INSTABILITIES OF RADIATIVE CONVECTIVE EQUILIBRIUM WITH AN INTERACTIVE SURFACE

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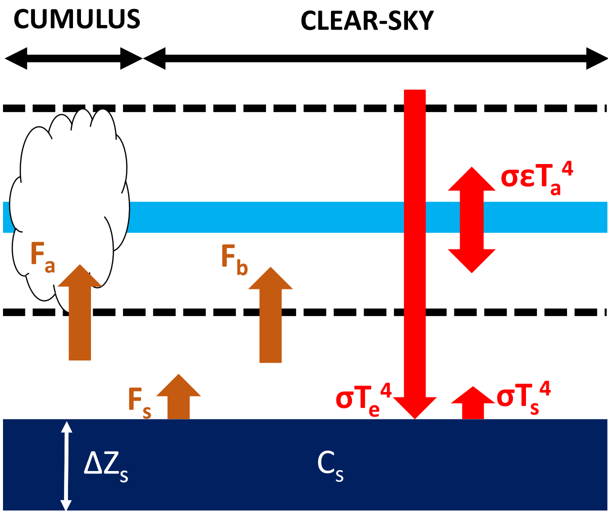

Fig. 1. One layer model of the tropical atmosphere. The surface is divided in two parts: the cumulus part, with deep moist convection, and the clear-sky part.

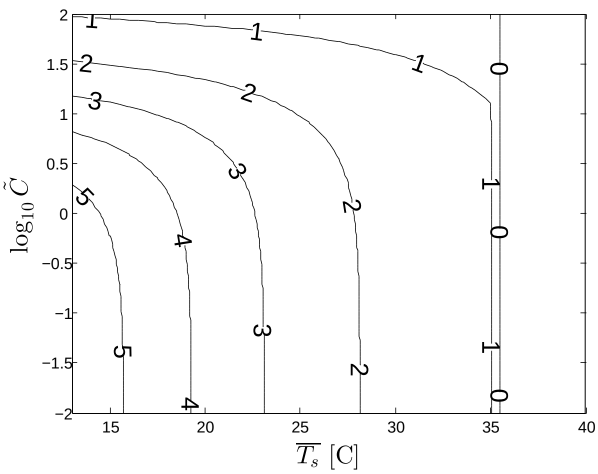

Fig. 2. $10^7\lambda_1$ in the parameter space: $\log_{10}\tilde{C}$ vs RCE surface temperature for the one layer model.

I. Introduction

One-dimensional radiative-convective models are arguably the simplest representations of the tropical atmosphere. Among them, moist radiative-convective equilibrium (RCE) is the equilibrium state of the atmosphere in the absence of lateral transport when radiation, convection, and water phase changes are taken into account.

It is accepted that RCE is unique when the greenhouse gas profile and the microphysics of the atmosphere are held fixed, except when the insolation is too high, in which case the runaway greenhouse effect can occur [Ingersoll, 1969], [Pujol and North, 2002]. However, when a large-scale circulation is allowed to develop, RCE has been shown to be conditionally unstable and to exhibit multiple equillibria [Renno, 1997], [Raymond and Zeng, 2000], even when homogeneous boundary conditions are imposed. The weak temperature gradient (WTG) framework [Sobel et al., 2001] has proven to be very useful to isolate this instability and study the possibility of multiple radiative-convective equillibria [Sessions et al., 2010], [Sessions et al., 2015], especially in a single column [Sobel et al., 2007], [Emanuel et al., 2014], where the effects of large-scale circulation are included by allowing large-scale vertical velocities to be generated.

The fact that RCE can go unstable has important climatic consequences. [Held et al., 1993], [Bony et al., 2015], [Bretherton et al., 2005], [Nolan et al., 2007] showed that aggregated convection leads to a warmer and drier troposphere in models, while [Tobin et al., 2012] observationally established that self-aggregation tended to dry the free troposphere and increase the outgoing longwave radiation. The dry bias of the RCE instability was further shown by the simulations of [Wing and Emanuel, 2014], [Boos et al., 2015], where the dry instability develops first, and the dry zone occupy a larger area than the moist one in the final state. Finally, the long relaxation times of coupled atmosphere-ocean RCE make simulations with cloud resolving models problematic. For that reason, all the previous studies have been realized with a fixed surface temperature, despite the importance of having an interactive surface to evaluate the RCE timescales [Cronin and Emanuel, 2013] and develop instabilities to large-scale circulations for a wider range of surface temperature [Nilsson and Emanuel, 1999].

This paper builds on the work of [Emanuel et al., 2014] and addresses two questions:

1) What are the different instabilities of RCE that can develop with an interactive surface?
2) What are the physical mechanisms of the dry bias of RCE instabilities?

It relies on key simplifying assumptions: We neglect the shortwave and the cloud feedbacks because we are mostly interested in the onset of the instability. We make the WTG

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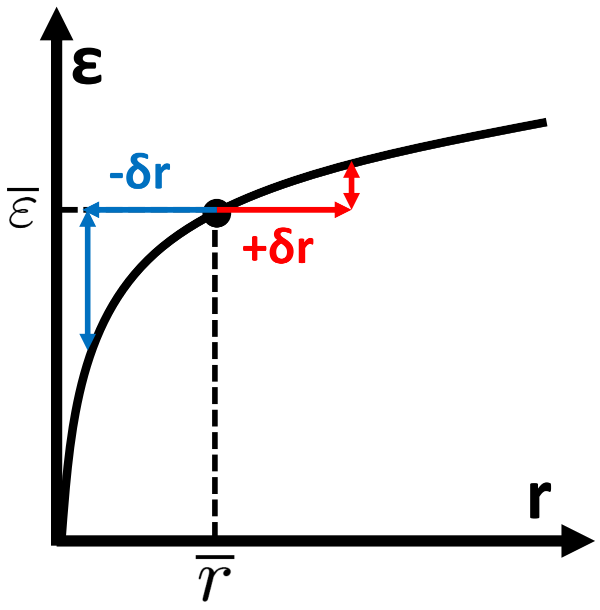

Fig. 3. Concave emissivity profile $\varepsilon(r)$. Starting from RCE (denoted by overlines), a positive moisture perturbation $+\delta r$ will result in a smaller absolute increase of the emissivity than the absolute decrease caused by a negative moisture perturbation $-\delta r$. Thus, dry perturbations to RCE will grow faster than moist perturbations to RCE.

approximation to focus on the key variable of the instability: the water vapor concentration. We also neglect convective gustiness, preventing the wind induced surface heat exchange effect, which can be important for self-aggregation.

II. Air-sea radiative-convective instability

The energy balance of the model presented on figure 1 is separated in two parts: convective (brown arrows) and radiative (red arrows). The convective fluxes are driven by the moist static energy (MSE) contrasts between the different levels of the model. We make the weak temperature gradient (WTG) approximation [Sobel et al., 2001], assume that the insolation is constant and that the atmosphere is transparent to shortwave radiation. The surface is assumed to be a homogeneous blackbody of temperature T_s and prescribed heat capacity per surface unit C_s. In the longwave domain, the atmosphere is assumed to be an isotropic gray body of temperature T_a and slab emissivity ε, which we take to be a given increasing function of the free-tropospheric water vapor mixing ratio r_a. The equations of the model are the energy budgets for each component of the system (surface heat equation, boundary quasi-equilibrium, vertically integrated MSE equation). We define RCE as the steady state of the equations with no large-scale vertical velocity. In this model, RCE is unique for a given insolation or equivalently a given RCE surface temperature. In RCE, the surface convective flux balances the radiative heating of the surface, equal to the radiative cooling of the atmosphere.

We are interested in the stability of RCE when the water vapor mixing ratio r_a and/or the sea surface temperature T_s are perturbed from their equilibrium values, and how the instability changes with the RCE surface temperature $\overline{T_s}$ and the heat capacity of the slab surface C_s. We perform the linear analysis of the equations of the one layer model, and find that the coupled atmosphere/ocean system is always linearly unstable. The growth rate of the instability is governed by the dimensionless surface heat capacity:

$$\tilde{C} \overset{\text{def}}{=} \frac{C_s T_{\text{ref}} g}{L_v r_{\text{ref}} \Delta p},$$

defined as the ratio of the typical internal heat content of the slab surface, to the typical latent heat content of the atmosphere. Figure 2 shows that the growth rate strictly decreases with the two free parameters of the one layer model: the surface heat capacity and the RCE surface temperature.

When the result of our linear analysis is compared to a full numerical integration of the one-layer model, we notice a small dry bias of the instability. We argue that the dry bias can be partially explained by the second order effect of the moisture perturbation on the surface heat equation, linked to the concavity of the emissivity as a function of the mixing ratio (see figure 3), a feature exhibited by all the theoretical and observational functions $\varepsilon(r)$.

The linear analysis helps us understanding the physical mechanisms of the air-sea instability. Figure 4 shows how an initial moisture perturbation can amplify and lead to a positive feedback between water vapor and surface temperature perturbations, explaining the occurrence of the air-sea instability, when the surface is allowed to be interactive.

III. Full radiative-convective instability

We generalize the one layer model by dividing the free-troposphere in two layers of equal mass per surface unit: the lower and the upper troposphere (cf figure 5). Allowing for an atmospheric vertical structure gives the possibility of an atmospheric radiative-convective instability. To isolate the atmospheric radiative-convective instability from the air-sea one studied in section II, we do not allow the air-sea instability to happen by fixing the surface temperature to its RCE value $\overline{T_s}$. We then study the stability of RCE when the lower and upper tropospheric mixing ratios are perturbed from their equilibrium values, and how this stability depends on the RCE surface temperature T_s. We perform the linear analysis of our system and show that RCE is always linearly unstable in our model. The growth rate is maximal for low and high RCE surface temperatures, with a minimum when the lower and upper net convective updrafts are of equal amplitude. We explain the different physical mechanisms of the instability for high (cf figure 6) and low RCE surface temperature. If we allow the surface temperature T_s to vary in time, both radiative-convective instabilities are present. We study the full instability as a function of the RCE surface temperature $\overline{T_s}$ and the dimensionless surface heat capacity $\tilde{C}$. We perform the linear analysis of the system; its growth rate, depicted on figure 7, is always positive, meaning that the system is always linearly unstable. Furthermore, the heat capacity of the surface only significantly affects the growth rate of the instability for low RCE surface temperatures,

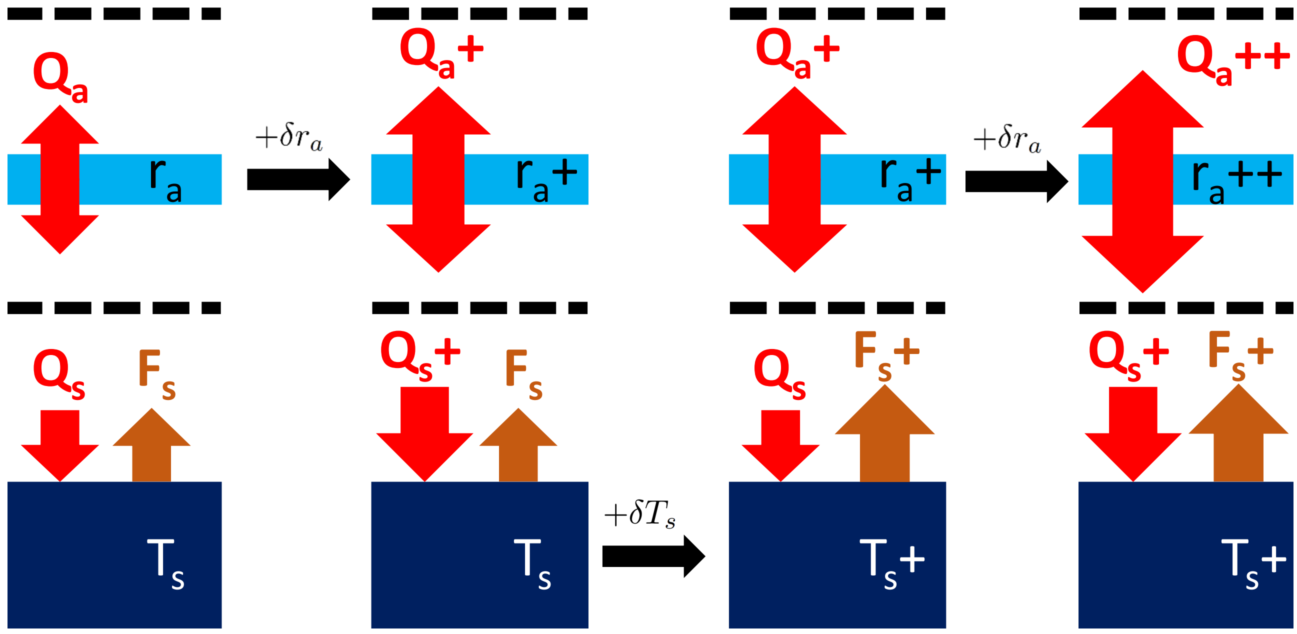

Fig. 4. Physical mechanism of the air-sea radiative-convective instability for a positive initial moisture perturbation.

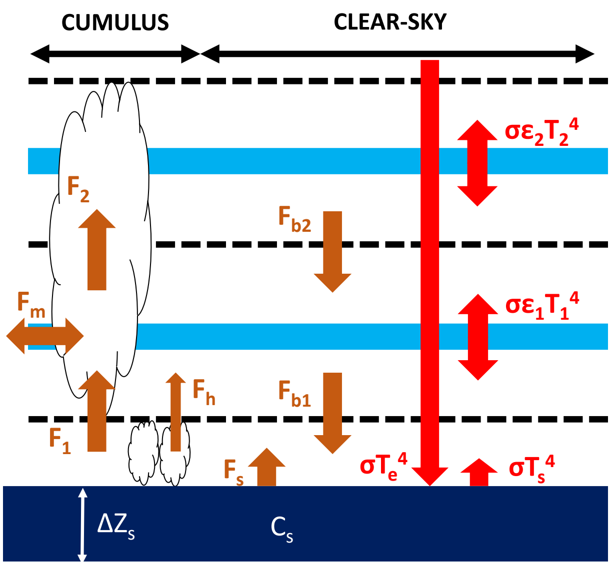

Fig. 5. Two layer model of the tropical atmosphere. The convective MSE fluxes are depicted in brown while the radiative ones are depicted in red. The surface is separated between the cumulus region, with deep moist convection, and the clear-sky region.

consistently with the fact that the air-sea instability described in section II is most intense for low temperatures.

IV. Single-Column Simulations

We conduct more than 120 experiments using the MIT single column model (MIT SCM) to examine the physics of the instability and its dry bias. We asks ourselves how allowing the surface to be interactive changes the occurrence of the instability, and show that the instability occurs for a wider range of surface temperatures if the surface is allowed to be interactive, independently of the heat capacity of the surface. By doing simple statistics on the final states of our experiments with an interactive surface, presented on figure 8, we note that when the atmosphere has reached a new statistically steady state, this state is more likely to be a dry state (low moisture content, large-scale descent) than a moist state (high moisture content, large-scale ascent). We show that the moist final states have much higher absolute vertical velocities than the dry final states. According to mass conservation, it means that the surface of the dry zones is typically more than twice the surface of the moist zones.

V. Conclusion

We have shown that under the WTG approximation, the presence of an interactive surface allowed a new instability to develop: the air-sea radiative-convective instability. For low RCE surface temperatures (or equivalently low insolation), this instability couples with the purely atmospheric radiative-convective instability, leading to a full radiative-convective instability with larger growth rates. Consistently with previous studies, negative moisture perturbations give rise to a dry and cool final state with large-scale descent, whereas positive moisture perturbations induce a moist and warm state, with large-scale ascent. This radiative-convective instability has a dry bias:

- In the sense that the dry instability grows faster than the moist instability, which is related to the concavity of the emissivity as a function of the water vapor mixing ratio.
- In the sense that the final moist state has higher absolute vertical velocities than the final dry state, which explains why it occupies a smaller area through mass conservation. This asymmetry is believed to be caused by the lower bounds on the clear-sky vertical velocity and the precipitation, that can be reached in the dry state.

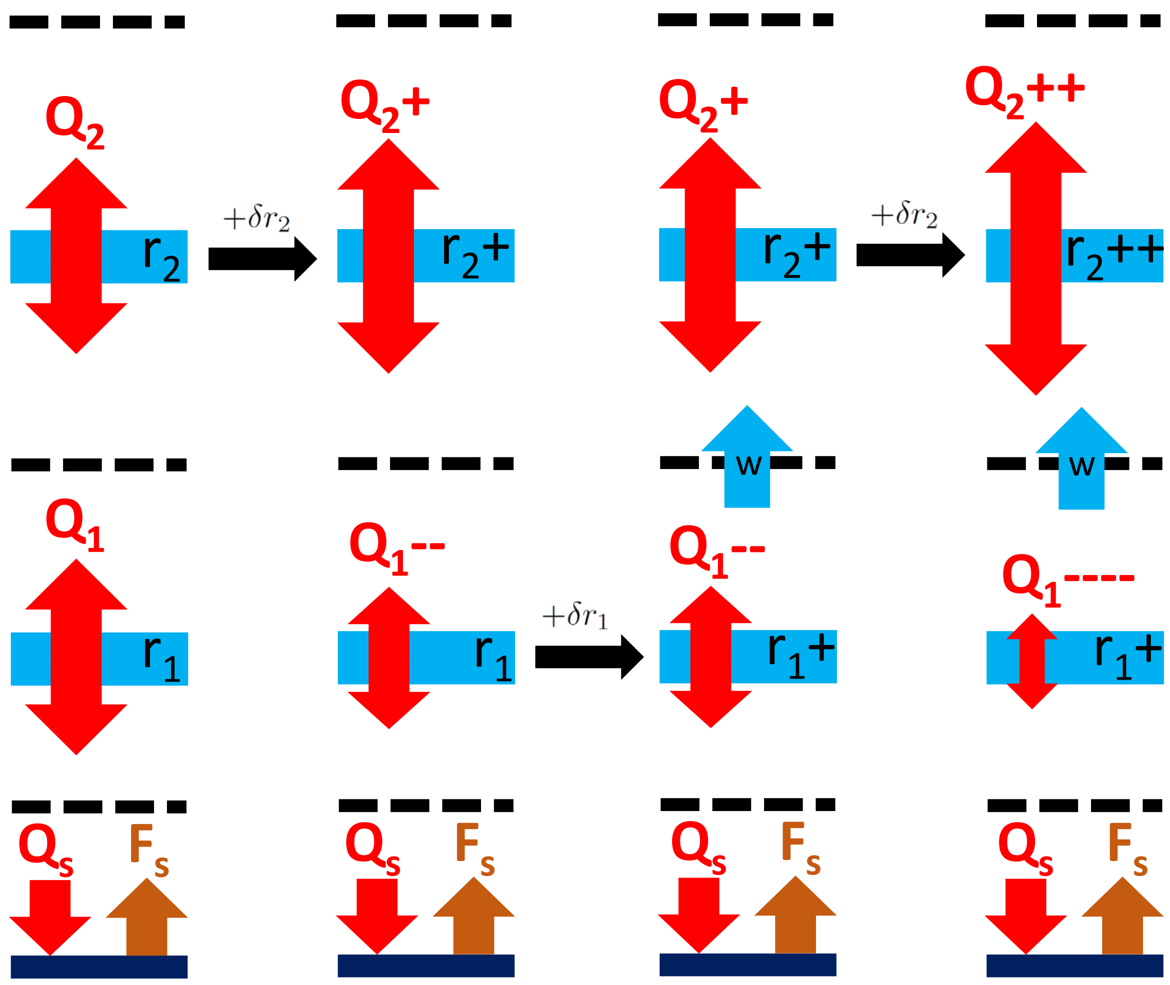

Fig. 6. Physical mechanism of the atmospheric radiative-convective instability with fixed surface temperature for a positive initial upper tropospheric moisture perturbation.

Acknowledgments

The first author was supported by the Neil and Anna Rasmussen Foundation Fund, as well as under NSF grant AGS-1136480 and SERDP grant RC-2336. He thanks Dr Tim Cronin for precious advice on how to use the MIT SCM.

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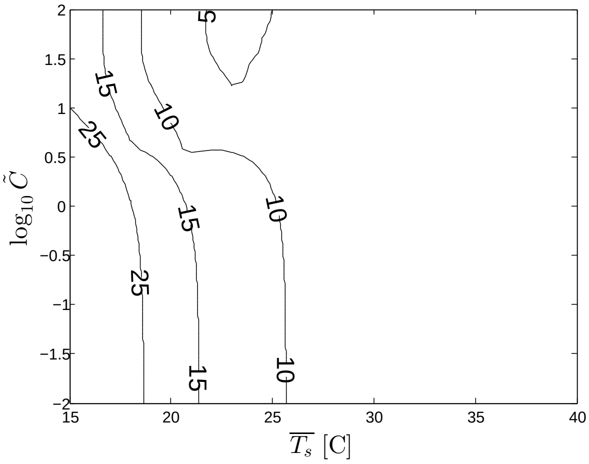

Fig. 7. 10^7 growth rate in the parameter space: $\log_{10}\tilde{\mathrm{C}}$ vs RCE surface temperature for the two layer model.

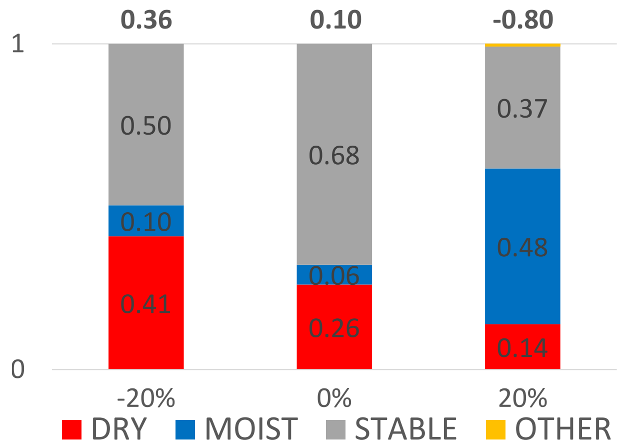

Fig. 8. Proportion of simulations which final state is deemed dry, moist, or stable for a moist perturbation (+20%), a zero perturbation, and a dry perturbation (−20%) in WTG mode. Above each perturbation is written the average mid-tropospheric vertical velocity at the end of the simulation in pressure coordinates ($\mathrm{hPa.hr}^{-1}$), where the average has been taken over all our experiments.

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