Impacts of the Basic Potential Vorticity Profiles on the Stability of Shallow-Water Vortices

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Introduction

The dynamic instability are thought to be an important internal mechanism governing hurricane structure and intensity change. Two-dimensional, hurricane-like vortices can be constructed by smoothly connecting piecewise constant vorticity profiles, with relatively low vorticity in the eye, high vorticity in the eyewall annulus, and then very low vorticity in the far field. Such vortices have been used in the study of two-dimensional vortex stability and evolution by Schubert et al. (1999, here after S99), Nolan and Montgomery (2000, 2002), Kossin and Schubert (2001), and Hendricks et al. (2009, here after H09). The initial condition consists of an axisymmetric vorticity ring defined by,

$$\bar{\zeta}(r)=\begin{cases}\zeta_1 & 0\le r\le r_1\\ \zeta_1 S\left(\frac{r-r_1}{r_2-r_1}\right)+\zeta_2 S\left(\frac{r_2-r}{r_2-r_1}\right) & r_1\le r\le r_2\\ \zeta_2 & r_2\le r\le r_3\\ \zeta_2 S\left(\frac{r-r_3}{r_4-r_3}\right)+\zeta_3 S\left(\frac{r_4-r}{r_4-r_3}\right) & r_3\le r\le r_4\\ \zeta_3 & r_4\le r\le r_5\\ \zeta_3 S\left(\frac{r-r_4}{r_5-r_4}\right)+\zeta_3 S\left(\frac{r_5-r}{r_5-r_4}\right) & r_5\le r\le r_6\\ 0 & r_6\le r<\infty\end{cases}$$

S99 showed that barotropic instability is possible in a vortex that has an annular band of maximum vorticity, and hypothesized that frequently observed polygonal and mesovortices are the byproducts of wave breaking caused by barotropic instability.

The structure characteristics of the enhanced rings are quantified by introducing two vortex parameters (S99, H09), with the first parameter defining the hollowness of the vortex,

$$\delta=(r_1+r_2)/(r_3+r_4)$$

i.e., the ratio of eye to inner-core relative vorticity.

The second parameter defining the thickness of the ring,

$$\gamma=\zeta_1/\zeta_{av}$$

i.e., the ratio of the inner and outer radii of the ring.

Numerical Model

In this study, the effect of the shape of basic-state potential vorticity (PV) profile on the stability and wave characteristics of hurricane-like vortices is discussed in the two-dimensional (2D) Shallow Water Vortex Perturbation Analysis and Simulation model designed by Nolan et al. (2001).

A sequence of 170 numerical simulations are conducted by setting the two structural parameters of basic-state PV to cover the thickness–hollowness (d, g) parameter space described above at regular intervals.

In the left panels in fig.1, the mean relative vorticity profiles, are shown for the [g= 0.0, d=0.00, 0.05, . . . , 0.85] rings. This illustrates how varying the ring thickness affects the curve while holding the hollowness fixed, and thicker curves represent thinner rings. Similarly, the initial conditions for the [g= 0.0, d 5 0.00, 0.05, . . . , 0.85] rings are shown in the right panels. This illustrates how the three curves change as the hollowness parameter g is varied while holding the thickness parameter d fixed, also thicker curves represent more filled rings.

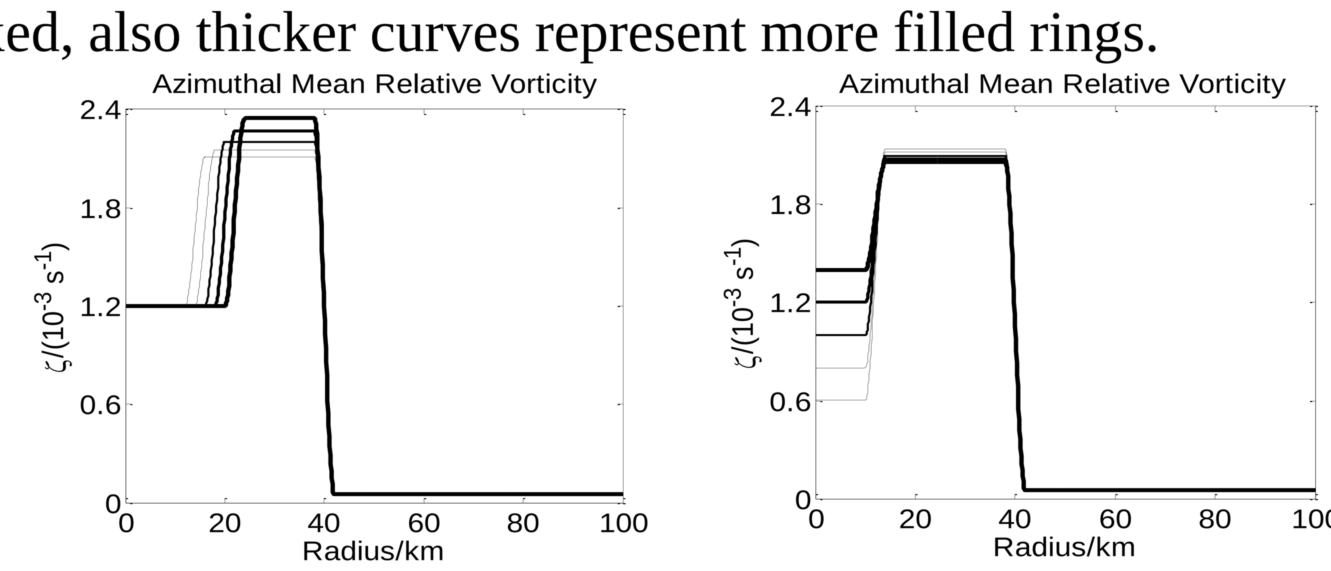

Fig.1. Radial distribution of relative for various PV rings as the basic state. On the left, γ = 0.60 and δ = (0.35,0.40,……,0.55), and thicker lines indicate increasing δ; on the right, δ = 0.30 and γ = (0.30,0.40,……,0.70), and thicker lines indicate increasing γ.

Results

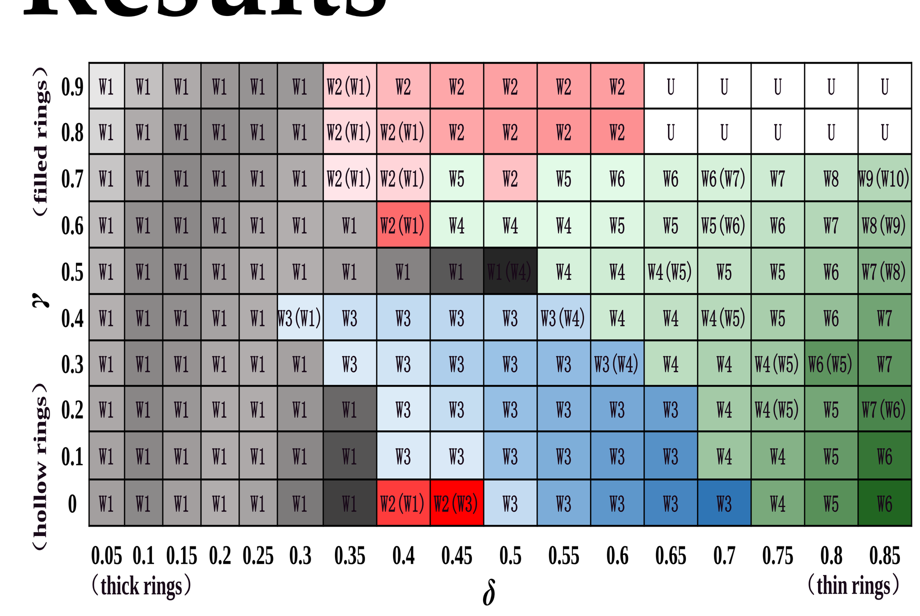

Fig. 2. Distribution of the most unstable wavenumber (MUWN) and growth rates in the (δ, γ)-parameter space, δ is abscissa, γ is ordinate.

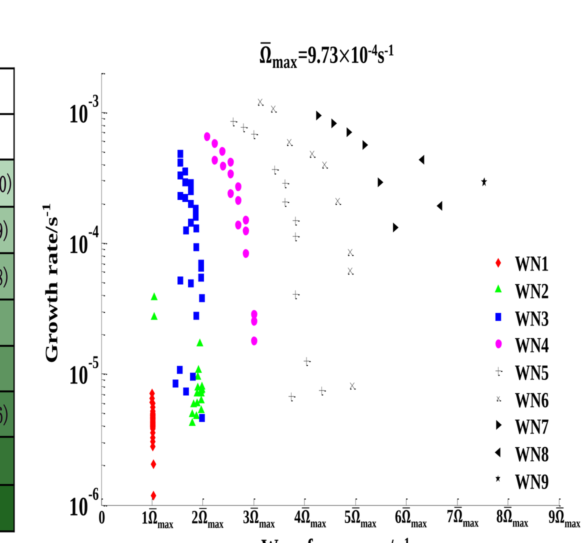

Fig.3. Eigenfrequencies-growth rates scatterplots for the MUMSs of various PV rings, different signs represent different the MUWNs.

In fig.2, the numbers in brackets denote the results from non-divergent barotropic model (H09), the points without brackets indicate that the results of two models are consistent. The four different colors represent different azimuthal wavenumbers (WNs). The deeper color levels represent the faster growth rates for the most unstable mode in systems (MUMSs).

The eigenfrequency in this paper represents the Doppler-shifted frequency, namely, $\tilde{\omega}=n\bar{\Omega}+\bar{\omega}$, n is the azimuthal WN, $\bar{\Omega}$ is the basic state angular velocity, and $\bar{\omega}$ is the dimensional intrinsic frequency. So the intrinsic frequency can be estimated by the difference between the eigenfrequency $\tilde{\omega}$ and advective frequency ($n\bar{\Omega}_{max}$). The eigenfrequency at WN n is closer to the advective frequency, corresponding the intrinsic frequency is lower.

Case Analysis

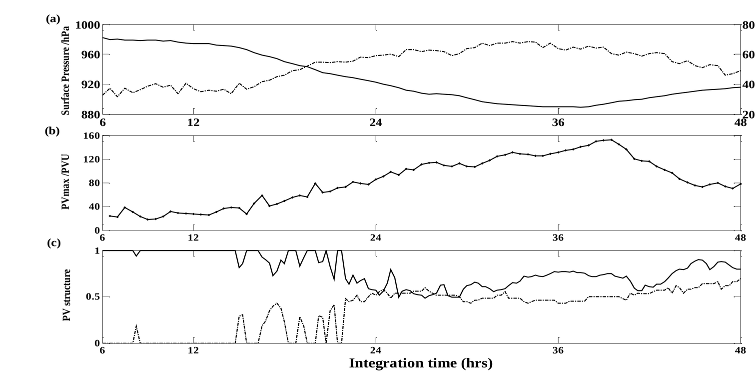

Fig. 4 Time series of (a) minimum sea level pressure (solid, hPa) and surface maximum tangential wind (dashed, m s-1), (b) the maximum PV at z=3 km, and (c) two structural parameters, in which thickness and hollowness are denoted by solid and dot-dashed line, respectively.

The evolution of the potential vorticity (PV) structure at different stages are investigated with the high resolution simulation. Before rapid intensification occuring, the PV increase slightly, and its structure is a monople. PV increases rapidly and the hollow structure becomes narrower and more hollow during the rapid intensification period. Later on, the high PV being mixed into eye results in PV ring becoming thicker and more filled.

Table 1 The structural parameters of PV rings and the MUWNs of Hurricane Wilma at different integration times.

Times	15:00	23:00	44:30
(δ，γ)	(0.29，0.81)	(0.44，0.67)	(0.58，0.81)
MUWN	WN1	WN4	WN2

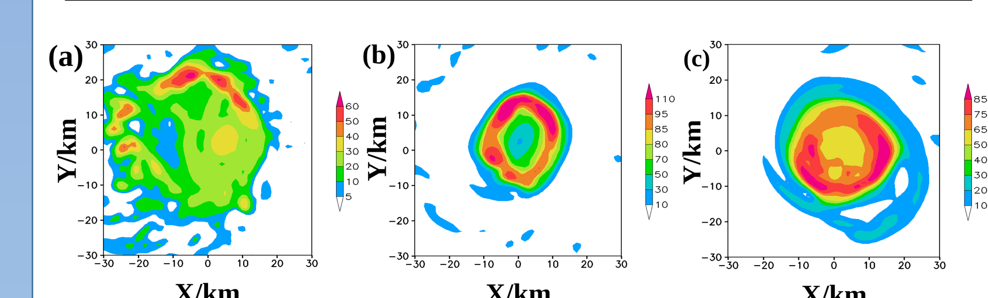

Fig. 6 Horizontal maps of predicted PV (shaded, PVU) at z=3 km over a subdomain of 30 km×30 km at the 15h, 23h and 44h30min model integration, respectively.

The asymmetry is obvious at 15:00 in model time. The PV value is significantly increased, and the hollow structure is completely formed, which presents the irregular square shape at 23:00. While the maximum value of PV is reduced at 44:30, the PV in eye increase. The structure is an ellipse of high vorticity at the same time, which is closely related to its most unstable mode occurs for WN 2.

Conclusion

The results show that (1) for thicker rings, the dynamic instability is more prone to lower WNs, and corresponding most unstable mode possess lower intrinsic frequencies and small growth rates; (2) thinner rings are more prone to higher wavenumber growth, and the more filled the rings become, the higher the most unstable WN is; and (3) for thin and hollow rings, its most unstable mode possess high frequencies and large growth rates.