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FORCED, BALANCED MODEL OF TROPICAL CYCLONE INTENSIFICATION

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1. INTRODUCTION

"If we should observe a hurricane, we might ask ourselves, 'Why did this hurricane form?'," wrote Edward Lorenz (1960). He concluded that the formation of a hurricane in a model with the exact equations and perfect initial conditions does not address the original question. To unravel the nature of atmospheric phenomena such as tropical cyclones, we must understand the relative importance of terms retained and omitted in simplified systems of equations.

This principle illustrates that the tropical cyclone intensification mechanism for an inviscid, axisymmetric vortex is the radial contraction of absolute angular momentum (M) surfaces by a diabatically forced transverse circulation (see Fig. 1). The Eliassen (1951) balanced vortex equations show this process in the context of a vortex that evolves from one gradient balanced state to another. Eliassen's equations provide meaningful insights into the distribution of inertial stability's influence on intensification rate (e.g., Shapiro and Willoughby 1982; Schubert and Hack 1982) and the existence of vortex features (e.g., eyewall tilt and warm ring thermal structures).

To gain additional understanding into tropical cyclone intensification processes, analytical solutions of the time evolving balanced vortex would be useful. Schubert et al. (2016) present time-dependent solutions of an axisymmetric, one-layer, balanced vortex using the wave-vortex approximation (Salmon 2014). Section 2 presents an overview of how that model is derived. Section 3 contains results from a two-region (core and far-field) model to examine "incubation" and a multi-region model to examine the inward movement of M surfaces across the radius of maximum wind and the rate of contraction and intensification. Section 4 concludes with remarks on the insight gained from and future extensions to this work.

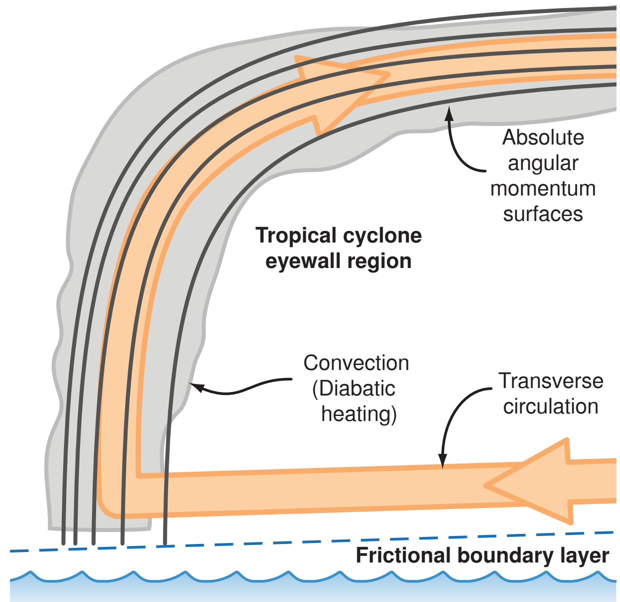

FIG. 1. A schematic showing a cross-section of a tropical cyclone for the inviscid flow above the frictional boundary layer. The convectively forced (diabatic heating) transverse circulation drives a radial contraction of absolute angular momentum (M) surfaces.

2. FORCED, BALANCED MODEL

The axisymmetric shallow water primitive equations on an f-plane are

$$\frac{\partial u}{\partial t} - (f+\zeta)v + \frac{\partial}{\partial r}\left[gh + \frac{1}{2}\left(u^2+v^2\right)\right] = 0, \quad (1)$$

$$\frac{\partial v}{\partial t} + (f+\zeta)u = 0, \text{ and} \quad (2)$$

$$\frac{\partial h}{\partial t} + \frac{\partial (ruh)}{r\,\partial r} = 0, \quad (3)$$

where u is the radial wind, v the tangential wind, h the fluid depth, g the acceleration due to gravity, f the Coriolis parameter, and $\zeta = \partial(rv)/r\partial r$ the relative vorticity.

To derive the wave-vortex approximation, the shallow water dynamics are expressed in the equivalent Hamiltonian form. The kinetic energy in the fluid column, $\frac{1}{2}\left(u^2+v^2\right)h$, is then approximated by replacing h with $\bar{h}$, the fluid depth as $r \to \infty$. This assumption constrains interpretation to flow speeds much less than $c = (g\bar{h})^{1/2} =$ 250 m s^{-1}, i.e., low Froude number. Next, a mass sink term, S, is added to the continuity equation and inertia-gravity waves are filtered. The forced, balanced model takes the form

$$Pv = g\frac{\partial h}{\partial r}, \quad (4)$$

$$\frac{\partial v}{\partial t} + (f+\zeta)u = 0, \text{ and} \quad (5)$$

$$\frac{\partial h}{\partial t} + \frac{\partial (ruh)}{r\,\partial r} = -hS, \quad (6)$$

where P is the potential vorticity defined as

$$P = \frac{\bar{h}}{h}(f+\zeta) \quad (7)$$

and the potential vorticity equation is

$$\frac{DP}{Dt} = SP. \quad (8)$$

Analytic solutions have been derived for a two-region model (see Schubert et al. 2016) and multi-region models

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in which an M surface is the interface separating regions. The position, $r_j(t)$, of each M surface, j, is initially undetermined. To find the time-dependent solutions for the M surfaces, the tangential wind, and fluid depth, the potential vorticity invertibility principle is solved analytically. The model is initialized by selecting initial radii for the M surfaces as well as prescribing a mass sink, S_j, for each region.

3. RESULTS AND DISCUSSION

The first experiment, see Fig. 2, uses the two-region model initialized with $r_1(0) = 400$ km and $S_1 = (48\text{ h})^{-1}$. As the potential vorticity increases between $r = 0$ and $r_1(t)$, the M surface is advected inward by the transverse circulation. The vortex takes ~5-days to develop Category 1 strength winds (~33 m s^{-1}) and then experiences rapid intensification. The long duration until the vortex intensifies can be considered "incubation." Ooyama (1969, section 15) discusses how a system can go through an extended period of incubation before experiencing a pressure drop and intensification, the hallmarks of tropical cyclogenesis. More recently, studies using full-physics models have explored convective aggregation and found similar incubation periods (e.g., Davis 2015). In the context of the two-region model, the contraction of the vortex is initially slow while potential vorticity develops.

The second experiment, see Fig. 3, demonstrates that how vortices traverse the (v, r)-plane to conserve M is dependent on the prescribed mass sink distribution. The orange vortex, with more convection in the outer regions, intensifies faster and develops a hollow potential vorticity structure. This could cause the vortex to stall its intensification until the inner M surfaces continue to contract, a scenario indicative of a system with little convection or dissipating convection in the core. While the blue vortex is contracting and intensifying slowly, outer M surfaces are not impeded from contracting by surfaces close to the core. The contrast in these two cases could affect whether a tropical cyclone reaches its potential intensity (see Emanuel 1986).

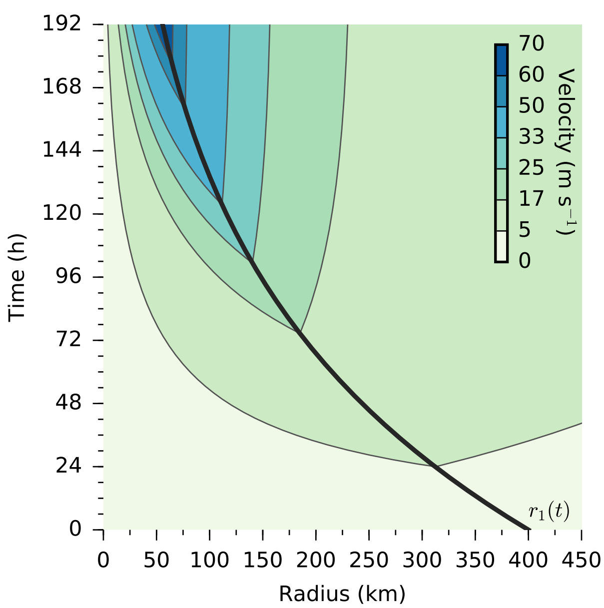

FIG. 2. Shaded contours of the tangential wind, $v(r,t)$, for the two-region model evolving from a resting state with the initial conditions $r_1(0) = 400$ km and $S_1 = (48\text{ h})^{-1}$. The thick solid line show the inward movement of the M surface, $r_1(t)$.

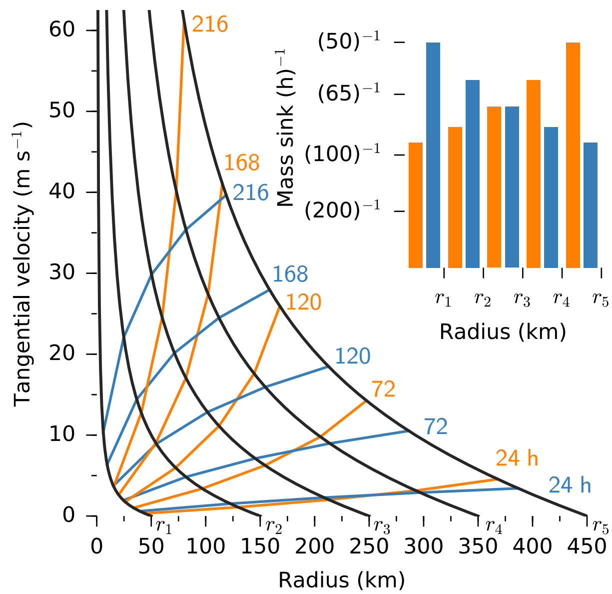

FIG. 3. A plot of M surfaces on a (v, r)-plane (black curves) with the orange and blue curves representing two vortex development cases at $t = 24$, 72, 120, 168, 216 h. The inset shows the distribution of the mass sink between M surfaces for each case (longer bars represent a stronger mass sink).

4. CONCLUSIONS

Time-dependent solutions of a forced, balanced model are offered to demonstrate tropical cyclone incubation and dynamical controls on intensification rate. As noted by Ooyama (1969), long incubation times can be problematic for tropical cyclogenesis. Often, tropical cyclone development is preceded by a preexisting vorticity anomaly. This effect could be demonstrated with the two-region model if a non-zero initial vorticity is used to "jump-start" the intensification process. During intensification, the radius of maximum wind does not need to be on a fixed M surface. Stern et al. (2015) note that the radius of maximum wind ceases to contract before peak intensity. Since there is not a one-to-one relationship between M and the radius of maximum wind, Smith and Montgomery (2015) explain that M surfaces can continue to contract. The multi-region model can be used to test the consistency of these arguments.

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