Estimation of noise induced variances in the dual-pol moments using simple parametric model of the recorded signal.

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Why do we have to model received signal?

The statistical properties of the received signal and derived moments are quite well analyzed in the literature [Bringi 2001, Skolnik 1990, Doviak 1993, Melnikov 2004]. The direct analysis is quite complex and mostly limited to estimation of the variances using the first order approximation.

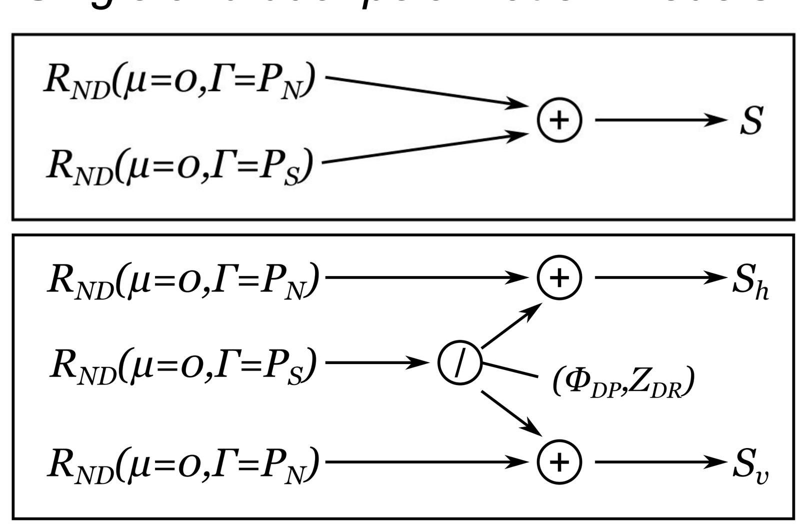
The ability to tabulate additional statistical parameters, such as confidence intervals, would be beneficial to the practical applications. The multi-equation dual-pol QPE would be one of those applications.

The Model

The first step toward that goal is to produce a simplest possible model of the received signal. The proposed model utilizes the following three approximations:

- he distribution of the random complex voltages S_N (aka I&Qs) of the noise is the circular symmetric complex normal distribution *with* $\Gamma = P_N$, where P_N is a noise power $P_N = E(S^2{}_N)$
- he complex voltages S_S of the signal from the precipitation are also described by the circular symmetric complex normal distribution with $\Gamma = P_S$, where P_S is a signal power $P_S = E(S^2{}_S)$
- the signal in the vertical(V) channel is the exact copy of the signal in the horizontal(H) channel, but scaled and phase-shifted to simulate Z_{DR} and Φ_{DP}
- the noise signals in H and V channels are independent/ uncorrelated.
- the spectral characteristics of the signal are irrelevant to all dual-pol moments.

Single and dual polarization models.



How it can be used?

The model is defined in terms of just three sources of normally distributed random complex numbers controlled by just four parameters – P_N, P_S, Z_{DR}, and Φ_{DP}. Almost all moments expected values and variances can be determined analytically within the framework of this model.

The model can be used as a core of the dual-polarization weather radar simulation, but, for completeness, one would have to apply a spectral filter to the white spectrum simulated signal in order to have realistic Doppler velocities and the Doppler width (W) moment.

The single most important use of the model is a study of moments properties via the Monte-Carlo simulation.

Why do we need a Monte-Carlo simulation?

The detection thresholds are determined by the required max percentage of false positive, which is a corresponding quantile of the moment values distribution. All computed dual-pol moments are not normally distributed, and, even if the variance of the moment is expressible analytically, the quantile function of the distribution is not known or tabulated.

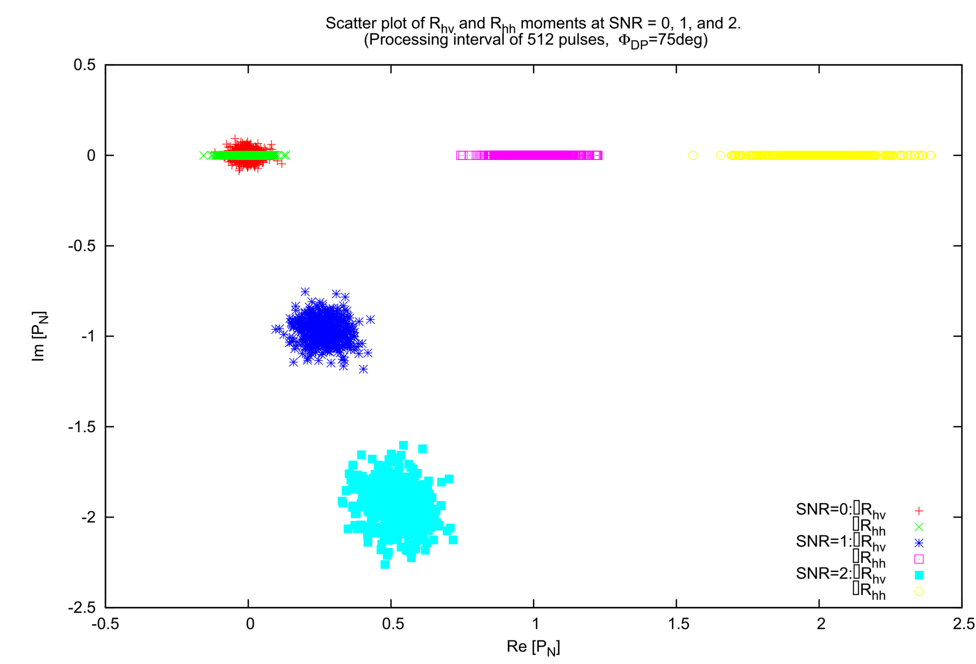
Some of the post-processing stages rely on the fuzzy logic classification to switch between computational algorithms. The Monte Carlo simulation is the only feasible and expedient way to validate those data processing stages.

Terms/Notation used here.

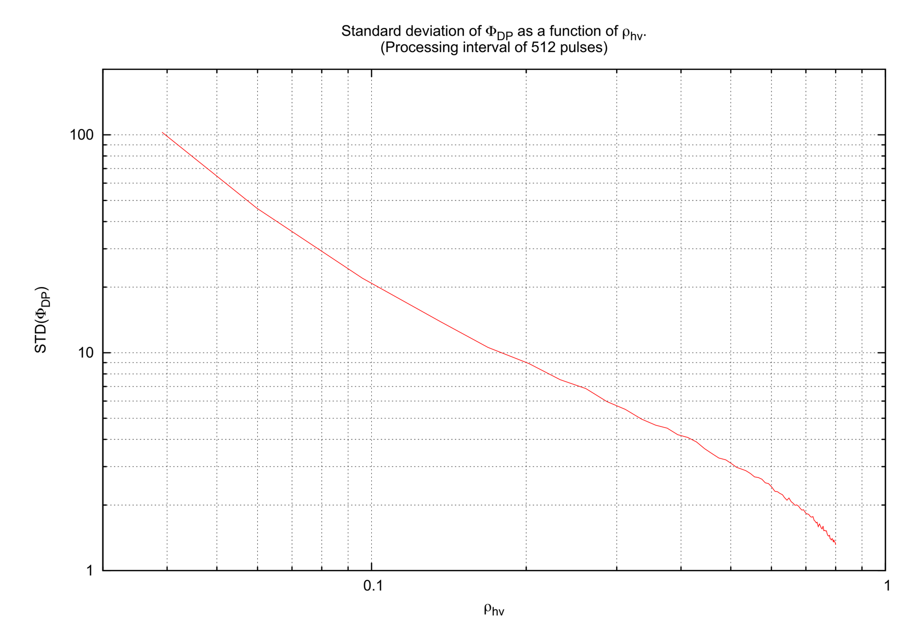
Each transmitted pulse produces two sequences of the per-range bin complex voltages – one for each polarization, H_i and V_i, where H and V were previously reffered as S_h and S_v. All dual-polarization moments and reflectivity Z are computed from three basic moments:

$$R_{hh} = (\Sigma_{i=0...N} H_i H_i^*)/N - P_N$$
$$R_{vv} = (\Sigma_{i=0...N} V_i V_i^*)/N - P_N$$
$$R_{hv} = (\Sigma_{i=0...N} H_i V_i^*)/N$$

Demo #1: R_{hh} and R_{hv} at different SNR



Demo #2: Standard deviation of Φ_{DP}



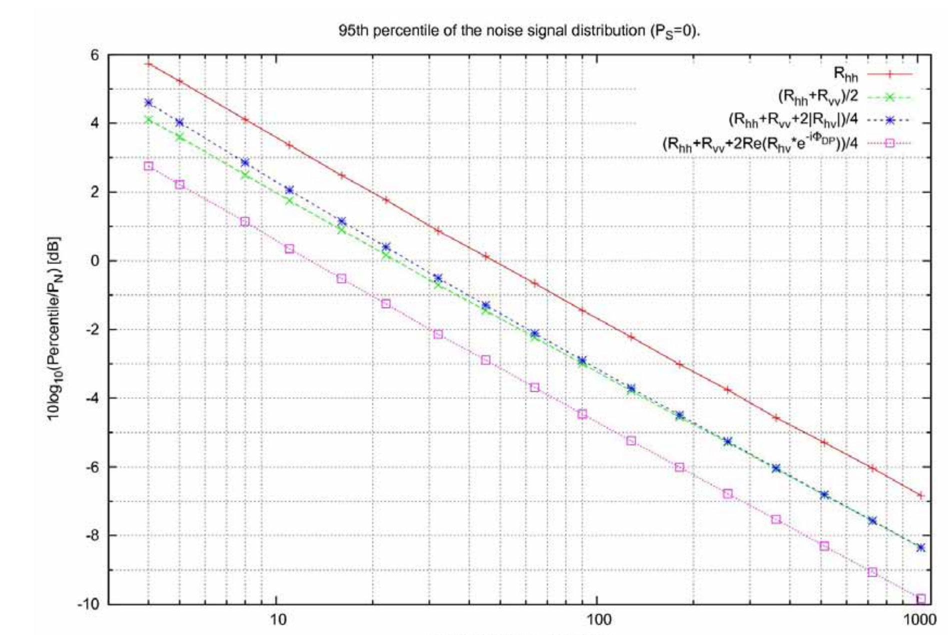
Demo #3: Analysis of the detection thresholds.

The dual-polarization weather radars have a multitude of advantages over single-polarization radars, but there is one small problem – the most important moment, reflectivity, is measured in just one of two channels. As a result, in the simultaneous transmission mode (STAR), the effective transmitted power is a half of the power transmitted by the single-polarization radar equipped with the same transmitter. The two-fold drop in the effective power results in the 3dB drop in the sensitivity. One can combine reflectivity from horizontal and vertical channels, but that combination can, in theory, get us back only 1.5dB. An obvious next step is to incorporate the cross term, R_{hv}, in the sum. But it is a complex number! There are two ways to reduce the full sum

$$R = (R_{hh} + R_{vv} + 2R_{hv})/4$$

to a real number. One way is to replace R_{hv} with $|R_{hv}|$, another is to replace R_{hv} with Re($R_{hv} exp(-i\Phi_{DP})$). Attempts to use the first approach did not produce a desirable result. One reasonable explanation is that taking absolute value of R_{hv} introduces upward bias when the signal is low. The second approach results in 3dB gain in reflectivity, effectively compensating for losses from the splitting power into two channels.

The use of Φ_{DP} to improve reflectivity data is feasible – it is possible to determine Φ_{DP} in all data bins in the first, preliminary processing stage and then to use obtained data to enhance reflectivity data in the secondary processing stage.



References

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