Clutter mitigation in a phased array radar system using the MMSE formulation

Tomoo Ushio, E. Yoshikawa, S. Yoshida, T. Morimoto, Z. I. Kawasaki (Osaka University)
F. Mizutani, M. Wada (Toshiba Corporation)
S. Satoh, and T. Iguchi (NICT)

Abstract

A new phased array radar system for meteorological application has been developed by Toshiba Corporation and Osaka University under a grant of NICT. It is now well known that rapidly evolving severe weather phenomena (e.g., microbursts, severe thunderstorms, tornadoes) are a threat to our lives particularly in a densely populated area and the number of such phenomena tends to increase as a result of the global warming. Over the past decade, mechanically rotating radar systems at the C-band or S-band have been proved to be effective for weather surveillance especially in a wide area more than 100 km in range. However, rapidly evolving weather phenomena have temporal and spatial scales comparable to the resolution limit (-10 min. and -500m) of typical S-band or C-band radar systems, and cannot be fully resolved with these radar systems. In order to understand the fundamental process and dynamics of such fast changing weather phenomena, volumetric observations with both high temporal and spatial resolution are required. The phased array radar system under developing has been required to have the unique capability of scanning the whole sky with 100m and 10 second resolution up to 30 km in a cost effective manner. To achieve this goal, the system adopts the digital beam forming technique for elevation scanning and mechanically rotates the array antenna in azimuth direction within 10 seconds. The radar transmits a broad beam of several degrees with 24 antenna elements and receives the back scattered signal with 128 elements digitizing at each elements. Then by digitally forming the beam in the signal processor, the fast scanning is realized. Although the phased array radar system using the digital beam forming technique can estimate the 3 dimensional structure of the precipitation system within 10 seconds with 100 meter resolution, the received signal may also be seriously contaminated by the relatively high received power from ground clutter and strong precipitation echoes through the side lobes of the transmitting beam. To avoid this problem, a beam forming technique using the MMSE (Minimum Mean Square Error) formulation has been proposed and tested in this paper. This approach can adaptively mitigate the masking interference that results from the standard digital beam forming method in the vicinity of ground clutter and strong precipitation area. The proposed method is compared with the standard beam forming technique and Capon's method. The simulation results show that the proposed technique can correctly estimate the precipitation echo within a few dB even in the presence of a strong ground clutter that is more than 30 dB higher than the precipitation echo with 15 pulse repetition number. The MMSE based technique is shown to be superior to the standard DBF scenarios under the small number of pulse repetitions to achieve the rapid scanning.

Introduction

In order to detect rapidly developing hazardous weather phenomena and provide the warning information for the hazard, weather radar system using a phased array antenna system, which achieves high speed scan, has been attracting attention. Collaborative Adaptive Sensing of Atmosphere (CASA) has proposed a phased array radar network to efficiently observe precipitation by an electronic scan steering beams for weather phenomena adaptively [1]. In [2], a multi-function phased array radar for not only detecting weather phenomena but controlling air traffic and tracking non-cooperative airplanes also by using a rapid scan has proposed.

At present we are developing a new phased array weather radar (PAWR) to rapidly scan and finely detect hazardous weather phenomena, such as tornadoes and downbursts whose lifetime is about several min and the horizontal scale is from tens to hundreds of meters, with 10 sec per volume scan (VoS) and 100 m 3-D mesh. The PAWR equips 1-D array antenna consisting of 128 elements for elevation as shown in Fig. 1. A fan beam (a beam width of roughly 30 deg) is transmitted due to feeding power into only 20 elements (the number of feeding elements is variable for central elevation angles). After receiving and sampling back scattered signals from precipitation particles with all 128 elements into 128 analog-to-digital converters (ADC), adaptive digital beam forming (DBF) is performed to sharpen the fan beam to a beam width of about 1 deg. Thus, precipitation in several elevation angles are simultaneously observed and temporal resolution is drastically improved 10 times higher than a general configuration using sharp beams in both transmission and reception in this case. This configuration achieves 10 sec VoS while transmitting 15 pulses for a direction. The array element is a slotted waveguide antenna whose beam widths are 80 and 1 deg for elevation and azimuth, respectively. In the demanded coverage of 30 km, 100 m 3-D mesh is achieved by the 1deg beams.

A significant problem of the fan beam transmission is huge side-lobes of strong echoes from strong precipitation cells or clutters, which is two times more than a sharp beam transmission and reception in dB. For example, while a sharp beam transmission and reception has a first side-lobe level of -39.2 dB from the main-lobe, a configuration of uniform and sharp beams has a first side-lobe level of -18.6 dB in the same angle. For the side-lobe reduction in phased array antenna system, adaptive DBF methods have been proposed. In atmospheric radars observing a scattering volume filled with particles, Capon method is used[3]. Though the Capon weighting function has been proved to be effective, the Capon approach needs sufficient number of samples (pulses) to estimate the reflectivity accurately and cannot work well by 15 pulses in the PAWR. On the other hand, an approach for solid direction-of-arrival (DOA) based on minimum mean-square error (MMSE) framework has been proposed [4], [5]. This approach does not employ spatial sample covariance information, which degrade DOA estimation in Capon or some other DOA method in case that correlated signals are scattered from different directions, and therefore, the number of pulses are not essential. In this paper, a MMSE approach for side-lobe reduction and resolution enhancement for scattering volume filled with precipitation particles with fewer pulses on the PAWR.

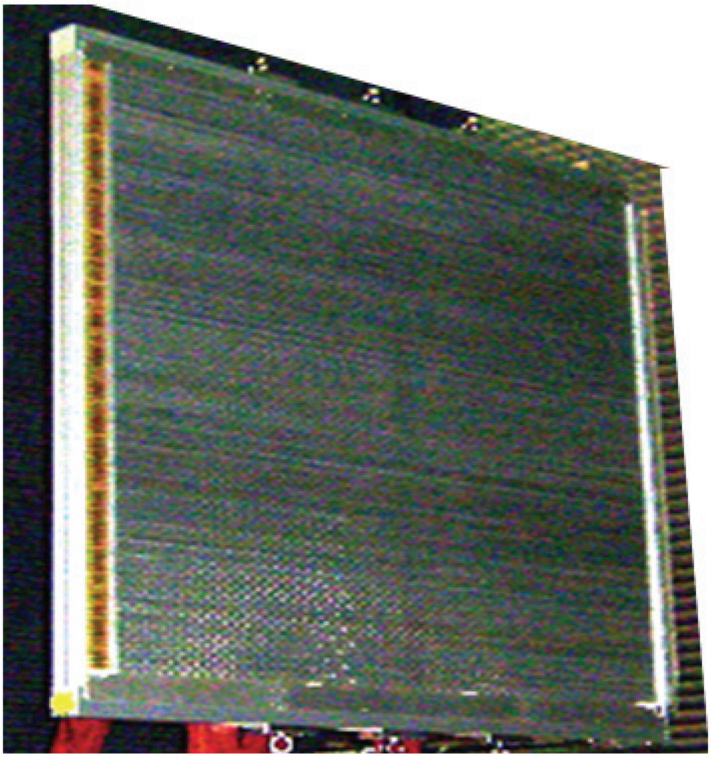

Figure 1 Photograph of the phase array antenna. 128 slot antenna element are aligned vertically for the rapid scanning.

Methodology

A. Signal Model

Assuming that a linear-spaced phased array antenna with N antenna elements, l th time sample of a received complex amplitude $\mathbf{y}_l$ ($N\times 1$-vector) is expressed by an associated complex amplitude $\mathbf{x}_l$ ($M\times 1$-vector, $M\gg N$), an $N\times M$-vector $\mathbf{S}$ which consists of spatial steering vector $\mathbf{s}(\theta)$, and an additional Gaussian noise vector $\mathbf{v}_l$ as

$$\mathbf{y}_l = \mathbf{S}\mathbf{x}_l + \mathbf{v}_l, \qquad (1)$$

where

$$\mathbf{y}_l = \begin{bmatrix} y_{l,0} & y_{l,1} & \cdots & y_{l,N-1} \end{bmatrix}^T , \qquad (2)$$

$$\mathbf{x}_l = \begin{bmatrix} x_{l,0} & x_{l,1} & \cdots & x_{l,M-1} \end{bmatrix}^T , \qquad (3)$$

$$\mathbf{S} = [\mathbf{s}(\theta_0) \quad \mathbf{s}(\theta_1) \quad \cdots \quad \mathbf{s}(\theta_{M-1})] , \qquad (4)$$

$$\mathbf{a}(\theta) = [1 \quad \exp(-j\omega_1(\theta)) \quad \cdots \quad \exp(-j\omega_{N-1}(\theta))]^T , \qquad (5)$$

and

$$\omega_n(\theta) = (2\pi/\lambda) dn \sin\theta . \qquad (6)$$

$[\bullet]^T$ is transpose. In the PAWR, θ means elevation angle, and $\mathbf{x}$ corresponds M-separated precipitation profiles in elevation angles from 0 to 90 deg.

In adaptive array signal processing, estimated precipitation profiles is calculated as

$$\hat{\mathbf{x}}_l = \begin{bmatrix} \hat{x}_{l,0} & \hat{x}_{l,1} & \cdots & \hat{x}_{l,M-1} \end{bmatrix}^T , \qquad (7)$$

$$\hat{x}_{l,m} = \mathbf{w}_m^H \mathbf{y}_l , \qquad (8)$$

where $\mathbf{w}$ is an $N \times 1$ complex weighting vector for received complex amplitudes of each antenna element. $[\bullet]^H$ is complex-conjugate transpose.

B. Fourier Beam Forming (FR)

Fourier beam forming (FR) is the most basic method in phased array radars to steer a beam for a direction by uniform phase shift. In FR, weighting complex vector (FR weight) is expressed as

$$\mathbf{w}_{FRm} = \mathbf{s}(\theta_m) . \qquad (9)$$

$[\bullet]^*$ is complex-conjugate. Precipitation profile estimated by using FR weight is equivalent to a result of Fourier transform of $\mathbf{y}_l$.

C. Capon Beam Forming (CP)

Capon beam forming method (CP) was proposed for atmospheric research in middle and upper (eg. Palmer et al., 1998). CP minimizes received power subject to the constraint in a desired direction is constant. In CP, weighting complex vector is expressed as

$$\mathbf{w}_{CP\,m} = \frac{\mathbf{R}_y^{-1}\mathbf{s}(\theta_m)}{\mathbf{s}^H(\theta_m)\mathbf{R}_y^{-1}\mathbf{s}(\theta_m)}, \qquad (10)$$

where

$$\mathbf{R}_y = \frac{1}{L}\sum_{l=1}^{L}\mathbf{y}_l\mathbf{y}_l^H . \qquad (11)$$

L is the number of time samples.

To calculate CP weight in a stable manner and fast, steepest descent method is used as in other applications.

D. MMSE Beam Forming

To estimate received power accurately, we apply a concept of gain-constrained APC, whose constrained cost function is

$$J_{l,m} = E\left[\left|x_{l,m} - \mathbf{w}_{MMSE\,m}^H\mathbf{y}_l\right|^2\right], \qquad (12)$$

where $E[\bullet]$ is expectation, $\mathrm{Re}\{\bullet\}$ is the real part of the argument, and λ is a Lagrange multiplier. Minimizing Eq. (12), weighting complex vector of MMSE beam forming (MMSE weight) is expressed as

$$\mathbf{w}_{MMSE\,m} = \left(\mathbf{S}\mathbf{R}_x\mathbf{S}^H + \mathbf{R}_v\right)^{-1} E\left[\left|x_{l,m}\right|^2\right]\mathbf{s}(\theta_m), \qquad (13)$$

$\mathbf{R}_x$ is a covariance matrix of $\mathbf{x}$ as

$$\mathbf{R}_x = E\left[\mathbf{x}\mathbf{x}^H\right] \approx \left[\frac{1}{L}\sum_{l=1}^{L}\mathbf{x}_l\mathbf{x}_l^H\right] * \mathbf{I}_{M\times M} . \qquad (14)$$

* is the Hadamard product and $\mathbf{I}_{M\times M}$ is an $M\times M$ identity matrix. Eq. (15) is on the assumption that signal souses from different positions are temporally uncorrelated. $\mathbf{R}_v$ is a noise covariance matrix expressed as

$$\mathbf{R}_v = E\left[\mathbf{v}\mathbf{v}^H\right] \approx \sigma_v\mathbf{I}_{M\times M} \qquad (15)$$

where σ_v is variance of thermal noise.

MMSE weight and solution are calculated iteratively with a use of a prior information as follows.

1) Prior Information

As a prior information, solution of FR is substituted to Eq. (14).

$$\mathbf{R}_x\Big|_0 = \left[\frac{1}{L}\sum_{l=1}^{L}\hat{\mathbf{x}}_{FR\,l}\hat{\mathbf{x}}_{FR\,l}^{H}\right] * \mathbf{I}_{M\times M} \quad , \qquad (16)$$

where

$$\hat{\mathbf{x}}_{FR\,l} = \left[\hat{x}_{FR\,l,0} \quad \hat{x}_{FR\,l,1} \quad \cdots \quad \hat{x}_{FR\,l,M-1}\right]^T , \qquad (17)$$

$$\hat{x}_{FR\,l,m} = \mathbf{w}_{FR\,m}^{H}\mathbf{y}_l \qquad (18)$$

2) Determination of MMSE Weights

M MMSE weights are determined by i-iterative $\mathbf{R}_x$.

$$\mathbf{w}_{MMSE\,m}\Big|_i = \left(\mathbf{S}\mathbf{R}_x\Big|_i\mathbf{S}^H + \mathbf{R}_v\right)^{-1}\mathbf{R}_x^{(m,m)}\Big|_i\mathbf{s}(\theta_m) \qquad (19)$$

where $\mathbf{R}_x^{(m,m)}$ is the *(m,m)* th element of $\mathbf{R}_x$

3) Computation of MMSE solutions

M MMSE solutions are calculated by i-iterative $\mathbf{w}_{MMSE\,m}$.

$$\hat{\mathbf{x}}_{MMSE\,l}\Big|_i = \left[\hat{x}_{MMSE\,l,0}\Big|_i \quad \hat{x}_{MMSE\,l,1}\Big|_i \quad \cdots \quad \hat{x}_{MMSE\,l,M-1}\Big|_i\right]^T , \quad (20)$$

where

$$\hat{x}_{MMSE\,l,m}\Big|_i = g^{-1}\mathbf{w}_{MMSE\,m}^{H}\Big|_i\mathbf{y}_l \,. \qquad (21)$$

g is gain-control factor defined as

$$g = \frac{\mathbf{w}_{MMSE\,m}^{H}\Big|_i\mathbf{SBB}^H\mathbf{S}^H\mathbf{w}_{MMSE\,m}\Big|_i}{\mathbf{w}_{FR}^{H}\mathbf{SBB}^H\mathbf{S}^H\mathbf{w}_{FR}} \qquad (22)$$

where in matrix B, diagonal elements contributing main-lobe of FR are 1 and the others are 0.

4) Re-iteration

i+1-iterative $\mathbf{R}_x$ is calculated by i-iterative MMSE solutions.

$$\mathbf{R}_x\Big|_{i+1} = \left[\frac{1}{L}\sum_{l=1}^{L}\hat{\mathbf{x}}_{MMSE\,l}\Big|_i\hat{\mathbf{x}}_{MMSE\,l}^{H}\Big|_i\right] * \mathbf{I}_{M\times M} \quad , \qquad (23)$$

And then, go back to 2).

Numerical Simulations

In order to assess the basic performance of the proposed method based on the MMSE formulation, one example of the simulation results for point target is shown in Figure 2. In this simulation, one dimensional array antenna with 128 elements spacing 16.5 mm between each element is used and the noise power is assumed to be 10 dB. The number of pulse repetition is 15 in this case, and the target is located at the 2 degree and -2 degree direction with the receiving power of 40 dB and 2.5 dB, respectively. The upper panel in Figure 2 shows the result from the conventional beam former method, while the middle and bottom panels are from Capon and MMSE formulation, respectively. As is clearly shown here, in the beam former method the targets located at 2 degree is detected but the weak echo nearby (in this case -2 degree direction) is obscured by the side lobes and cannot be detected. On the other hand, Capon and MMSE approach can clearly detect the two point targets and succeed in discriminating the adjacent echo pairs. Comparing the Capon and MMSE method, the narrower beam width is achieved in the MMSE formulation which enables the high resolution detection of the target.

Comparison of the three methods for the precipitation signals with ground clutter echoes is shown in Figure 3. The truth data is shown in black line in this figure, and the strong ground clutter echoes are distributed in 30 degree direction with maximum power of 84 dB. Note that the MMSE based method (red line) can detect both the precipitation and ground clutter echoes properly. The orange line is the result of the MMSE method without any iteration while the red line is obtained by repeating the procedure 4 times. It is shown that the iterative scheme proposed in this paper is effective to lower the side lobe level due to the strong ground clutter or strong precipitation echoes nearby.

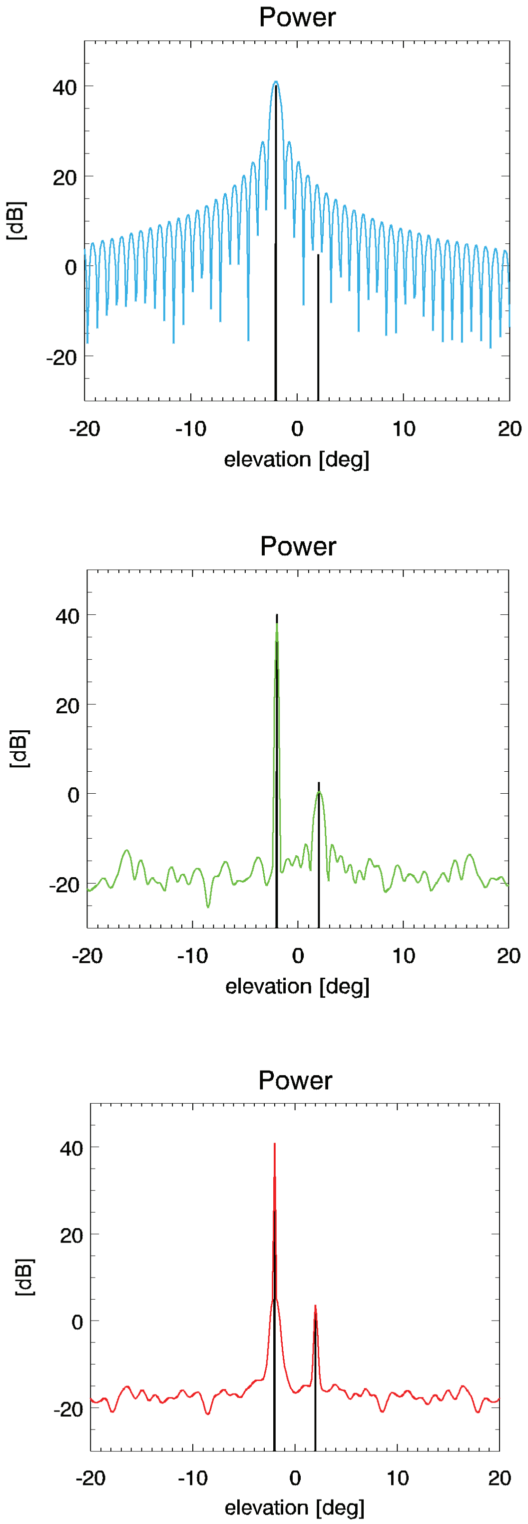

Figure 2 Simulation results for the point target. Upper panel shows the result from the conventional beam former method, and the middle panel shows the result from the Capon method. The bottom

panel is from the MMSE approach.

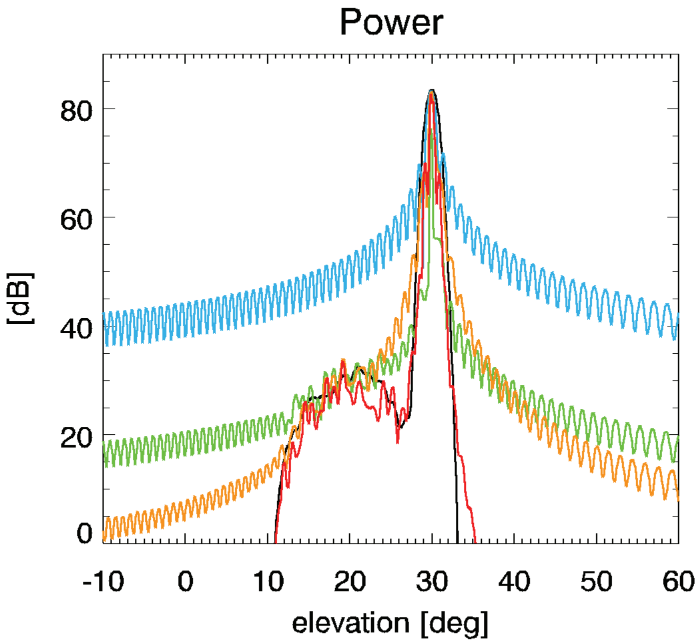

Figure 3 Simulation result for the precipitation and ground clutter echoes. Black line is the truth data and the ground clutter is distributed around 30 degrees while the precipitation echoes are from 10 to 30 degrees. The blue line is thre result from the conventional beam former method, and the green line is from the Capon method. The orange line is from the proposed method with no interation. The red line is from the proposed method with 4 iterations.

References

[1] F. Junyent, and V. Chandrasekar, “Theory and characterization of weather radar networks,” *J. Atmos. Ocean. Technol.*, vol. 26, pp. 474-491, 2009.

[2] D. S. Zrnic, J. F. Kimpel, D. E. Forsyth, A. Shapiro, G. Crain, R. Ferek, J. Heimmer, W. Benner, T. J. McNellis, and R. J. Vogt, “Agile-Beam Phased Array Radar for Weather Observations,” *Bulletin of the American Meteorological Society*, Vol. 88, pp. 1753-1766, 2007.

[3] R. Palmer, S. Gopalam, T.-Y. Yu, and S. Fukao, “Coherent radar imaging using Capon's method,” *Radio Sci.*, 33(6), 1585-1598, 1998

[4] S. D. Blunt and K. Gerlach, “Adaptive Pulse Compression via MMSE Estimation,” *IEEE Transactions on Aerospace and Electronic Systems,* Vol. 42, pp. 572-584, 2006.

[5] S. D. Blunt, T. Chan, and K. Gerlach, “A New Framework For Direction-of-Arrival Estimation”, *proceedings on Sensor Array and Multichannel Signal Processing Workshop*, 2008, pp. 81-85.