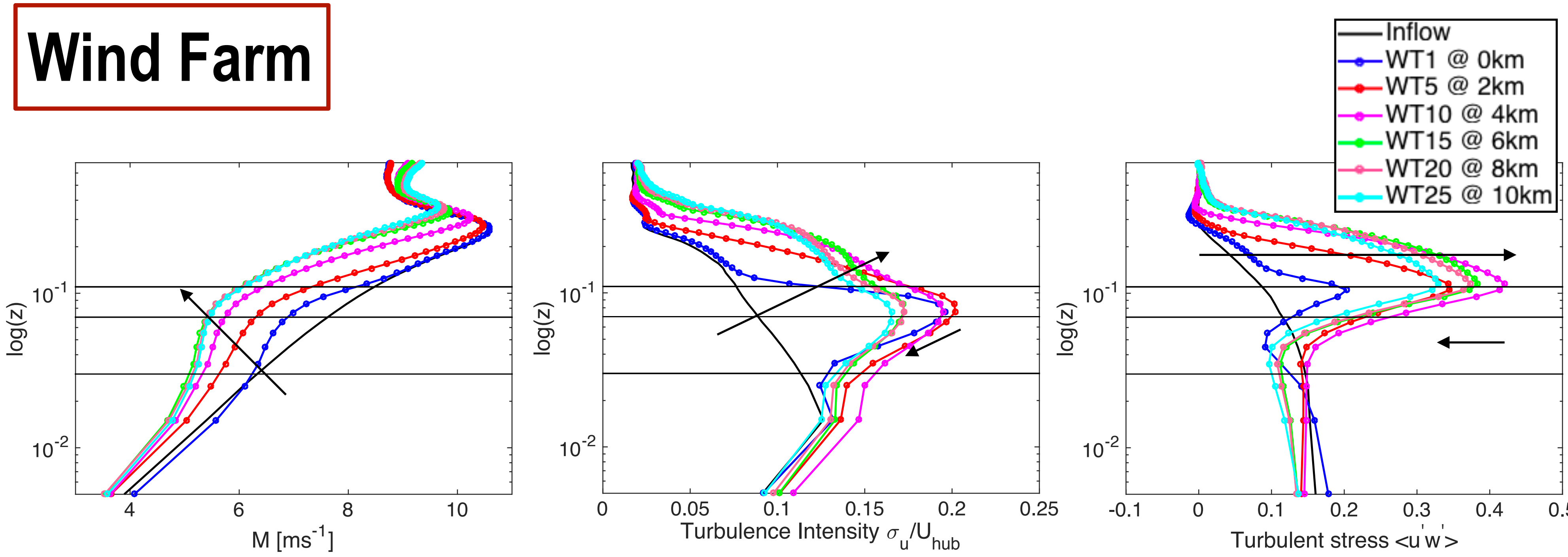
Vertical Wave Length

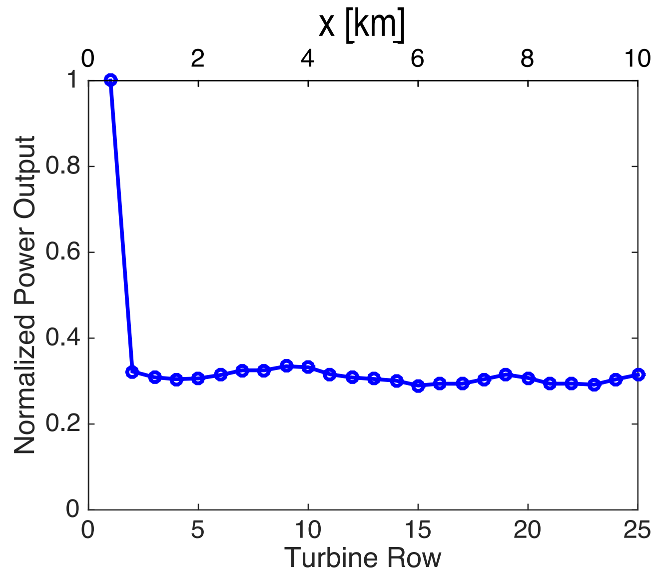
~3000m

Similar to linear mountain wave approximation

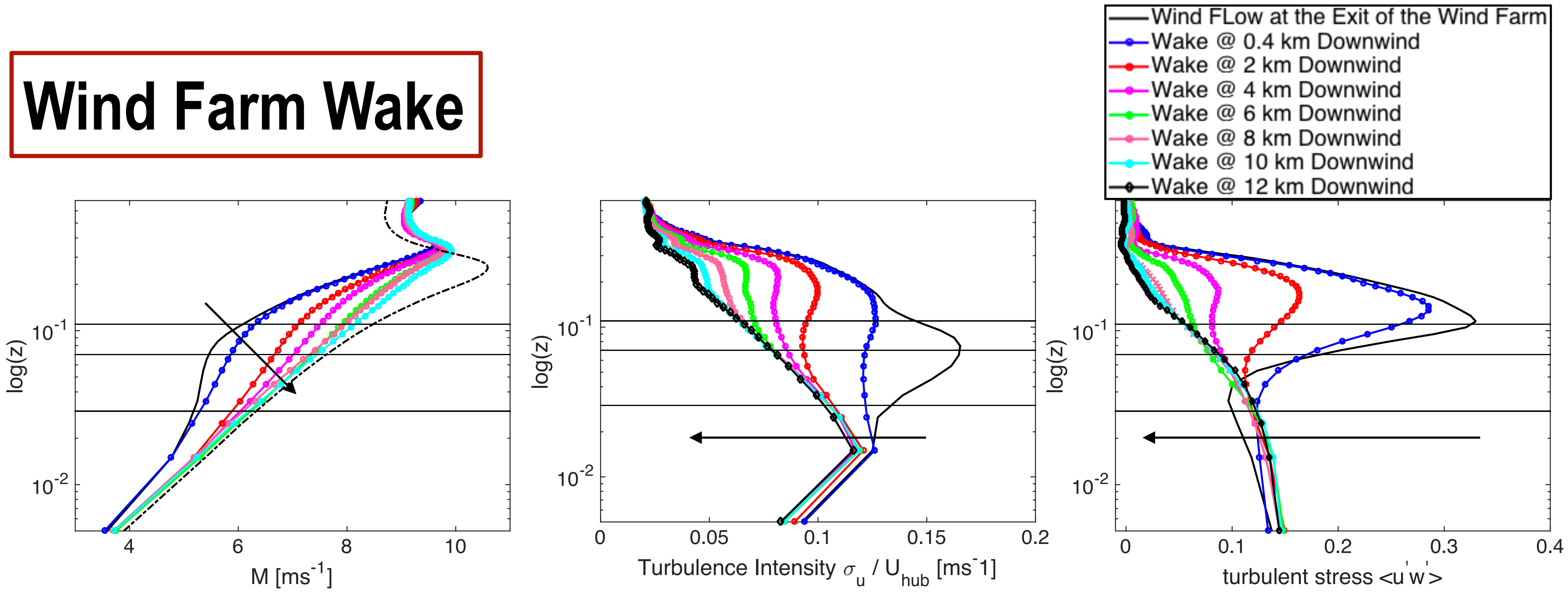
Flow Statistics of the Wind Farm and its Wake

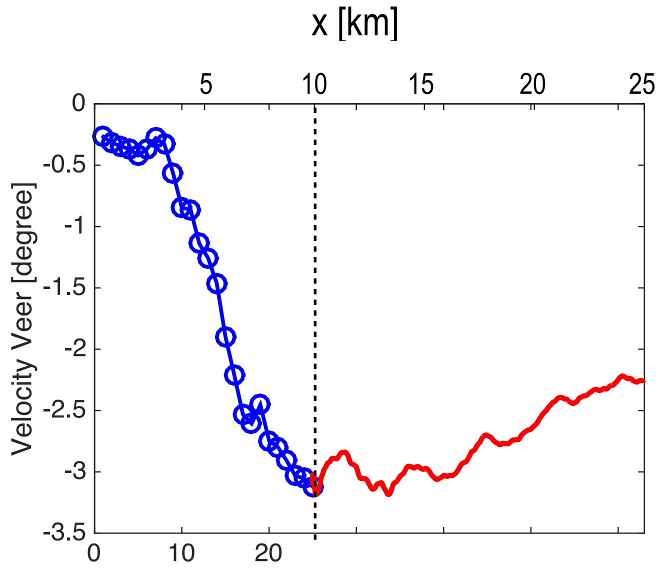
Wind Farm





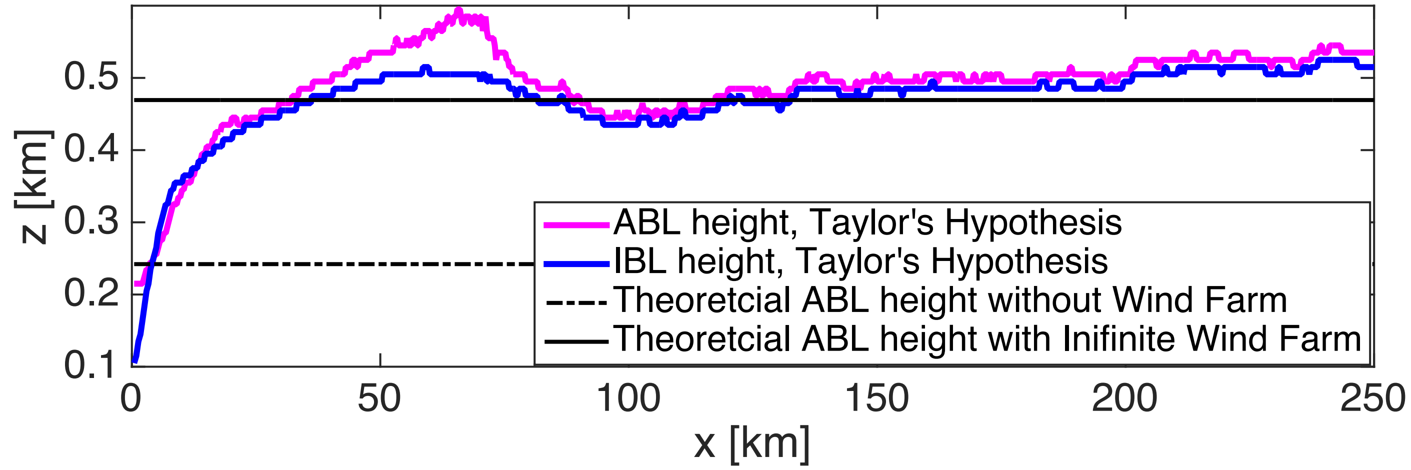
Wind Farm Wake

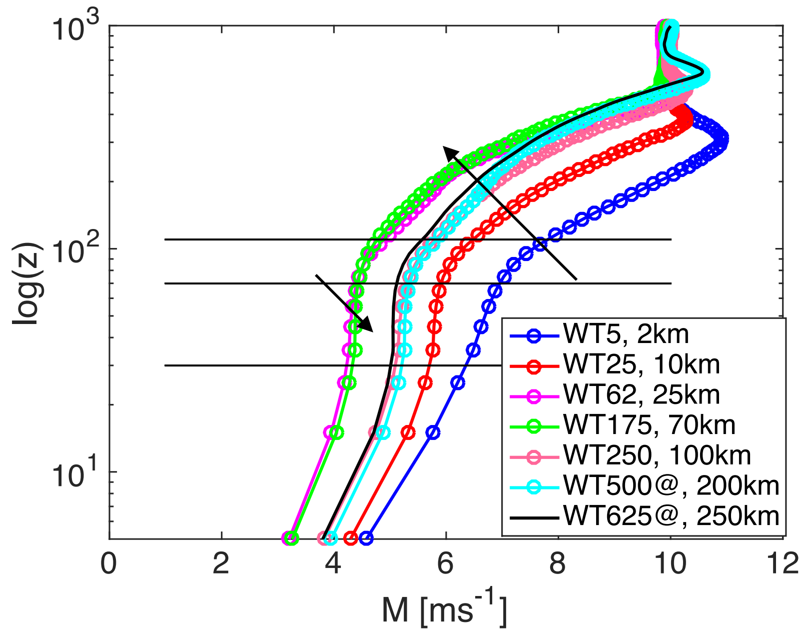


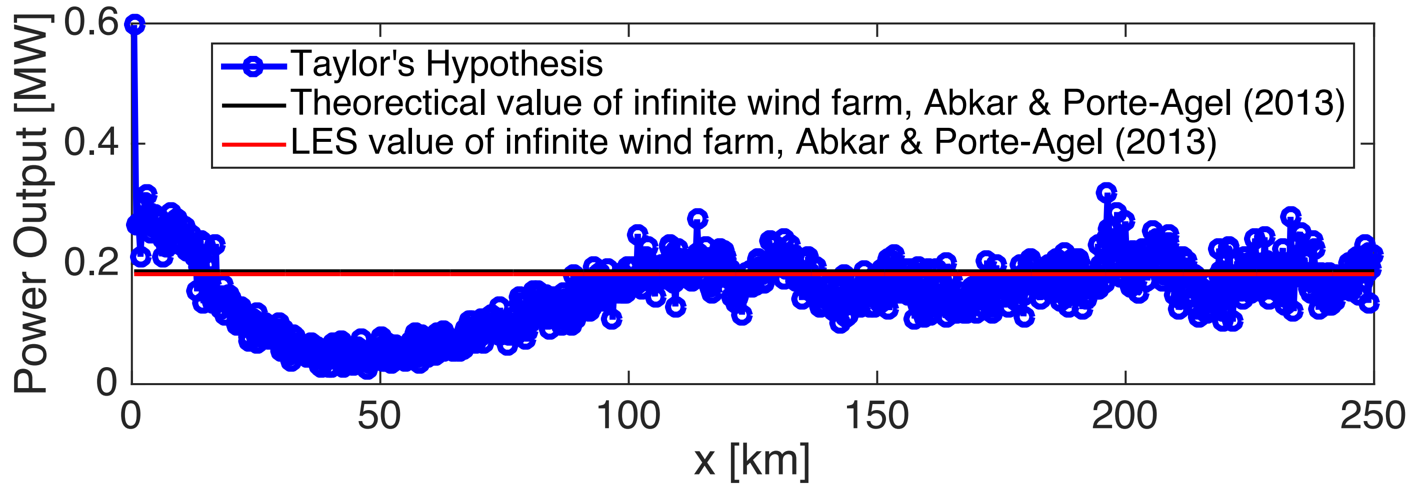


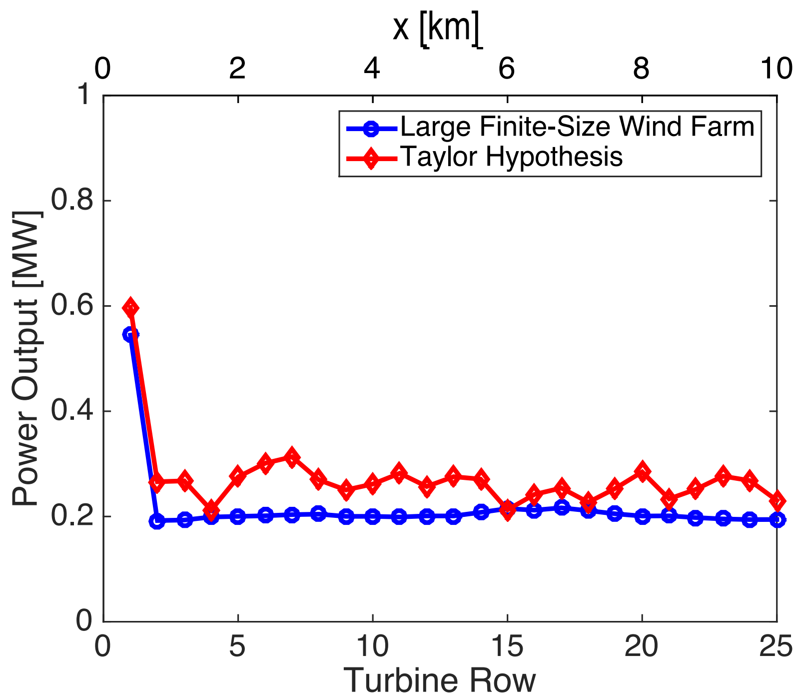
Boundary Layer Growth Estimation

Taylor's Hypothesis: connect the scale of space with time using wind advection speed at the hub height of wind turbines





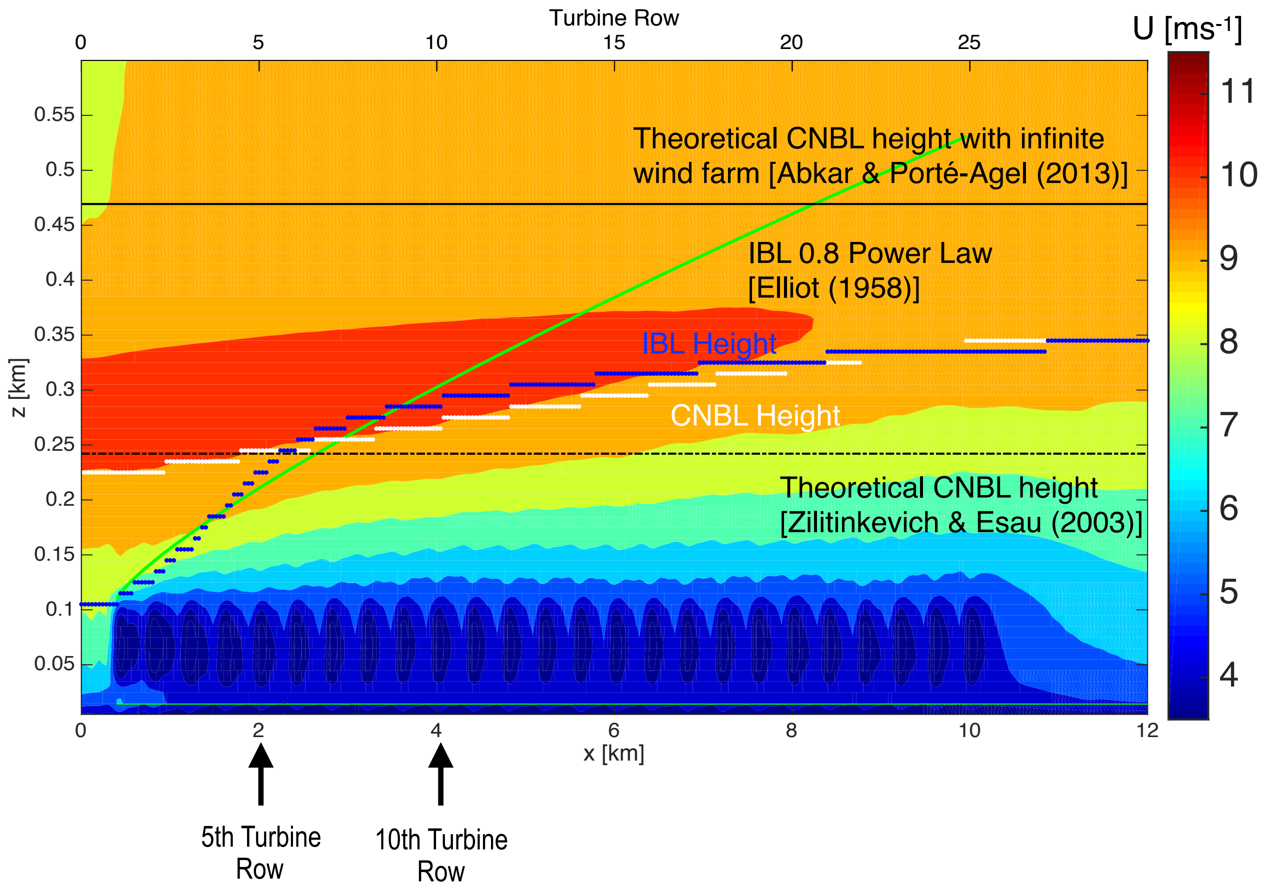




Conclusions and Future Work

- Boundary Layer Growth:

  The IBL first grows following Elliot's 0.8 Power Law. When it reaches the height of the CNBL, the CNBL acts as a lid and damps the IBL growth rate. After a transient period, the CNBL and the IBL grow together at a steady rate that is lower than the initial IBL growth rate
- Large finite-size wind farm's CNBL height is lower than the theoretical infinite wind farm's CNBL height
- Gravity waves are induced by the large finite-size wind farm



At 10 km downstream of the large finite-size wind farm:

- ~2.4% velocity deficit comparing to the inflow
- ~7% wind power loss

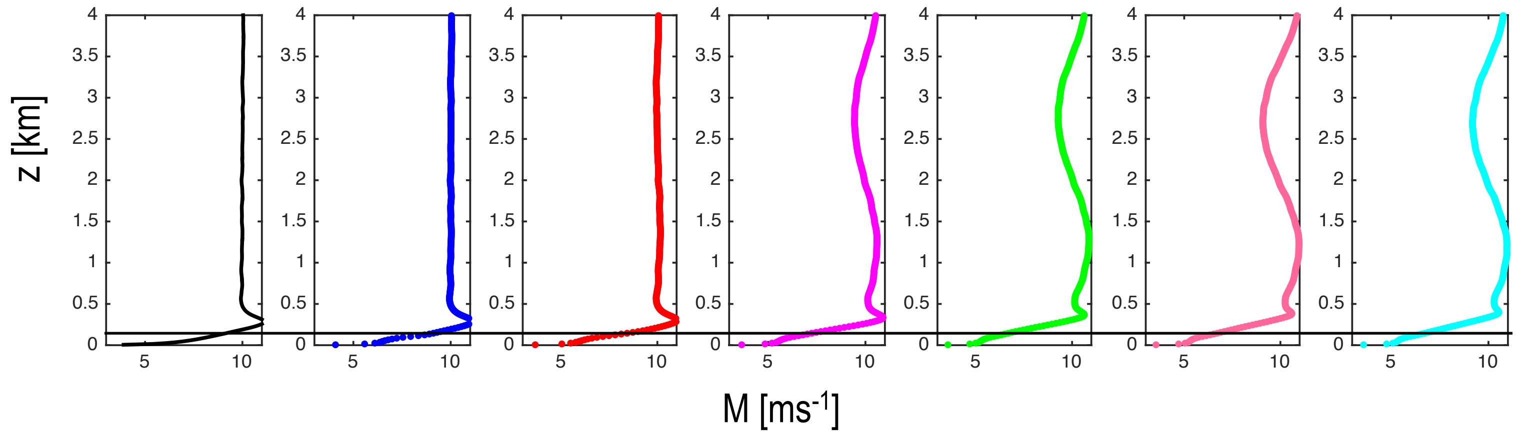
Future Work

- Investigate large finite-size wind farms that are longer than 15 km
- Develop an one-dimensional analytical model to predict velocity profiles inside the large finite-size wind farm and the wake of the wind farm

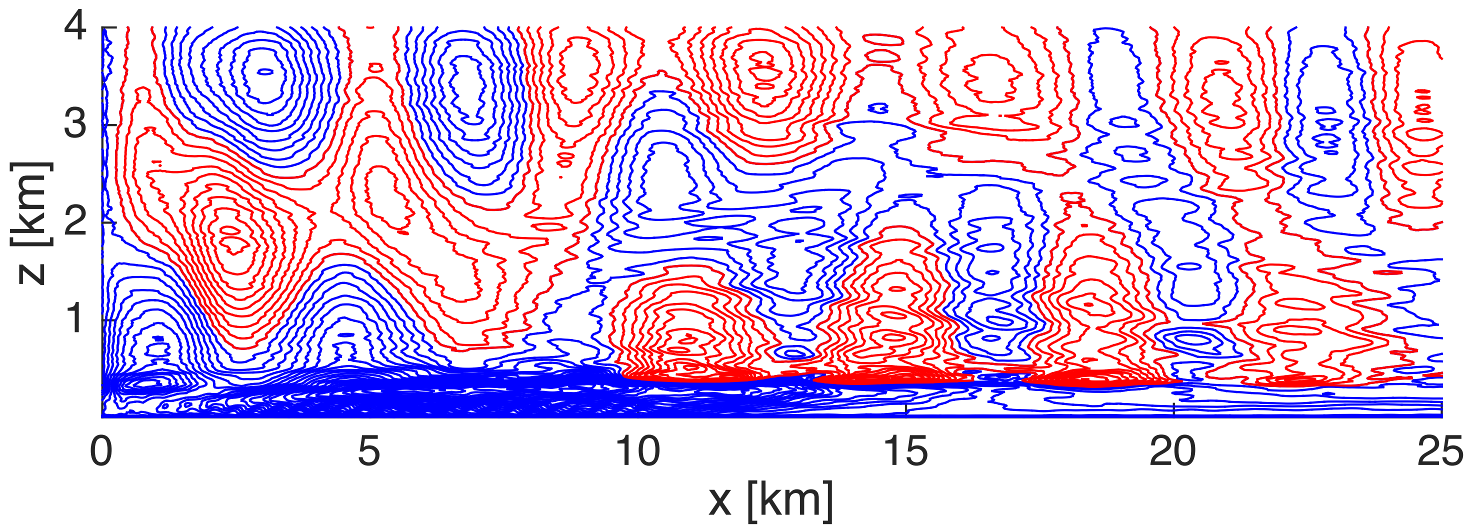
Questions?

Presentation Outline

Introduction	• Large Finite-Size Wind Farms • Objectives
Methodology	• Large-Eddy Simulations • Inflow Conditions and Setup
Boundary Layer Growth	• Atmospheric and Internal Boundary Layers (ABL & IBL) • Gravity Wave Generation
Flow Statistics	• Inside the Large Finite-Size Wind Farm • In the Wake of the Wind Farm
Boundary Layer Growth Forecast	
Conclusions	



z [km]
M [ms-1]



z [km]
x [km]

References

1. Abkar, Mahdi, and Fernando Porté-Agel. "The effect of free-atmosphere stratification on boundary-layer flow and power output from very large wind farms." *Energies* 6.5 (2013): 2338-2361.
2. Elliott, William P. "The growth of the atmospheric internal boundary layer." *Eos, Transactions American Geophysical Union* 39.6 (1958): 1048-1054.
3. Zilitinkevich, S. S., V. L. Perov, and J. C. King. "Near−surface turbulent fluxes in stable stratification: Calculation techniques for use in general−circulation models." *Quarterly Journal of the Royal Meteorological Society* 128.583 (2002): 1571-1587.