12C.8 EVALUATION OF A TIME-DEPENDENT MODEL FOR THE INTENSIFICATION OF TROPICAL CYCLONES

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1. INTRODUCATION

Recently, a theoretical model for axisymmetric tropical cvclone (TC) intensification with an approximate analytical solution is proposed by Emanuel (2012, E12). According to the solution of the approximate system of E12 [his (19)], the intensification rate is proportional to C_k and $1/C_d$, where C_k and C_d are the surface exchange coefficients for entropy and momentum, respectively. However, some previous studies (e.g., Rosenthal, 1971; Craig and Gray, 1996; Montgomery et al., 2010) show that the intensification rate is relatively insensitive or proportional to the change of C_d . Given the somewhat contrasting results to be found among numerical studies and the theoretical model of E12, this study uses an axisymmetric nonhydrostatic cloud model (CM1), set up to follow closely the physical content of the E12 model, to assess the latter's predications and further clarify its dynamics. Of particular interest here are the effects of the surface-exchange coefficients on TC intensification as predicted by the E12 model.

2. NUMERICAL SIMULATIONS

Figure 1a shows maximum azimuthal velocity V_m from simulations with $C_k/C_d = 1$ and C_k varying from 0.5×10^{-3} to 4×10^{-3} . These results support the idea that the TC intensification rate is proportional to



Fig. 1. Time series of maximum azimuthal velocity V_m [m s⁻¹] from simulations with different (C_k , C_d , 10^{-3}) for the simulations with (a) $C_k/C_d = 1$ and different values of C_k and (b) $C_k = 4 \times 10^{-3}$ and different values of C_d .

 C_k , which is consistent with the E12 theory and other previous studies (e.g., Craig and Gray, 1996). Figure 1b shows sensitivity of the intensification rate to the variation of C_d when C_k equals to 4×10^{-3} . These results are similar to the sensitivity of intensification rate to changes in C_d found in Montgomery et al. (2010), which indicate the intensification rate of the vortex increased with increasing C_d until a certain threshold value is attained and then remained

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Fig. 2. Entropy *s* (contour interval = 10 J kg⁻¹ K⁻¹, green lines) and angular momentum *M* (contour interval = 0.2×10^6 m² s⁻¹, black lines) from the experiment with a) $C_k = C_d = 0.5 \times 10^{-3}$, b) $C_k = 4 \times 10^{-3}$, $C_d = 0.5 \times 10^{-3}$ and c) $C_k = C_d = 4 \times 10^{-3}$. The red line is the *M* surface that passes through the location of maximum tangential wind.

relatively constant. In summary, the numerical-model evidence presents here and in past studies strongly suggest that the intensification rate is proportional to C_d for small values while the prediction from E12 model is for an intensification rate proportional to $1/C_d$.

3. EVALUATION OF E12

3.1 Moist slantwise neutrality

The E12 model assumes that saturation-entropy (s) is a function of angular-momentum (M) alone in the free atmosphere during the intensification period. According to the numerical-model results, the intensification of the TC can be divided into two periods, Phase I and Phase II. During Phase I, the TC intensifies while the *M* and s^* surfaces evolve from nearly orthogonal to almost congruent (not shown). During Phase II, the *M* and s^* surfaces in the

eyewall in the interior free atmosphere and TC outflow are congruent as the TC intensifies (Fig. 2), which is consistent with E12 assumption of moist slantwise neutrality. Based on these analyses, we conclude that an approximate condition of moist slantwise neutrality is achievable in the eyewall and outflow of a numerical simulation during the intensification in Phase II.

3.2 Gradient-wind balance

Based on hydrostatic and gradient-wind balance and the assumption of slantwise moist neutrality, the approximate diagnostic equation for the wind speed in E12 is

$$V_e^2 = -(T_b - T_o)M \frac{\partial s^*}{\partial M}, \qquad (1)$$

where V_e is the azimuthal velocity at the top of the boundary layer, T_b is the temperature at the top of boundary layer, T_o is the outflow temperature. In Fig. 3, it is found that although the tendency diagnosed by



Fig. 3. Time series of V_m (m s⁻¹) from the control simulation (black line), the velocity calculated from (1) (V_e , red line) and V_a from (2) which includes V_e and the non-gradient-wind-balance term (blue line) for a) $C_k = C_d = 1 \times 10^{-3}$, b) $C_k = C_d = 4 \times 10^{-3}$ and c) $C_k = 4 \times 10^{-3}$.

E12 is qualitatively similar to the numerical-model result during Phase II, it is not quantitatively similar.

To determine the effect of the unbalanced term, we use a derivation with similar approximations to those used in BR09a and express a modified maximum wind speed (V_a) containing the unbalanced terms,

$$V_a^2 = V_e^2 + r_b \eta_b w_b,$$
 (2)

where the subscript *b* denotes evaluation at the top of the boundary layer, *w* is the vertical velocity component and $\eta = \partial u / \partial z - \partial w / \partial r$ is the azimuthal vorticity. It is clear that the evolution of V_a is in considerably better agreement with V_m (Fig. 3), which means the neglect of non-gradient-wind effects in E12 may be the reason for the quantitative difference between V_m and V_e .

The non-gradient-wind balance term is strongly related to surface drag, which in turn is related to C_d . The present analysis suggests that the neglect of non-gradient-wind effects in the diagnostic equation for wind speed of E12, (1), may be the reason for the different dependence of intensification rate on C_d between the E12 model and the present numerical simulations.

3.3 Self-stratification of TC outflow

The self-stratification of the outflow temperature used in the E12 model is determined by small-scale turbulence that limits the Richardson number to a critical value for the onset of turbulence, which is proposed by Emanuel and Rotunno (2011):

$$\frac{\partial T_o}{\partial M} \cong -\frac{Ri_c}{r_t^2} (\frac{dM}{ds^*}),\tag{3}$$

where Ri_c is the critical Richardson number and r_t is the physical radius at which the Richardson number first attains its critical value. Figure 4 shows the relationship between dT_o/dM and -dM/ds from hourly output in the control simulation during Phase II. Fitting a straight line to the data between dT_o/dM and -dM/ds gives a slope of 3.1×10^{-10} m⁻², which is close to the value of Ri_c/r_t^2 (2.7×10^{-10} m⁻²) when $Ri_c = 1$ and $r_t \approx 60$ km. Therefore, the self-stratification hypothesis of outflow in the E12 model is consistent



Fig. 4. The relationship between dT_o/dM (K m⁻² s) and -dM/ds (K s) for hourly output in control simulation during Phase II.

with our numerical simulations during the growth stage in Phase II.

4. CONCLUSIONS

According to the numerical-model results, the intensification of the TC can be divided into two periods, Phase I and Phase II. During Phase I, the TC intensifies while the *M* and s^* surfaces evolve from nearly orthogonal to almost congruent. During Phase II, the *M* and s^* surfaces in the eyewall and outflow are congruent as the TC intensifies, which is consistent with E12. Therefore, the condition of moist slantwise neutrality and the hypothesis of outflow self-stratification of E12 are sufficiently satisfied throughout the intensification in Phase II. It is also found that the sensitivity of the intensification rates to C_k matches the theoretical result, however the relationship between intensification rate and C_d is essentially the inverse of the theoretical result. Furthermore, the present analysis finds the inclusion of non-gradient-wind effects in the theoretical framework of E12 produces an intensification rate that is quantitatively similar to the numerical-model results. The neglect of non-gradient-wind effects in E12 may be the reason for the different dependence of its intensification rate on C_d compared to that of the numerical model.

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