Degree of Polarization at Simultaneous Transmit: Theoretical Aspects

Michele Galletti[1], Dusan S. Zrnic[2]

[1] Brookhaven National Laboratory – Department of Environmental Sciences, Upton, NY

[2] National Oceanic and Atmospheric Administration – National Severe Storms Laboratory (NOAA-NSSL)

120 David L. Boren Blvd. Norman, OK

Corresponding Author: mgalletti@bnl.gov

Abstract— We consider weather radar measurements at simultaneous transmission and simultaneous reception of horizontal and vertical polarizations, and show that the degree of polarization at simultaneous transmit (p_s) is related to differential reflectivity and copolar correlation coefficient at simultaneous transmit (namely, Z_{DR}^{s} and ρ_{hv}^{s}). We evaluate the potential of degree of polarization at simultaneous transmit for weather radar applications. Ultimately, we explore the consequences of adjusting the transmit polarization state of dual-pol weather radars to circular polarization.

Keywords: copolar correlation coefficient, differential reflectivity, degree of polarization at simultaneous transmit, simultaneous transmission mode.

I. Simultaneous Transmission and Simultaneous Reception of H and V (STSR Mode)

The so-called STSR mode (Simultaneous Transmission – Simultaneous Reception, also known as hybrid mode or Z_{DR} mode) consists in transmitting a polarization state (χ), lying on the circular/slant circle of the Poincare sphere (1) and receiving the backscattered signal in the horizontal and vertical polarimetric channels. This mode of operation was chosen for the operational implementation of polarimetry in the US NEXRAD network, so that not only spectral moments (reflectivity Z_H^S, velocity V, spectrum width σ_v) but also polarimetric moments (differential reflectivity Z_{DR}^S, copolar correlation coefficient ρ_{hv}^S and differential phase Φ_{DP}^S) can be made available, both as real-time products and as archived data. The superscript s stands for simultaneous transmission, and reminds the polarimetric mode used to retrieve the moments of interest.

The phase difference β between the signals injected in the H and V ports is constant from pulse to pulse and is determined by the radar architecture. This phase difference ultimately establishes the actual radiated polarization state.

$$\boldsymbol{\chi} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ e^{i\beta} \end{bmatrix}$$

(1)

Since the signal is simultaneously received in the horizontal and vertical polarization channels, the Coherency matrix $\mathbf{J}_{\boldsymbol{\chi}}^{\mathbf{HV}}$ (the superscript **HV** indicates the receive polarization basis is horizontal-vertical) is measured [2- 4].

$$\mathbf{J}_{\boldsymbol{\chi}}^{\mathbf{HV}} \equiv \begin{bmatrix} \langle |s_{h\chi}|^2 \rangle & \langle s_{h\chi} s_{v\chi}^* \rangle \\ \langle s_{v\chi} s_{h\chi}^* \rangle & \langle |s_{v\chi}|^2 \rangle \end{bmatrix}$$

(2)

For a general target with scattering matrix **S**,

$$\mathbf{S} = \begin{bmatrix} s_{hh} & s_{hv} \\ s_{vh} & s_{vv} \end{bmatrix}$$

(3)

The entries of the Coherency matrix at Simultaneous Transmission can be expressed as:

$$\mathbf{J}_{\boldsymbol{\chi}}^{\mathbf{HV}} \equiv \begin{bmatrix} \langle |s_{h\chi}|^2 \rangle & \langle s_{h\chi} s_{v\chi}^* \rangle \\ \langle s_{v\chi} s_{h\chi}^* \rangle & \langle |s_{v\chi}|^2 \rangle \end{bmatrix} = \begin{bmatrix} \langle |s_{hh} + s_{hv}|^2 \rangle & \langle (s_{hh} + s_{hv})(s_{vv} + s_{vh})^* \rangle \\ \langle (s_{vv} + s_{vh})(s_{hh} + s_{hv})^* \rangle & \langle |s_{vv} + s_{vh}|^2 \rangle \end{bmatrix}$$

(4)

From the matrix $\mathbf{J}_{\boldsymbol{\chi}}^{\mathbf{HV}}$, reflectivity ($Z_H^S$), differential reflectivity (Z_{DR}^S), copolar correlation coefficient (ρ_{hv}^S), degree of polarization at simultaneous transmission (p_S) and differential phase ($\Phi_{hv}^S + \delta_{hv}^S$) can be evaluated for radars operating at hybrid mode:

$$Z_H^S \propto \langle |s_{h\chi}|^2 \rangle$$

(5)

$$Z_{DR}^S \equiv \frac{\langle |s_{h\chi}|^2 \rangle}{\langle |s_{v\chi}|^2 \rangle}$$

(6)

$$\rho_{hv}^S \equiv \frac{|\langle s_{h\chi} s_{v\chi}^* \rangle|}{\sqrt{\langle |s_{h\chi}|^2 \rangle \langle |s_{v\chi}|^2 \rangle}}$$

(7)

$$\left(\Phi_{hv}^{S} + \delta_{hv}^{S}\right) \equiv \arg\langle s_{h\chi} s_{v\chi}^{*} \rangle \tag{8}$$

$$p_S = \sqrt{1 - \frac{4\det\left[\mathbf{J}_{\chi}^{\mathbf{HV}}\right]}{\left(\mathrm{trace}\left[\mathbf{J}_{\chi}^{\mathbf{HV}}\right]\right)^2}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \tag{9a}$$

$$\mathrm{trace}\left[\mathbf{J}_{\chi}^{\mathbf{HV}}\right] \equiv \langle |s_{h\chi}|^2 \rangle + \langle |s_{v\chi}|^2 \rangle = \lambda_1 + \lambda_2 \tag{9b}$$

$$\det\left[\mathbf{J}_{\chi}^{\mathbf{HV}}\right] \equiv \langle |s_{h\chi}|^2 \rangle \langle |s_{v\chi}|^2 \rangle - |\langle s_{h\chi} s_{v\chi}^{*} \rangle|^2 = \lambda_1 \cdot \lambda_2 \tag{9c}$$

In (9), λ_1 and λ_2 are the eigenvalues of $\mathbf{J}_{\chi}^{\mathbf{HV}}$**.**

Simultaneous transmission implies that in presence of cross-polarizing scatterers ($s_{hv} > 0$), differential reflectivity and copolar correlation coefficient will differ from the corresponding variables measured at Alternate Transmit and Simultaneous Reception mode (ATSR mode) [2].

$$\rho_{hv}^{S} \equiv \frac{|\langle (s_{hh} + s_{hv})(s_{vv} + s_{vh})^{*} \rangle|}{\sqrt{\langle |s_{hh} + s_{hv}|^2 \rangle \langle |s_{vv} + s_{vh}|^2 \rangle}} \neq \frac{|\langle s_{hh} s_{vv}^{*} \rangle|}{\sqrt{\langle |s_{hh}|^2 \rangle \langle |s_{vv}|^2 \rangle}} \equiv \rho_{hv} \tag{10}$$

$$Z_{DR}^{S} \equiv \frac{\langle |s_{hh} + s_{hv}|^2 \rangle}{\langle |s_{vv} + s_{vh}|^2 \rangle} \neq \frac{\langle |s_{hh}|^2 \rangle}{\langle |s_{vv}|^2 \rangle} \equiv Z_{DR} \tag{11}$$

Interest in the degree of polarization at simultaneous transmission [5-8] is motivated by the fact that it is not intrinsically biased by cross-polarizing scatterers, that is, its physical meaning is preserved across the spectrum of all possible scatterers, both with low and high LDR (linear depolarization ratio). We manipulate the definition in (9) to obtain an important theoretical relationship, valid in general [3, 4].

$$(1 - p_S^2) = \frac{4 \cdot Z_{DR}^{S}}{\left[1 + Z_{DR}^{S}\right]^2} \left(1 - \left[\rho_{hv}^{S}\right]^2\right) \tag{12}$$

The relation in (12) shows that the degree of polarization at simultaneous transmission (p_S) can be obtained from differential reflectivity Z_{DR}^{S} and copolar correlation coefficient ρ_{hv}^{S}. This identity is important for both theoretical and practical reasons. The most prominent practical consequence of the identity in (12) is that the degree of polarization at simultaneous transmission can be computed from processed polarimetric moments (Z_{DR}^{S} and ρ_{hv}^{S}), that is, access to the raw I and Q samples is not strictly necessary. Further, we have that

$$\frac{4 \cdot Z_{DR}^{S}}{\left[1 + Z_{DR}^{S}\right]^2} = \left[\frac{\sqrt{\langle|s_{h\chi}|^2\rangle \langle|s_{v\chi}|^2\rangle}}{\frac{\langle|s_{h\chi}|^2\rangle + \langle|s_{v\chi}|^2\rangle}{2}}\right]^2 \leq 1 \tag{13}$$

Since the ratio of geometrical to arithmetical mean is always less or equal than 1, it follows that, for any type of scatterers (prolate, oblate or isotropic), the following relation holds:

$$0 \leq \rho_{hv}^{S} \leq p_S \leq 1 \tag{14}$$

The relation in (14) shows that the degree of polarization at simultaneous transmit (p_S) is always larger than the copolar correlation coefficient at STSR mode (ρ_{hv}^{S}). For the specific case of isotropic weather scatterers (light rain, hail or graupel), for which intrinsic Z_{DR} is equal to 1 (linear units), differential reflectivity at simultaneous transmit is also 1 ($Z_{DR}^{S} = 1$), regardless of the intrinsic LDR value of the scatterers. So, for the particular case of isotropic scatterers, we obtain that the copolar correlation coefficient at simultaneous transmission is equal to the degree of polarization.

$$p_S = \rho_{hv}^{S} \tag{15}$$

This theoretical result is relevant since it permits to assign a physical meaning to the copolar correlation coefficient at simultaneous transmit in presence of isotropic, depolarizing scatterers ($s_{hv} > 0$).

In Fig. 1 we report plots of the identity in (12). On the abscissa is Z_{DR}^{S}, differential reflectivity in logarithmic units (generally used in practical radar meteorological analysis), on the ordinate is the difference between degree of polarization at simultaneous transmit and copolar correlation coefficient at simultaneous transmit. The different curves are for different values of ρ_{hv}^{S} (copolar correlation coefficient at simultaneous transmit), indicated on the right of the panels. Fig. 1 confirms the result in (15), that is, for isotropic targets (Z_{DR} = 0 dB) the copolar correlation coefficient at STSR mode is equal to the degree of polarization.

For scatterers with high copolar correlation coefficient (> 0.99, rain, ice crystals) the difference between p_S and ρ_{hv}^S is in practice negligible (Fig. 1A). Differences between p_S and ρ_{hv}^S are expected only for large (absolute value) differential reflectivity and low copolar correlation coefficient, like in the case of heavy rain mixed with irregularly shaped hail, melting band or biological scatterers (birds, bugs, bats). The analysis of the identity in (12) suggests that degree of polarization and copolar correlation coefficient will often display similar patterns, consistently with what reported in [5, 6] where rain and ice crystals are analyzed. Note however that the degree of polarization (p_S) always adheres to its physical meaning (ratio of polarized to total power), whereas the copolar correlation coefficient (ρ_{hv}^S), in presence of cross-polarizing scatterers (LDR > 0) departs from its intended physical meaning (coherence between the horizontal and vertical polarimetric channels).

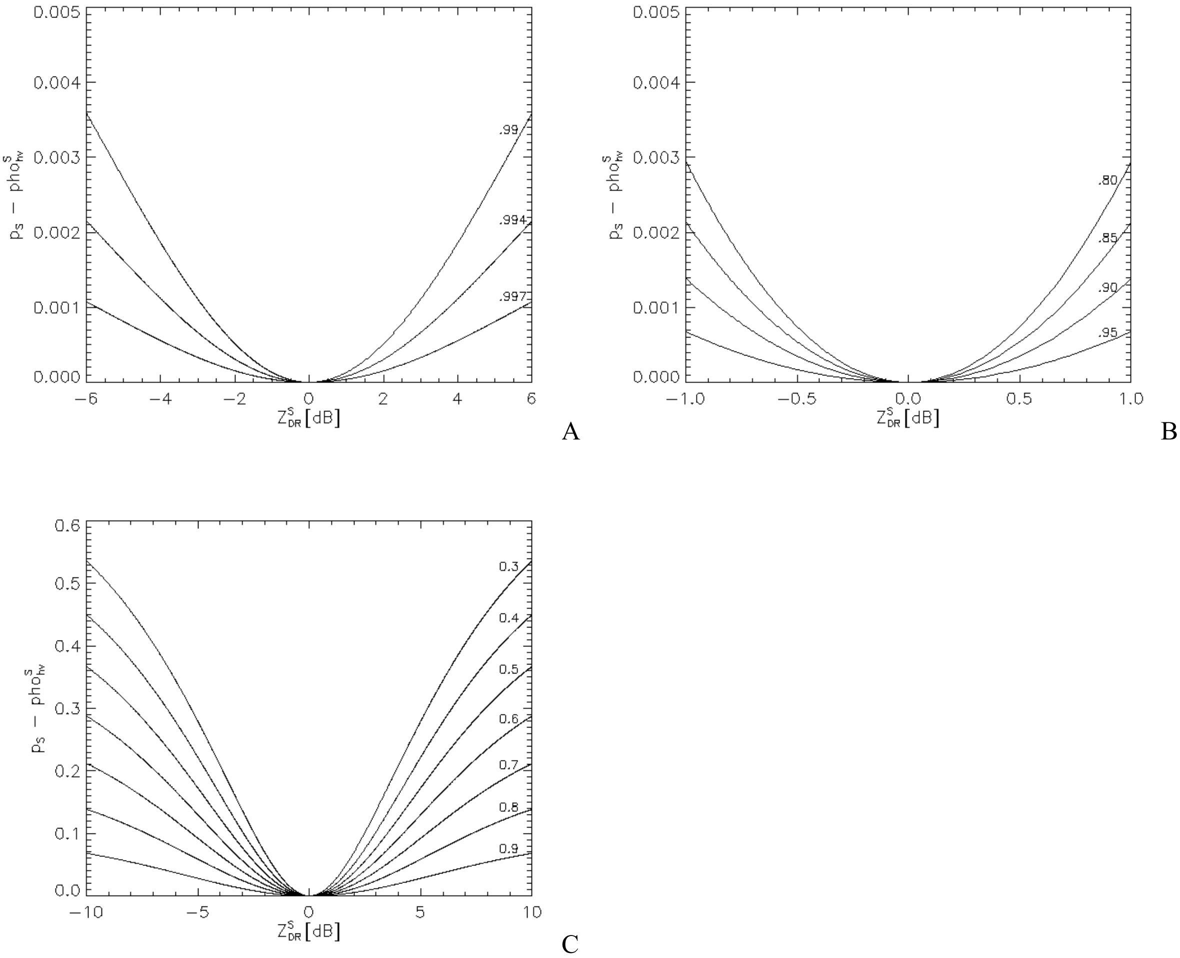

Fig 1. Plots of the identity in (12). On the abscissa is differential reflectivity at STSR mode (logarithmic units), on the ordinate is the difference between degree of polarization at simultaneous transmit (p_S) and copolar correlation coefficient at simultaneous transmit (ρ^S_{hv}). The different curves correspond to different numerical values of ρ^S_{hv}, indicated on the right of each curve. For isotropic targets (Z_{DR} = 0 dB), the degree of polarization is equal to the copolar correlation coefficient. For high ρ^S_{hv} scatterers (> 0.99) the difference between p_S and ρ^S_{hv} is in practice negligible (1A). Differences between the degree of polarization and the copolar correlation coefficient are to be expected for targets with low ρ^S_{hv} and large Z^S_{DR} (1C).

II. Circular Polarization Transmit

In the rest of this letter we discuss the effects of the system transmit differential phase (parameter β in (1)) on polarimetric measurements. In order to minimize the bias in ρ_{hv}^{S} and Z_{DR}^{S}, this phase should be chosen to be either 0° or 180°, that is, transmission of slant linear polarization is preferable [11 -15]. For example, this choice will minimize the appearance of radial stripes in Z_{DR}^{S} due to coherent forward scattering from aligned ice crystals [14]. Also, the system transmit differential phase (combined with the propagation differential phase) has a significant impact on the degree of polarization at simultaneous transmission [5, 6]. Adjusting the system differential phase to a desired value is generally achievable with phased array antennas, but is more challenging with parabolic reflectors. Even though transmission of slant linear polarization (β = 0°, 180°) is preferable to minimize the bias in polarimetric variables, in the following we consider the particular case of circular polarization transmission (β = ±90°). Such implementation of dual-polarization technology is found both in weather radars (circular transmit, horizontal and vertical receive), and air traffic control radars (circular transmit, right-hand and left-hand circular receive).

A. *Dual- polarization radar at circular transmit*

We consider a dual-polarization radar transmitting circular polarization, with simultaneous reception of left-hand circular (LHC) and right-hand circular (RHC) polarizations. Such radars were used in the early days of radar meteorology [17, 18] and are operationally used nowadays for airport and air route surveillance by the ASR-9 and the ARSR-4 radars. This polarimetric mode permits the measurement of the Coherency matrix at circular polarization [2].

$$\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}} = \begin{bmatrix} \langle |S_{ll}|^2 \rangle & \langle S_{ll} S_{rl}^* \rangle \\ \langle S_{rl} S_{ll}^* \rangle & \langle |S_{rl}|^2 \rangle \end{bmatrix} \rightarrow \begin{cases} Z_C \\ \mathrm{CDR}, \mathrm{ORTT}, p_c \\ \mathrm{ALD} \end{cases} \tag{16}$$

From the Coherency matrix, reflectivity at circular polarization (Z_C), circular depolarization ratio (CDR), orientation parameter (ORTT), alignment direction (ALD) and degree of polarization at circular transmit (p_C) can be evaluated [2]:

$$Z_C \propto \langle |S_{rl}|^2 \rangle$$

(17)

$$\mathrm{CDR} \equiv \frac{\langle |S_{ll}|^2 \rangle}{\langle |S_{rl}|^2 \rangle}$$

(18)

$$\mathrm{ORTT} \equiv \frac{|\langle S_{ll} S_{rl}^* \rangle|}{\sqrt{\langle |S_{ll}|^2 \rangle \langle |S_{rl}|^2 \rangle}}$$

(19)

$$\mathrm{ALD} \equiv \frac{1}{2}\left(\arg\langle S_{ll} S_{rl}^* \rangle - \pi\right)$$

(20)

$$p_C = \sqrt{1 - \frac{4\det[\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}}]}{\left(\mathrm{trace}[\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}}]\right)^2}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}$$

(21a)

$$\mathrm{trace}\left[\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}}\right] = \langle |S_{ll}|^2 \rangle + \langle |S_{rl}|^2 \rangle = \lambda_1 + \lambda_2$$

(21b)

$$\det\left[\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}}\right] = \langle |S_{ll}|^2 \rangle \langle |S_{rl}|^2 \rangle - |\langle S_{ll} S_{rl}^* \rangle|^2 = \lambda_1 \cdot \lambda_2$$

(21c)

In (21), λ_1 and λ_2 are the eigenvalues of $\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}}$.

Manipulation of the definition in (21) yields the following result [3, 4]:

$$(1 - p_C^2) = \frac{4 \cdot \mathrm{CDR}}{[1 + \mathrm{CDR}]^2}(1 - \mathrm{ORTT}^2)$$

(22)

For the particular case of isotropic scatterers (for which intrinsic Z_{DR}=0), we have that ORTT = 0 and we obtain the following result:

$$p_C = \frac{1 - \mathrm{CDR}}{1 + \mathrm{CDR}}$$

(23)

In particular, if CDR is small (quasi-spherical scatterers) a Taylor expansion yields the following relation:

$$p_C = 1 - 2\,\mathrm{CDR}$$

(24)

B. STSR Mode with Circular polarization Transmit

For dual-pol weather radars operating at STSR mode, the actual transmit polarization state can be chosen between slant linear or circular. In general, slant linear polarization is recommended to minimize the bias in polarimetric variables, sometimes visible as radial stripes of positive and negative differential reflectivity [14]. However, if for some reason the transmit polarization state is adjusted to circular, a unitary transformation applied to the Coherency matrix at H and V receive basis ($\mathbf{J}_{\mathbf{C}}^{\mathbf{HV}}$, STSR mode with circular transmit) yields a Coherency matrix as measured by a circular polarization radar ($\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}}$ circular transmit, dual-pol circular receive), see reference [2].

$$\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}} = \mathbf{U} \cdot \mathbf{J}_{\mathbf{C}}^{\mathbf{HV}} \cdot \mathbf{U}^{-1} \tag{25}$$

The equation in (25) has two consequences.

1. If the transmit polarization state of radars operating at STSR mode can be adjusted to circular, polarimetric variables at circular polarization ($\mathrm{Z_C}$, CDR, ORTT, ALD) are also available, provided we effect a unitary transformation on the retrieved Coherency matrix at H - V receive.
2. The eigenvalues of $\mathbf{J}_{\mathbf{C}}^{\mathbf{RHC-LHC}}$ and $\mathbf{J}_{\mathbf{C}}^{\mathbf{HV}}$ are the same, and, consequently, the degree of polarization $\mathrm{p_C}$ is also the same. The degree of polarization only depends on transmit polarization state (indicated by the subscript c) but not on the polarization basis used in the receiver. Therefore, the degrees of polarization obtained by systems with different receive polarization bases are then directly comparable, with no need to effect a unitary transformation on the measured Coherency matrix.

For the particular case of circular polarization transmit and isotropic scatterers ($\mathrm{Z_{DR}^S} = 1$ and ORTT = 0); from the combination of (12) and (22), we obtain that:

$$\mathrm{p_C} = \mathrm{\rho_{hv}^S} = \frac{1 - \mathrm{CDR}}{1 + \mathrm{CDR}} \tag{26}$$

If, in addition, CDR is small (quasi-spherical scatterers) a Taylor expansion yields the following relation:

$$\mathrm{p_C} = \mathrm{\rho_{hv}^S} = 1 - 2\ \mathrm{CDR} \tag{27}$$

Eigenvalue-derived variables obtained at circular polarization transmit (trace of the Coherency matrix and degree of polarization) are the same regardless of the polarization basis used in the receiver. For such polarimetric variables, no unitary transformation is needed to obtain comparable quantities from systems with different receive polarization bases (linear or circular).

III. CONCLUSIONS

For weather radars operating at simultaneous transmission, we show that the degree of polarization is a function of differential reflectivity and copolar correlation coefficient. In particular, in the case of isotropic weather scatterers (Z_{DR}^{S}=0) we show that the degree of polarization and the copolar correlation coefficient are equal.

If the transmit polarization state of the radar can be adjusted to circular polarization, then, besides polarimetric variables at STSR mode (Z_{H}^{S}, Z_{DR}^{S}, ρ_{hv}^{S}, Φ_{DP}) polarimetric variables at circular polarization (Z_{C}, CDR, ORTT, ALD) are also available after a change of polarization basis. Further, since eigenvalue-derived variables are polarization basis invariant, the trace of the Coherency matrix and the degree of polarization are the same for weather radars (circular transmit, horizontal and vertical receive) and air traffic control radars (circular transmit, right-hand and left-hand circular receive) and are therefore directly comparable.

The degree of polarization can be expressed as a function of Z_{DR}^{S} and ρ_{hv}^{S} when the linear receive basis is used, and to CDR and ORTT when the circular receive basis is used. For the particular case of circular polarization transmit and isotropic scatterers (Z_{DR}=0 and ORTT=0), we show that the copolar correlation coefficient at simultaneous transmit is not only equal to the degree of polarization, but is also one-to-one related to the circular depolarization ratio (CDR).

The present study highlights some theoretical aspects leading to a better understanding of the physical meaning of polarimetric weather radar variables at simultaneous transmission. We show that, often, the degree of polarization possesses the same discrimination capabilities of the copolar correlation coefficient. However, with respect to the copolar correlation coefficient, the degree of polarization has two advantages:

- The degree of polarization preserves its physical meaning for every type of scatterers including cross-polarizing scatterers with LDR > 0.
- If the transmit polarization state of a weather radar operating at STSR mode can be adjusted to circular, then eigenvalue-derived variables (trace of the Coherency matrix and degree of polarization) are the

same as those evaluated from a dual-polarization air traffic control radar (circular polarization transmit, right-hand and left-hand circular receive).

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