NUMERICAL EXPERIMENTS WITH A VARIATIONAL SCHEME FOR ATTENUATION-CORRECTION FOR X-BAND RADAR

Leonid Tolstoy[1], V.N. Bringi[1] and Robin J. Hogan[2]

[1]Colorado State University

[2]University of Reading, UK

Abstract

The variational scheme developed and evaluated by Robin Hogan for S-band radar for estimating rain rate and detecting hail, is used here specifically for X-band application to correct the measured Z_h and Z_{dr} for attenuation due to mixed phase precipitation (rain mixed with wet ice/hail) which is an especially severe problem for lower power, short range X-band radars. While a number of algorithms for attenuation-correction (based on differential propagation phase) are available at X-band when only rain occurs along the propagation path, there is no stable algorithm as yet when rain is mixed with wet ice/hail. Hence this paper is aimed at applying the variational scheme, which in its formulation, estimates the attenuation due to both rain and hail along the propagation path. We report on several numerical experiments with the variational code to optimize the attenuation-correction in mixed phase precipitation at X-band.

The forward model was adapted from S to X-band based on scattering simulations. The measurement errors in Z_{dr} and differential propagation phase for the particular X-band radar which we used (one of the CASA IP1 radars in Oklahoma) were adapted based on reflectivity and copolar correlation coefficient along the beam. This adjusts the weights given to Z_{dr} and differential propagation phase in the cost function. We used the hail detection ratio (H_{dr}) based on (Z_h, Z_{dr}) to pre-identify hail along the path and thereby initialize the detection of hail in the variational scheme. This was followed by pre-estimating the "reflectivity weighted fraction of ice" using the "deviation from the rain line" methodology as an initialization for the "fraction of ice" in the variational scheme. The scheme then finds an optimal solution for attenuation-corrected Z_h and Z_{dr} due to rain and wet ice along the path. The preliminary improvements of the variational scheme using CASA X-band radar data in a convective storm with rain and wet hail are evaluated.

Variational method and forward model (FM)

The forward model, which is the essence of the variational method, uses the first guess of state vector consisting of the ln(a) for each gate of the beam, where coefficient a is the coefficient between reflectivity Z_h and the rainrate R:

$$Z_h = aR^b$$

where b is equal to 1.5.

Then these values are used as an input to the forward model to predict the observations at each gate (Z'_{dr} and Φ'_{dp}). The difference between predicted and observed variables is used to change the state vector for better fit with the observations in a least squares sense. This is done by minimization of the cost function, which is defined as

$$2J = \sum_{i=1}^{m} \frac{(Z_{dr,i} - Z'_{dr,i})^2}{\sigma^2_{Z_{dr}}} + \frac{(\phi_{dp,i} - \phi'_{dp,i})^2}{\sigma^2_{\phi_{dp}}} + \sum_{i=1}^{n} \frac{(x_i - x^a_i)^2}{\sigma^2_{x^a}}$$

where first two summations represent the deviation of the observations Z_{dr} and Φ_{dp} from the values predicted by the forward model Z'_{dr} and Φ'_{dp}, respectively, and the third summation represents the deviation of the elements of the state vector from some *a priori* estimate x^a (a priori a=200 $mm^6 m^{-3} (mm\ h^{-1})^{-1.5}$).

The terms σ_{Zdr} and $\sigma_{\Phi dp}$ are the root-mean-square observational errors, and σ_x^a is the error in the *a priori* estimate, *m* is number of the input gates in the beam, *n* is a set of basis functions, typically ~m/10. This minimization process would be repeated until convergence is reached.

This method also can be used to find gates with hail and estimate the fraction of reflectivity due to the hail. Using S-band data, Hogan (2007) has shown that the optimal estimation scheme produces good results for S-band, but application of it to X-band radar data (CASA radars) does not produce equally good results. The observed radar values of Z_{dr} and Φ_{dp} cannot be predicted by the FM (forward model) well enough in some cases, especially for beams which go through the storm core.

Variable observational errors in the cost function

One way to improve the performance of the optimal estimation scheme is to adjust the default errors assigned to Φ_{dp} and Z_{dr} values, which are used as an input data into the described algorithm.

For CASA radars, the root-mean-square observational error for Z_{dr} data, i.e., σ_{Zdr} has a default value of 0.5 dB. For low rain rate areas (drizzle) the values of Z_h are expected to be less than 20 dBZ, and there the observational error should be higher then this default value.

After examination of the CASA radar quality and some numerical experiments it was found that value of σ_{Zdr}, $\sigma_{\Phi dp}$ could be changed according to the empirical formulas

$$\sigma_{Zdr}(Z_h) = 0.5 * \begin{cases} -0.05 Z_h(dBz) + 2, & Z_h < 20 dBz \\ 1, & Z_h > 20\ dBz \end{cases}$$

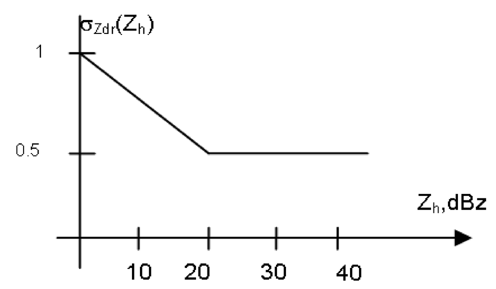

Fig.1 Empirically-based dependency of σ_{Zdr} on Z_h

$$\sigma_{\Phi_{dp}}(\rho_{hv}) = 3 * \begin{cases} -4.44\rho_{hv} + 5, & \rho_{hv} < 0.9 \\ 1, & \rho_{hv} \ge 0.9 \end{cases}$$

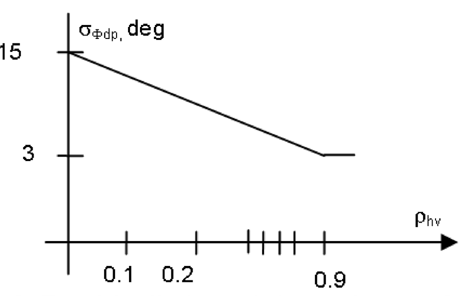

Fig 2. Empirically based dependency of $\sigma_{\Phi dp}$ on the copolar correlation coefficient, ρ_{hv}.

As the result of the above procedure of changing the default values of observational errors σ_{Zdr}, $\sigma_{\Phi dp}$ leads to the re-balancing the influence of that corresponding variable on the cost function. The overall effect is that corresponding forward-modeled range profiles (Z'_{dr} or Φ'_{dp}) tends to be close to the input variable, as in the case when only one variable (Z_{dr} or Φ_{dp}) was used in the input to the program (these variables can be used as an input to the scheme together or switched off if not available in the radar data).

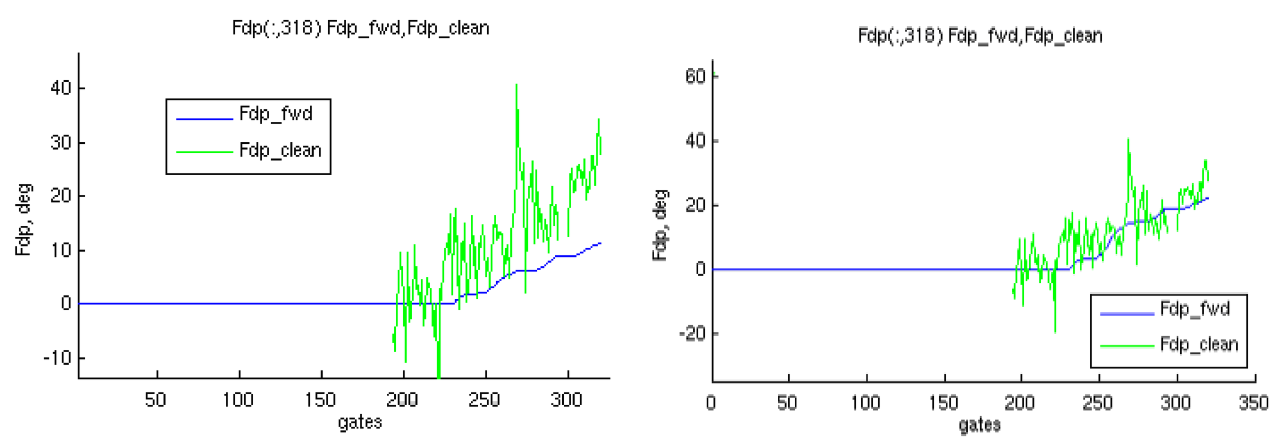

Fig 3. Φ_{dp} for one beam, generated by FM using constant observational errors (left pane) and variable errors σ_{Zdr}, $\sigma_{\Phi dp}$ (right pane).

Estimation of reflectivity-weighted fraction of ice in a rain-hail mixture

Another way to improve the optimal estimation algorithm is related to the problem of automatic detection of wet ice and hail in the observed precipitation. In its original form, the algorithm when used with X-band data, in some cases, converges to physically unrealistic set of output variables, which are far from the input and show saturation in Φ_{dp} and A_h, where they increase up to the maximum allowed values. It was found that this situation happens mostly for beams going through the core of the storm where occurrence of the hail or wet ice is highly probable.

One can "help" the algorithm by detecting the gates with wet ice and hail and supply them to the program. To find the gates where the probability to find hail is high, and estimate the "first guess" of fraction of reflectivity due to hail f_{ice} there, one can use the difference reflectivity factor Z_{dp} as it was proposed by Golestani et al. (1989):

$$Z_{dp} = 10 \log_{10}(Z_h - Z_v), \quad Z_h > Z_v,\ mm^6\ m^{-3}$$

$$Z_H/Z = f_{hail} = 1 - 10^{-0.1(\Delta Z)}$$

The "rain line" equation found from simulated Z_h, Z_{dp}:

$$Z_{dp} = 1.327 Z_h - 19.82$$

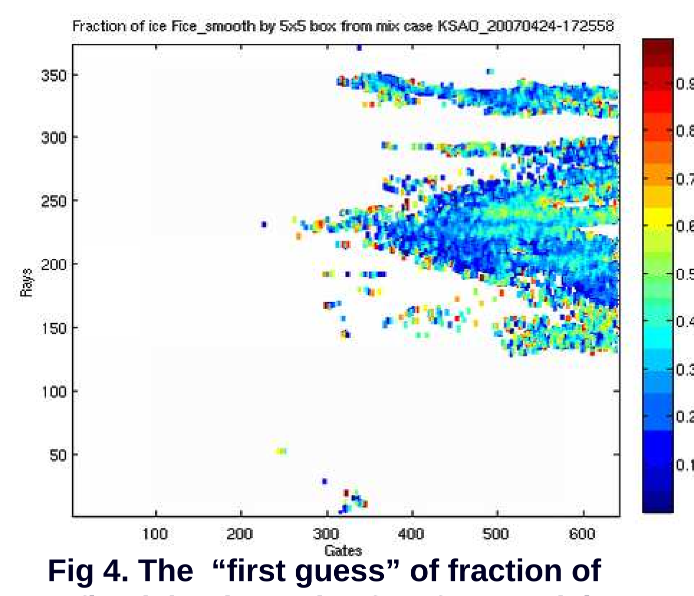

Fig 4. The "first guess" of fraction of reflectivity due to ice f_{ice} after spatial smoothing by 5x5 matrix, supplied to the FM.

The effect of modifications

One can see the general effect of the improvements discussed above: in the output of the modified version there are much less number of saturated gates (figure 5), the differential phase Φ_{dp} can be followed more precisely, as well as Z_{dr}, and f_{ice} looks much less spiky and more smooth, and hail is found in much more gates than before (figure 6):

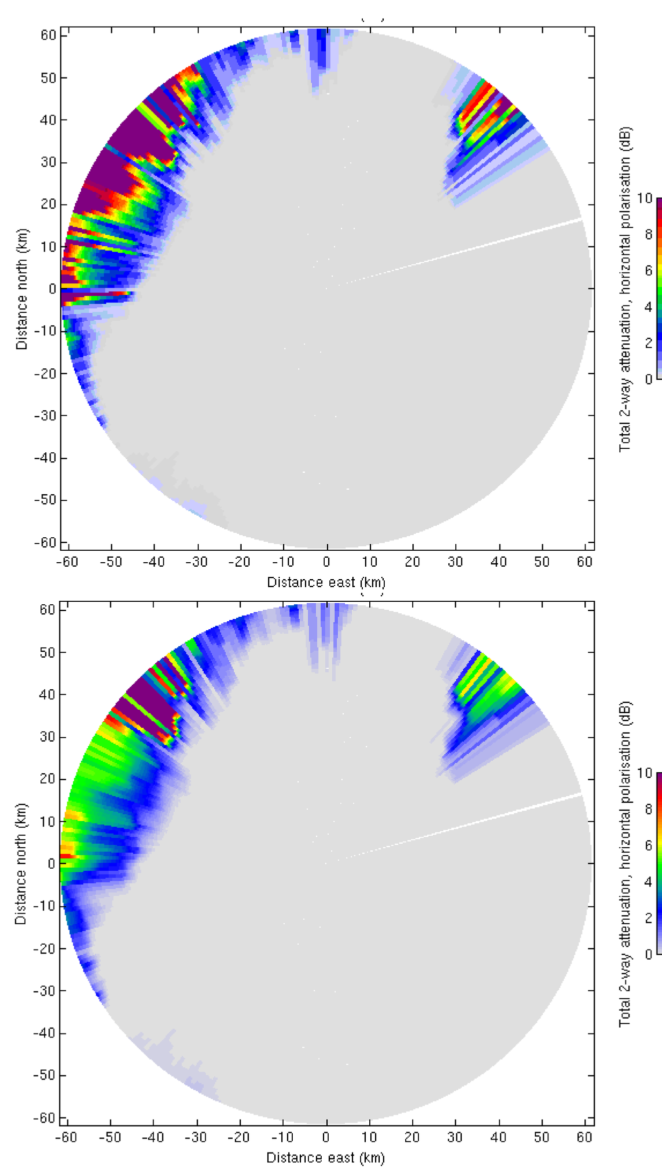

Fig 5. PIA, generated by original algorithm (top pane), and by modified algorithm (bottom pane). Dark brown color indicates "saturated" beams.

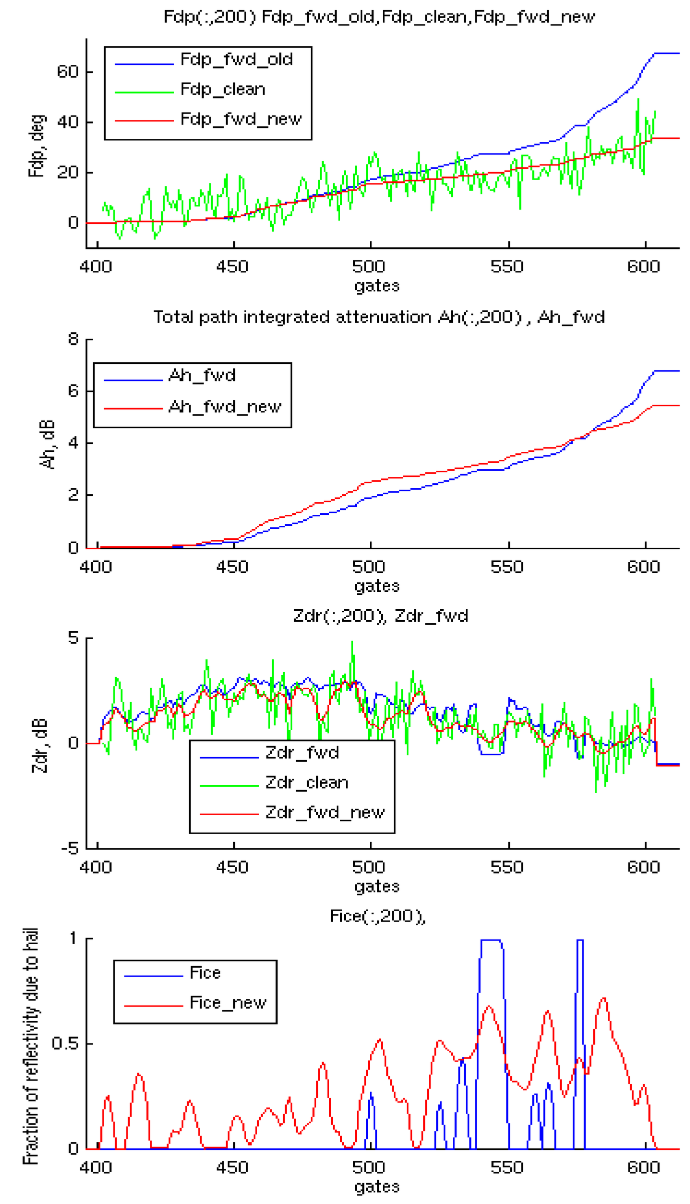

Fig 6. Comparison of the variables generated by original and modified algorithms for one beam: Φ_{dp}, A_h, Z_{dr}, f_{ice}.

Conclusions

The FM algorithm was adapted from S-band to X-band and applied to the CASA IP-1 dual-polarization radar data. The main goal was the correction of measured reflectivity in mixed phase precipitation (rain+hail) which cannot be achieved using differential phase constraints.

Several modifications to the original scheme were applied, with the general goal to increase the stability of the optimal estimation scheme. The principal ones being, (a) adjusting the observational errors based on Z_h and ρ_{hv}, (b) provide initial 'guess' for fraction of ice, and (c) sensitivity tests for the Z_h offset. Overall, it appears that the optimal estimation scheme can be adapted to X-band data for correction of attenuation due to mixed phase precipitation (rain mixed with wet ice/hail) provided the input data is well-calibrated (system offsets for Z_h, Z_{dr} and Φ_{dp}).

References

Bringi, V.N. and V. Chandrasekar, 2001: Polarimetric Doppler Weather Radar: Principles and Applications. Cambridge.

Golestani, Y., Chandrasekar, V., and Bringi, V. N. 1989: Intercomparison of multiparameter radar measurements. Prep., Radar Meteorol. Conf., 24th, 1989, 309-314.

Hogan R.J., 2007: A Variational Scheme for Retrieving Rainfall Rate and Hail Reflectivity Fraction from Polarization Radar. *J. Appl. Meteor. and Climat.*, 46, 1544.