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Neutral Equivalent Surface Stress

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Background

Sonic anemometers are not always mounted perfectly vertically during field campaigns. Correcting for the slight tilt is often important to obtaining more accurate turbulent flux calculations, especially momentum flux, which is particularly sensitive to instrument tilt. Over a planar surface, this correction is obtained from the WOS planar tilt correction (Wizcak et al 2001) using data from the entire deployment.

In a complex environment, the WOS method cannot be used. Other methods are problematic and none are currently as universally accepted as WOS. My goal has been to find a momentum flux computation for use in complex environments which does not need information about sonic tilt or surface slope information.

This study will focus on evaluating the new alternative over simple terrain using the CASES99 main tower data. This was chosen due to the high confidence in standard u_* computation which will be used as a ground truth point of reference for the performance of my alternative method.

CASES99 Field Campaign

The CASES99 field campaign's main tower was located in a flat rural area near Leon, KS, USA, and took place in Oct. 1999 (Poulos et al 2002). More info and data are available from NCAR https://www.eol.ucar.edu/projects/cases99/.

Surface stress/momentum flux

A textbook equation for surface stress is $\rho u_*^2 = -\rho\overline{u'w'}$. In atmospheric flows, to account for complexities such as the turning of the wind vector, we also use

$$u_*^2 = \left(\overline{u'w'}^2 + \overline{v'w'}^2\right)^{1/2} \tag{1}$$

This treats the two terms $\overline{u'w'}$ and $\overline{v'w'}$ as if they were orthogonal components of a 2D vector, when in fact they are just two of the six components of the Reynolds stress tensor. Within the Reynolds stress tensor, $\overline{u'w'}$ and $\overline{v'w'}$ behave like a 2D vector only under rotations around the vertical axis.

Matrix Properties

Symmetric matrices with real numbers for entries fall into the category of Hermitian matrices. This class of matrices has many physical applications, such as for quantum mechanics, and are therefore well studied. All eigenvalues of Hermitian matrices are real, and the eigenvectors corresponding to distinct eigenvalues are mutually orthogonal.

$$\overline{u'_i u'_j} = \begin{pmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{u'v'} & \overline{v'v'} & \overline{v'w'} \\ \overline{u'w'} & \overline{v'w'} & \overline{w'w'} \end{pmatrix} \Rightarrow \begin{pmatrix} \lambda_B & 0 & 0 \\ 0 & \lambda_M & 0 \\ 0 & 0 & \lambda_S \end{pmatrix}$$

For the Reynolds stress tensor, this means that the eigenvalues, λ_B, λ_M, and λ_S, may be thought of as fundamental variances acting in the directions of the corresponding eigenvectors, Λ_B, Λ_M, and Λ_S. Since these eigenvectors are mutually orthogonal, they form a coordinate system where the turbulent Reynolds tensor has only diagonal elements, no covariances.

This would seem to imply that there is a coordinate system where $\overline{u'w'} = 0$, however, $\overline{u'w'}$ is only equal to the surface stress for one very specific coordinate system, one in which w is perpendicular to the underlying surface. This is the reason that surface stress measurements are so sensitive to instrument tilt. If the sonic is tilted to more closely align with the eigen-coordinate system, the measured $\overline{u'w'}$ will be closer to 0. If the sonic is tilted away from the eigen coordinates, then the measured $\overline{u'w'}$ will be larger than the surface stress.

The purpose of planar tilt correction is to convert the data into a coordinate system where the direction of the w data aligns with the wall normal direction. This is relatively straight forward for locations over relatively flat terrain. In urban canyons and over complex terrain there is no single wall normal direction.

Another important property to remember is that the eigenvalues and eigenvectors are invariants. This means that the eigenvalues will be the same and eigenvectors will point in the same direction no matter what coordinate system your data is in. You can use the data in the form it comes off the sonic anemometer with u and v oriented in a way that is advantageous for the site, or you can use the same data after it has been rotated so that u is east-west and v is north-south, or you can use data rotated into a streamwise coordinate system with $\overline{V} = \overline{W} = 0$. The resulting eigenvalues will be identical for the same time block no matter what coordinate system the data is presented in. The eigenvectors will point the same direction in space no matter what coordinate system the data is presented in, but might look different since the reference axes are different for each coordinate system.

As a side note, another matrix invariant is the trace, the sum of the diagonal elements from upper left to lower right. The turbulence kinetic energy is an invariant of the Reynolds stress tensor since it is equal to half the trace.

Quantify Inclination

The relative inclination between the eigen-coordinate system and the streamwise system varies as a function of distance from the surface, averaging time used to calculate the variances and covariances, and how the inclination angle is measured. For laboratory flows, the commonly reported number is 17° (Hanjalic and Launder 1973). This is for 2D flows where there is only one angle to define the difference between the eigen coordinates and streamwise coordinates. Atmospheric flows are rarely 2D. Fig. 1 shows two methods of defining an angle of inclination. The angle between the eigenvector associated with the smallest eigenvalue and the wall normal direction, which is parallel to the gravity vector for the CASES99 data, is labeled α. The angle between the direction of the mean 3D wind vector and the plane defined by the eigenvectors associated with the two larger eigen-values is labeled β. Since atmospheric flows are rarely exactly 2D even for simple terrain, α and β are rarely equal to each other, but are often close in value. In general, β will be preferred because it is defined using only sonic anemometer data and does not need information about how the sonic is oriented with respect to its environment.

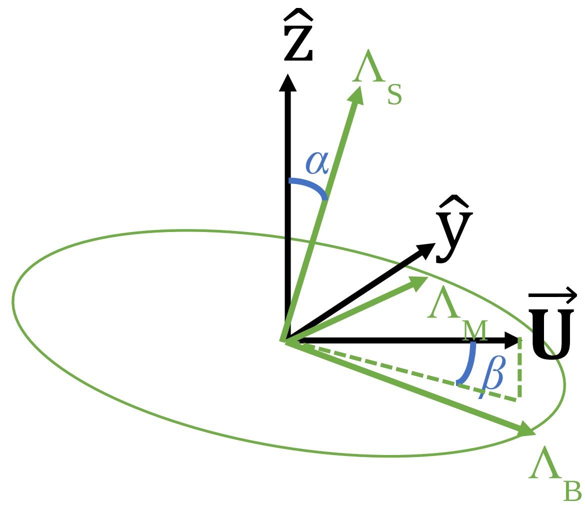

Figure 1: Streamwise coordinate directions compared to eigen coordinate directions and two possible ways to quantify the relative inclination angle

Ideal 2D flow

In 2D laboratory flows the Reynolds stress tensor is of the form

$$\begin{pmatrix} \overline{u'u'} & 0 & \overline{u'w'} \\ 0 & \overline{v'v'} & 0 \\ \overline{u'w'} & 0 & \overline{w'w'} \end{pmatrix}$$

which can be obtained from the diagonalized matrix by rotating the eigen coordinate system by β (or α) degrees around the cross-stream axis. For ideal 2D flow, this procedure recovers the streamwise coordinate system. The same procedure can be applied to diagonalized Reynolds stress tensors from 3D flows, yielding a term in the upper right (and lower left) of $(\lambda_S - \lambda_B)\sin\beta\cos\beta$. By equating this with the term $\overline{u'w'}$, we can now calculate a surface stress.

$$u_*^2 = -(\lambda_S - \lambda_B)\sin\beta\cos\beta \tag{2}$$

Note that since this calculation is based on invariants of the Reynolds stress tensor, the results are independent of sonic tilt. By using β instead of α, the results needed no information about how the sonic was oriented.

Data that closely resembles laboratory flows are used to compare eq. 2 to the traditional eq. 1. The data have near neutral stability as measured from a bulk temperature difference from the thermistors at 5m and 55m on the main CASES99 tower such that -0.005 C°m^{-1} < $(T_{55} - T_5)/50$ < 0.02 C°m^{-1}. In addition, only data from when the winds were greater than a threshold value are used. The threshold wind was determined from plots of β calculated from one hour blocks of data as a function of one hour mean wind speed. For wind speeds below threshold, β values vary from 0 to greater than 50°, while for wind speeds above threshold, β values are greater than 5° and are relatively close to the mean β value. The thresholds are listed in Table 1 and are best fit by a $U^{1/2}$ relationship. These threshold values are remarkably close to the thresholds reported in Sun et al (2012) derived from the same CASES99 data but by using plots of $TKE^{1/2}$ as a function of wind speed. Klipp (2014) also reports similar thresholds for 5m and 50m from anisotropy characteristics.

From the scatter plot (fig. 2) of neutral equivalent u_* (u_{*ne}, eq. 2) and standard u_* (u_{*st}, eq. 1) for neutral conditions, it is clear that the two are nearly identical. From the scatter plot of u_{*ne} and u_{*st} for above threshold winds with stable and unstable thermal stability (fig. 3), the equivalence is much lower, especially for unstable conditions. From the plot of u_{*ne} and u_{*st} for slower than threshold winds (fig. 4), it is clear that the correspondence does not hold except for the neutral stability data. This supports the idea that eq. 2 is a neutral equivalent stress.

Table 1: Threshold wind speeds

Elevation (m)	Threshold (ms^{-1})
0.5	1.6
1.5	2.1
5	3.4
10	4.3
20	6.0
30	7.1
40	8.2
50	9.2
55	10.3

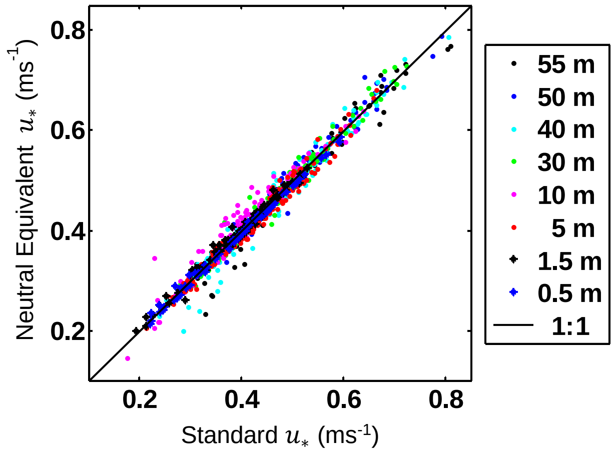

Figure 2: Scatter plot of neutral equivalent u_* (Eq 2) and standard u_* (Eq 1) for near neutral conditions with wind speeds faster than threshold

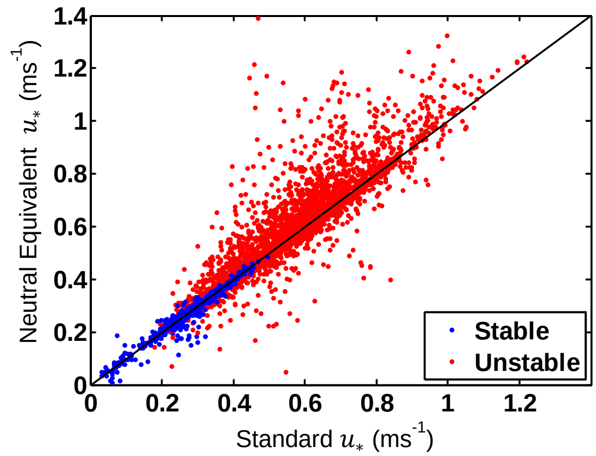

Figure 3: Scatter plot of neutral equivalent u_* (Eq 2) and standard u_* (Eq 1) for non-neutral conditions with wind speeds faster than threshold

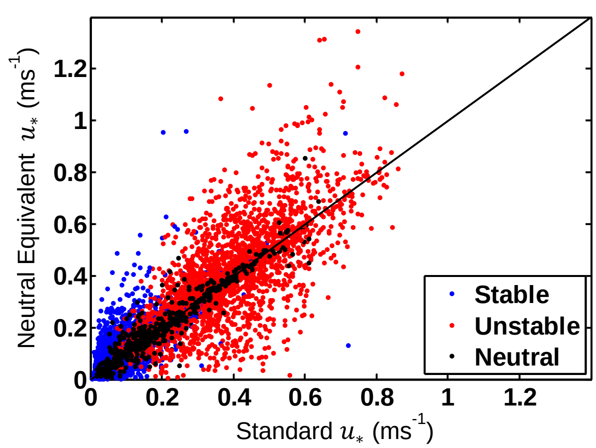

Figure 4: Scatter plot of neutral equivalent u_* (Eq 2) and standard u_* (Eq 1) for all stabilities with wind speeds slower than threshold

Monin-Obukhov Similarity

One major use of u_* is as a scaling term in Monin-Obukhov surface layer similarity. For the above threshold wind neutral data where $u_{*ne} = u_{*st}$ using either should give essentially the same results. For other conditions where the two terms are not identical, there could be a large impact on the flux gradient relationship in $\phi_m(z/L)$.

$$\phi_m = \frac{\kappa z}{u_*}\frac{\partial U}{\partial z}$$

$$\frac{z}{L} = \frac{\kappa z g}{\overline{T_s}}\frac{\overline{w'T'}}{u_*^3}$$

From the scatter in plots 5a and 5b, it is seen that for faster than threshold winds, the same relationship holds with comparable scatter whether one uses u_{*ne} or u_{*st} even for non-neutral conditions where $u_{*ne} \neq u_{*st}$. Plots 6a and 6b, show that even for winds slower

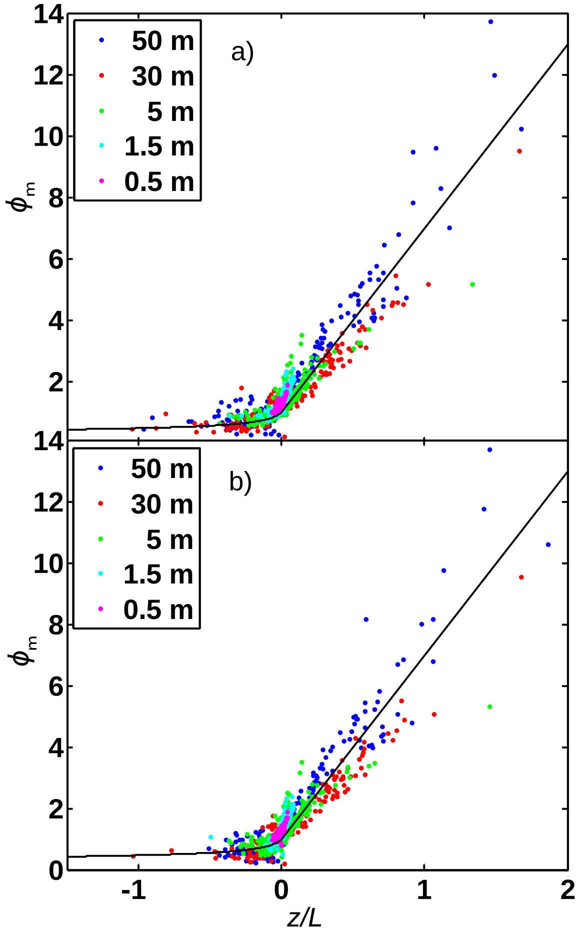

Figure 5: Monin-Obhukov dimensionless shear as a function of z/L for all stabilities with wind speeds faster than threshold a) Using standard u_* and b) Using neutral equivalent u_*

than threshold, the relationship and scatter is the same using either version of u_* in the scaling relationship. Note that only data from the CSAT3 anemometers are used. The line is from Högström (1988) and is used here to provide context.

Also note that use of eq. 2 instead of eq. 1 does not remove the possibility of self-correlation due to the same scaling term in the denominator of both ϕ_m and z/L (Klipp and Mahrt 2004).

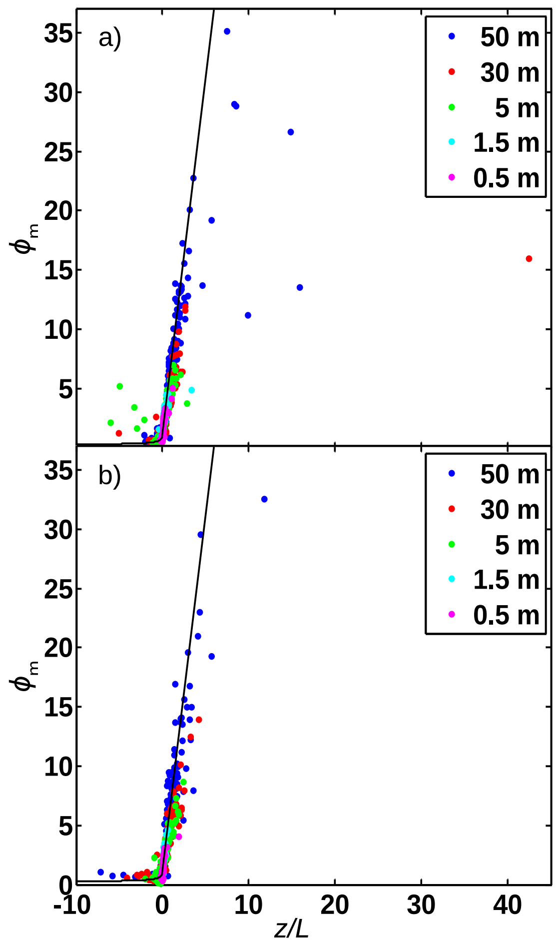

Figure 6: Monin-Obhukov dimensionless shear as a function of z/L for all stabilities with wind speeds slower than threshold a) Using standard u_* and b) Using neutral equivalent u_*

Complex, not difficult

Although it seems complex, the process is not significantly longer than the traditional method where one not only calculates fluxes, but must account for instrument or streamline tilt, and rotate into mean wind coordinates before calculating u_*.

1. Calculate variances and covariances (fluxes) as well as the 3D wind vector, preferably in sonic anemometer coordinates before any tilt correction. Although you can start with data in any coordinate system, it is important that the fluxes and wind vector be in the same coordinates. Also, use the same averaging time (don't mix 5 min fluxes and 30 min mean winds). It is easiest to either use the quality controlled data in the original sonic anemometer coordinates without any tilt correction or to rotate the quality controlled data into streamwise coordinates before calculating fluxes and means.
2. Use your favorite math package software (ie Matlab, NumPy, etc) to calculate the eigenvalues and eigenvectors using the variances and covariances.
3. Use the eigenvectors and 3D mean wind vector to calculate β.
4. Use β and the eigenvalues to calculate the neutral equivalent stress.

Conclusions

For data over relatively simple terrain, calculating u_* from either eq. 1 or eq. 2 yields the same value for near neutral stability. For non-neutral conditions, there is significant difference in the values from eq. 1 and eq. 2, especially for slower wind speed conditions. None the less, eq. 2 yields just as useful a scaling term as eq. 1 even when $u_{*ne} \neq u_{*st}$. Although calculation of eq. 2 is more complex, it does not require the researcher to try to rotate the sonic data into a wall normal coordinate system. This sets the stage for the neutral equivalent stress (eq. 2) to be used in situations where planar tilt correction cannot be done or where the wall normal direction is either poorly known with respect to the sonic anemometer orientation or more than one wall normal influences the flow.

One shortcoming is that this analysis only applies to calculating the local stress, and does not apply to calculating the flux of scalars.

Bibliography

Hanjalic, K., and B. E. Launder, 1972: “Fully developed asymmetric flow in a plane channel.” *J. Fluid Mech.*, **51**, 301 – 335.

Högström, U., 1988: Non-dimensional wind and temperature profiles in the atmospheric surface layer: A re-evaluation. *Bound.-Layer Meteor.*, **42**, 55 – 78.

Klipp, C. L., 2014: Turbulence anisotropy in the near-surface atmosphere and the evaluation of multiple outer length scales. *Bound.-Layer Meteor.*, **151**, 57 – 77.

Klipp, C. L. and L. Mahrt, 2004: Flux–gradient relationship, self-correlation and intermittency in the stable boundary layer. *Q. J. R.Meteorol. Soc.*, **130**, 2087–2104

Poulos, G. S., Blumen, W., Fritts, D. C., Lundquist, J. K., Sun, J., Burns, S. P., Nappo, C., Banta, R., Newsom, R., Cuxart, J., Terradellas, E., Balsley, B., and Jensen, M., 2002, "CASES-99: A Comprehensive Investigation of the Stable Nocturnal Boundary Layer," *Bulletin of the American Meteorological Society,* vol. 83(4), pp. 555-581.

Sun, J., L. Mahrt, R. M. Banta, and Y. L. Pichugina, 2012: Turbulence regimes and turbulence intermittency in the stable boundary layer during CASES-99. *J. Atmos. Sci.*, **69**, 338 – 351.

Wilczak, J. M., S. P. Oncley and S. A. Stage, 2001: Sonic anemometer tilt correction algorithms. *Bound.-Layer Meteor.*, **99**, 127-150.