Accounting for Errors when Projecting Data onto a TC-Centered Space

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Introduction

It is not uncommon for tropical cyclone (TC) analyses to be conducted on a **TC-centered, cylindrical grid.** Most data are not natively gridded in this way (e.g., reanalysis data is commonly gridded in latitude-longitude coordinates).

If we have data in Earth-relative coordinates, how do we convert the data into TC-relative coordinates? How do we get fields like tangential and radial wind?

A Simple Approach

One way, for example, to orient data in TC-relative space is to use great-circle distance between the data and the TC center for **radius**:

$$\boldsymbol{r(\lambda,\phi) = ac}$$

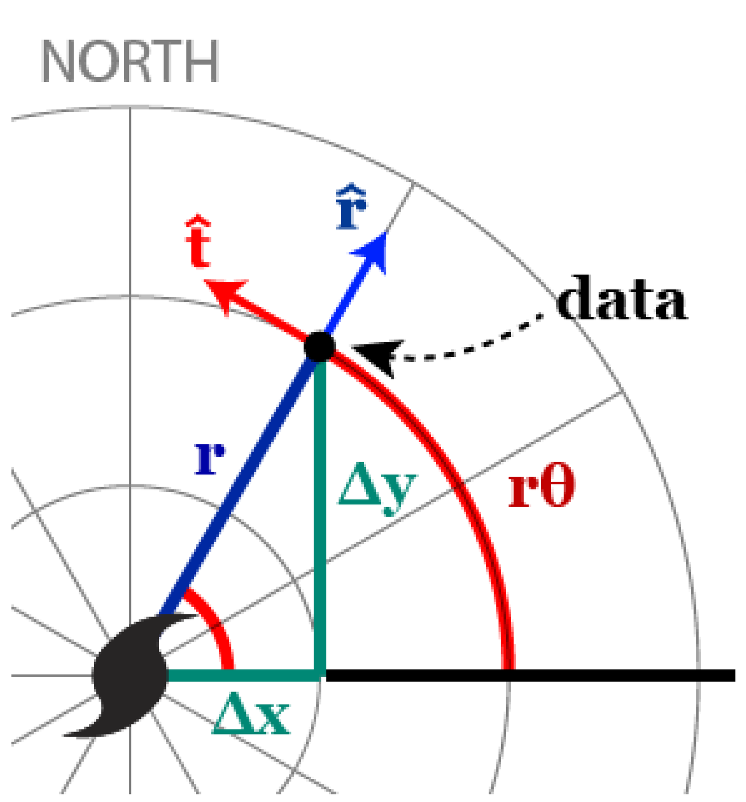
$$c = \cos^{-1}[\sin\phi_C \sin\phi + \cos\phi_C \cos\phi \cos(\lambda - \lambda_C)]$$

a : Earth's radius ; λ: longitude ; ϕ: latitude

(λ_C, ϕ_C) : Longitude, latitude at grid center

Azimuth is found using longitudinal and latitudinal distances $(\Delta x, \Delta y)$ between the data and TC-center:

$$\boldsymbol{\theta(\lambda,\phi) = \mathrm{atan2}(\Delta y, \Delta x)} \qquad \Delta x = a(\lambda - \lambda_C)\cos(\phi) \qquad \Delta y = a(\phi - \phi_C)$$



We will call this approach the **"great-circle transform."** With azimuth found, we can find tangential or radial wind using basic vector calculus.

$\hat{t}$ should be parallel to contours of radius, and $\hat{r}$ should be orthogonal to $\hat{t}$, as shown in Fig. 1.

Fig. 1: Schematic of the method described above. A polar grid with concentric contours of great-circle distance (radius) from the center is superimposed over a TC.

The problem with this method (and others) is that curvature in longitude and latitude is overlooked in Δx and Δy, which leads to errors in azimuth and any value therefrom.

Positioning data on an **azimuthal equidistant projection** is favorable for analyses that require a TC-relative cylindrical/polar grid, as this projection **preserves all angles and distances relative to the grid's center** (Snyder 1987).

The azimuth of a point on the projection is found by:

$$\boldsymbol{\theta(\lambda,\phi) = atan2(Y', X')}$$

$$X' = ac(\sin c)^{-1}\cos\phi\sin(\lambda - \lambda_C)$$

$$Y' = ac(\sin c)^{-1}[\cos\phi_C\sin\phi - \sin\phi_C\cos\phi\cos(\lambda - \lambda_C)]$$

The Curious Case: A TC-like Polar Low

A script written originally for analyzing primary and secondary circulations in TCs was used to analyze similar properties in a polar low. Quickly, it became obvious that **our use of the great-circle transform was severely problematic for high-latitude analysis.**

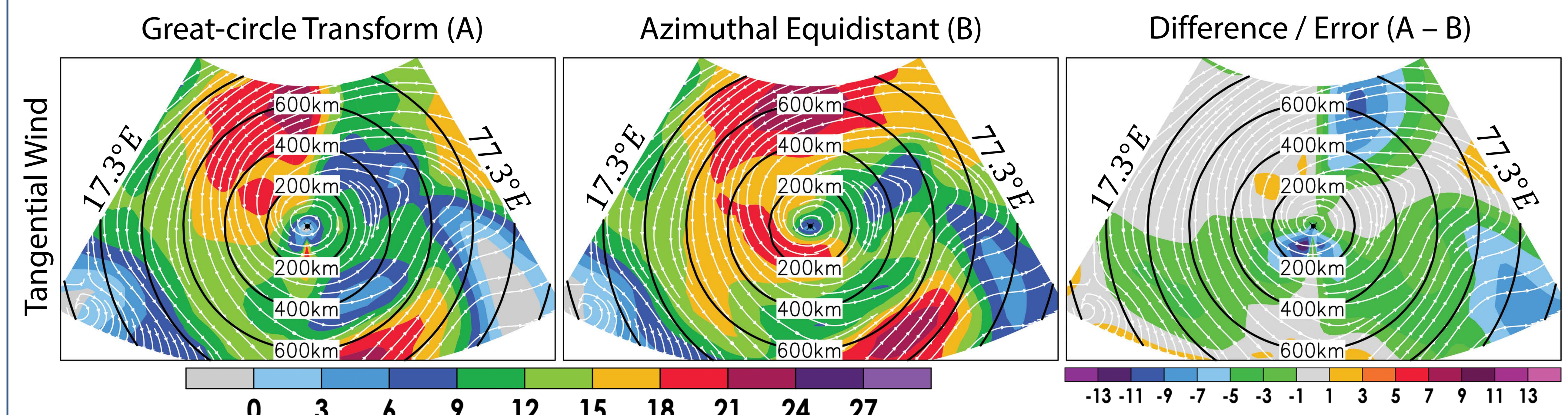


Fig. 2: Two 950-hPa tangential wind calculations (and their difference; $m\ s^{-1}$) at 12Z 19 Dec 2002 using ERA-Interim data. The domain is between 68.3°N and 80.3°N. White streamlines and black contours of great-circle distance are superimposed. Landmask is omitted.

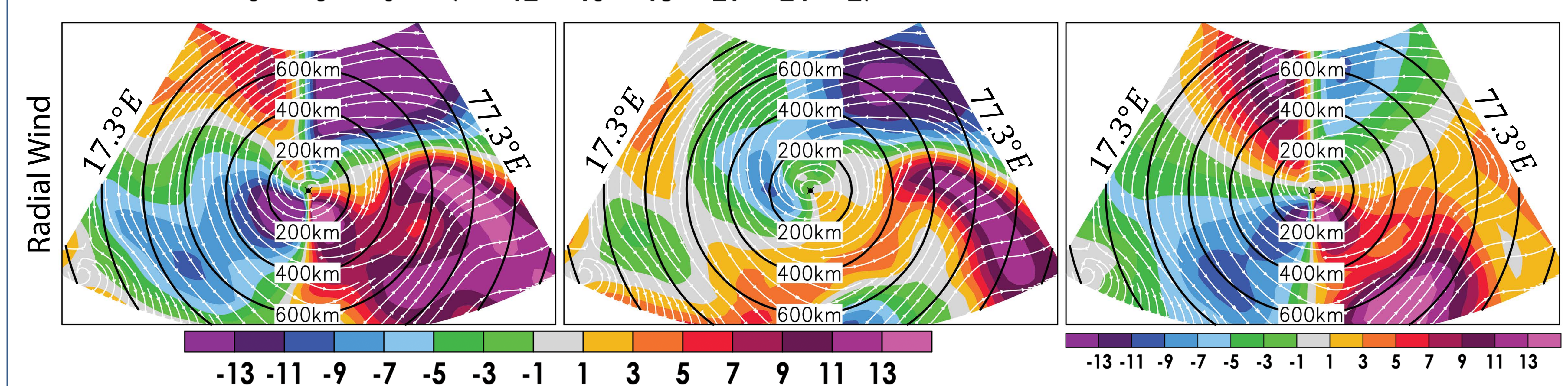


Fig. 3: Similar to Fig. 2, but for radial wind. The great-circle transform has regions where even the sign is incorrect (e.g., northwest of the center). Note the shape of the quadrupole evident in the error field, which is roughly separated by lines of latitude and longitude.

Error Analysis for Tropical Cyclones

These errors are highly prevalent in TC analysis. **Median error in azimuthal-mean radial wind approaches 60% at 2000 km radius.**

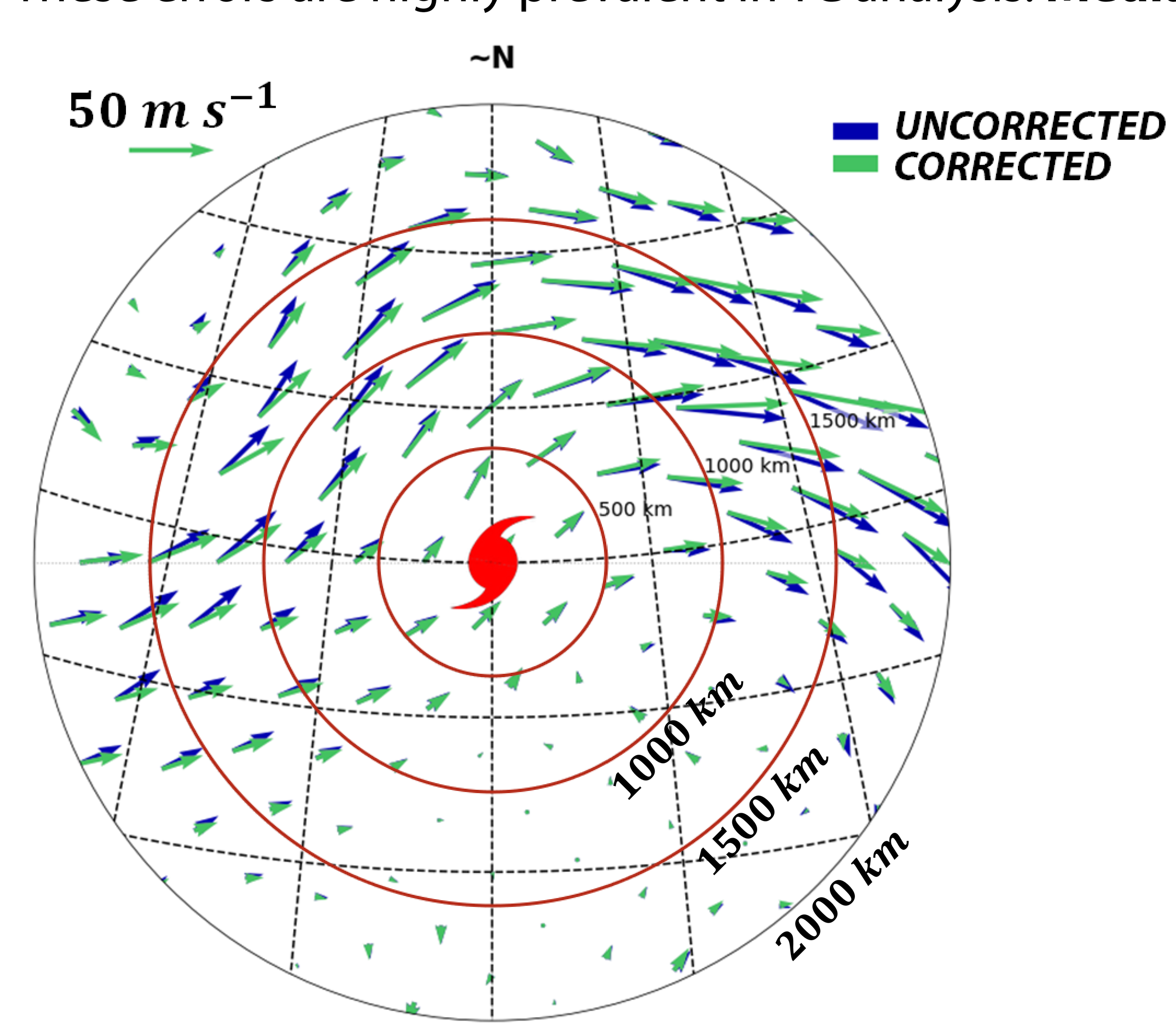


Fig. 4: 200-hPa wind vectors for Hurricane Debby (0Z 19 Sep 1982) plotted in r-θ space using the great-circle transform ("uncorrected," blue) and the azimuthal equidistant transform ("corrected," green). Dashed contours denote lines of constant latitude and longitude. Circular contours are great-circle distance (500 km interval).

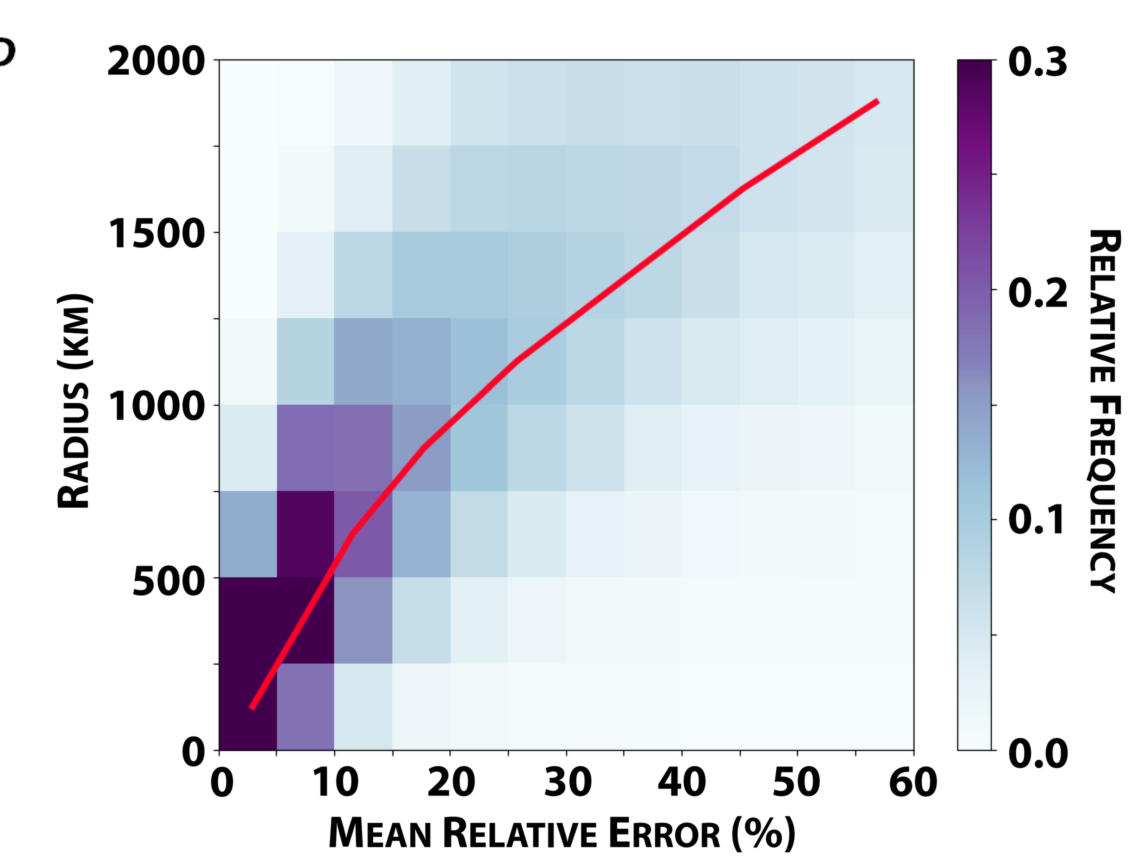


Fig. 5: Distribution of azimuthal-mean relative error in 200-hPa radial wind as a function of radius from 6934 six-hourly Atlantic TC samples during 1979–2015. Wind fields are obtained from ERA-Interim reanalysis, and tropical TC track points over water are taken from HURDAT for TCs of >34 kt intensity. Error is calculated as the radial wind from the great-circle transform relative to the azimuthal equidistant (correct) transform. The red line represents the median error.

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