12C.7 AVAILABLE POTENTIAL ENERGY DENSITY IN AN AXISYMMETRIC TROPICAL CYCLONE

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1 INTRODUCTION

The study of the sources of energy available for tropical cyclone (TC) intensification has focused on the roles of surface enthalpy fluxes and CAPE (e.g. Charney and Eliassen, 1964; Emanuel, 1986). Processes such as water vapour diffusion, heat conduction, and irreversible phase changes have received little attention; the classical view of a TC as a heat engine regards such irreversible processes as dissipative, since they are a source of entropy. However, from the perspective of Available Potential Energy (APE) theory, which describes the energy available to atmospheric motions by comparing the actual atmospheric state to a stable reference state (Lorenz, 1955), such processes can be either a source or a sink of APE. This means that it may be possible under some circumstances for irreversible processes to contribute towards intensification.

Investigating irreversible processes from an APE perspective requires an APE theory for a moist atmosphere that correctly accounts for their effects; such a theory has been historically difficult to construct (Tailleux, 2017). APE generation in TCs has been previously studied using sorting algorithms to construct a reference state by rearranging the air parcels in a 2D domain into an approximation of their minimum total potential energy configuration (Wong et al., 2016). However, this work used a global view of APE, which cannot correctly capture the local effects of irreversible processes. This study aims to develop a local theory of APE for moist atmospheres, which will be used in conjunction with the numerical axisymmetric TC model of Rotunno and Emanuel (1987), in order to allow us to establish whether irreversible processes may occasionally act as a source of energy for TC intensification.

2 THEORY

APE density is defined as the work that an air parcel needs to perform against buoyancy forces to move from a notional reference position to its actual position (Tailleux, 2013),

$$\epsilon_a = -\int_{z_r}^{z} b\left(\theta_l, q_T, z', t\right) \mathrm{d}z', \tag{1}$$

where b denotes buoyancy relative to an arbitrary reference state. Conservation of liquid potential temperature θ_l and total specific humidity q_T ensures that the movement of a parcel to a height z' is reversible adiabatic. The reference height z_r is defined as a level of neutral buoyancy, $b\left(\theta_l, q_T, z_r, t\right) = 0$, so that the material derivative of the APE density is

$$\frac{D\epsilon_a}{Dt} = -bw + G_a + G_t. \tag{2}$$

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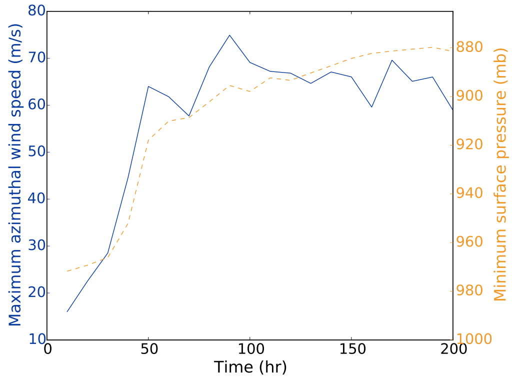

Figure 1: Intensity of the numerically modelled axisymmetric TC.

Here, w is the vertical wind speed; the term $-bw$ represents the exchange between APE density and vertical kinetic energy as mediated by buoyancy fluxes. G_a denotes the rate of local generation/dissipation of APE by diabatic processes, and G_t the rate of change of APE density due to the time evolution of the reference state.

If diabatic processes in a TC generate APE density, this can be converted to kinetic energy in the secondary circulation via buoyancy fluxes, and then in turn to kinetic energy in the primary circulation, resulting in vortex intensification.

The choice of reference state appears to be crucial to the insight that can be gained on TC intensification. If the generation of APE density is consistently larger than the conversion of APE to kinetic energy, then APE will be stored, and it becomes difficult to see a direct relation between the generation of APE and the intensification of the TC, which relies on the conversion to kinetic energy. It may therefore be useful to choose a reference state that minimises APE storage.

3 AXISYMMETRIC MODEL

In order to explore the behaviour of APE density in a TC, we have used the axisymmetric numerical model of Rotunno and Emanuel (1987), with modifications to the microphysics and boundary conditions as implemented by Craig (1996). This is a nonhydrostatic model which simulates the intensification of a weak initial vortex into a TC over an ocean with constant SST.

The model is run on a domain with 2.5km resolution in the radial direction, extending a total of 3600km, and 625m in the vertical, up to 27.5km. We initialise the model with the mean hurricane season sounding of Jordan (1958), and run

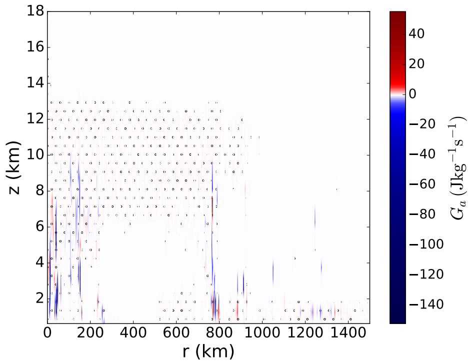

Figure 2: Diabatic generation/dissipation rate of APE, G_a, at 30 hours, taking the reference state to be the initial model state. Stippled regions show where $G_a > 0$, i.e. diabatic processes are generating APE.

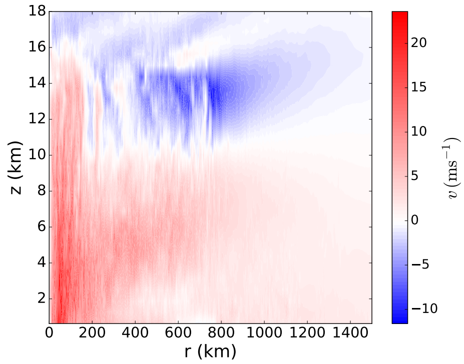

Figure 3: Azimuthal wind speed after 30 hours.

the model out to 200 hours. The intensification of the vortex in terms of both maximum azimuthal wind speed and minimum surface pressure is shown in Figure 1. The cyclone intensifies most quickly from approximately 30 to 50 hours. After 100 hours, the maximum wind speed becomes steady, though the surface pressure continues to decrease.

The buoyancy b is defined in the axisymmetric model equations as

$$b = g\left[\frac{\theta - \overline{\theta}}{\overline{\theta}} + 0.61\left(q_v - \overline{q_v}\right) - q_l\right], \tag{3}$$

where overbars refer to the value of a variable in the initial state of the model, which depends only on z.

The most natural definition of the APE reference state in this case is to take a time-independent reference state equal to the initial state, so $G_t = 0$. It appears that

$$G_a = -g\frac{D\theta}{Dt}\int_{z_r}^{z}\frac{dz'}{\overline{\theta}\left(z'\right)} - g\left(z - z_r\right)\left(0.61\frac{Dq_v}{Dt} - \frac{Dq_l}{Dt}\right). \tag{4}$$

However, writing G_a in this way implies that θ, q_v and q_l are all conserved when evaluating the integral in Eq. (1). Instead, the buoyancy should be rewritten in terms of θ_l and q_T, so that Eq. (1) may be calculated assuming reversible adiabatic motion of the parcel. This results in a diabatic APE generation/dissipation rate of the form

$$G_a = \Upsilon_{\theta_l}\frac{D\theta_l}{Dt} + \Upsilon_{q_T}\frac{Dq_T}{Dt}, \tag{5}$$

where Υ_{θ_l} and Υ_{q_T} are thermodynamic efficiencies not included here due to complexity.

4 RESULTS

Preliminary results using the initial state as the reference state show that the diabatic generation of APE during intensification occurs chiefly near the centre of the TC and along the sea surface, shown for the TC at 30 hours in Fig. 2. To provide a reference point for the structure of the TC at this time, the azimuthal wind speed is displayed in Fig. 3. This result is expected since APE can be generated by surface heat fluxes, and by latent heating during ascent in the eyewall.

However, it is evident from Fig. 4 that we have a high APE storage; APE increases throughout the domain from 30 hours, when the TC is intensifying, to 100 hours, after which further intensification does not occur. This indicates that the particular choice of APE reference state is not suited to studying intensification, since it results in high APE density, which is not converted into kinetic energy.

Further work on the APE density theory for this model will include testing alternative reference states, such as a time-evolving horizontal mean density state, to construct a framework that is useful for the study of the impact of diabatic processes on intensification. We will also investigate each term of G_a to ascertain which processes contribute most strongly, in particular to study whether irreversible processes can generate APE in a way that aids intensification.

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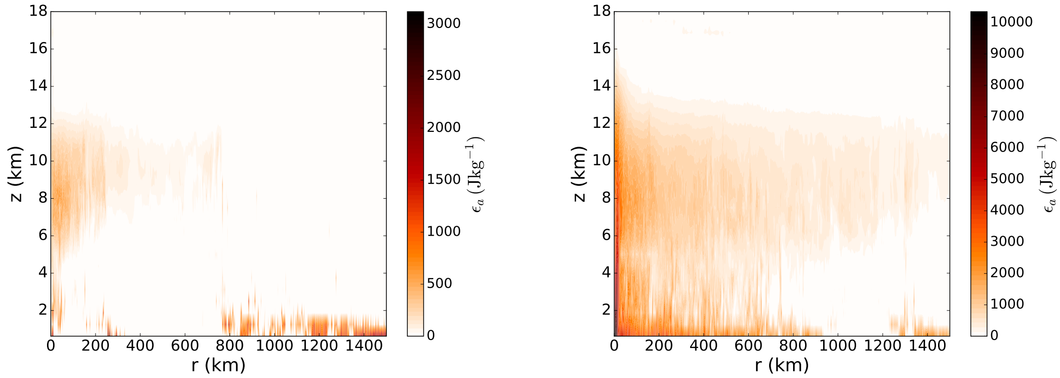

(a) 30 hours

(b) 100 hours

Figure 4: APE density in the axisymmetrically modelled TC, taking the reference state to be the initial model state at all times. Note the different scales for ϵ_a.

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