15A.2 **STATISTICAL QUALITY OF SPECTRAL POLARIMETRIC VARIABLES FOR WEATHER RADAR**

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1. Introduction

In addition to Doppler measurements, application of polarimetry to meteorological observations is one of the most important advancements in weather radar. Typical polarimetric measurements include differential reflectivity, co-polar correlation coefficient, differential phase, and linear depolarization ratio, which contain information integrated over the radar resolution volume. However, the size of radar resolution volume can be too large. Spectral polarimetry is to combine Doppler and polarimetric measurements so that the the distribution of polarimetric variables as a function of radial velocity within the radar resolution volume. Spectral polarimetry has been used for improving data quality, especially in clutter identification and suppression. For example, Bachmann and Zrnić (2007) applied spectral polarimetric variables to filter out the contamination from biological clutter and consequently, better wind estimates from clear air are obtained. The spectral co-polar correlation coefficient and the texture of spectral differential reflectivity and spectral differential phase were used to discriminate different types of clutter using a fuzzy logic inference system for improved estimation of Doppler and polarimetric moments (Moisseev and Chandrasekar 2009; Moisseev et al. 2010). In addition to clutter mitigation, a technique of velocity dealiasing for alternative transmission was developed by using spectral differential phase (Unal and Moisseev 2004). Moreover, spectral polarimetry has been used to obtain microphysical information of precipitation as well as environmental parameters such as background wind and turbulence within the radar resolution volume (Yanovsky et al. 2005; Spek et al. 2008; Dufournet 2010). The hydrometeor classification is carried out in the spectral domain using all the three spectral polarimetric variables (Moisseev and Chandrasekar 2007; Moisseev et al. 2008). Although spectral polarimetry has shown promising results, the quality of these spectral polarimetric estimators has not been studied comprehensively. In this work, the statistical error of spectral differential reflectivity and spectral co-polar correlation coefficient were derived using perturbation method and verified using simulations.

This paper is organized as follows. In Section 2, the estimation of spectral differential reflectivity and spectral co-polar correlation coefficient is defined. In Section b, the bias and SD of these two estimators are derived and dis-

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cussed. The derived statistical errors are subsequently verified using simulation in Section 3. Finally, a summary and conclusions are presented in section 4.

2. Statistical analysis of spectral polarimetric variables

a. *Estimation of spectral polarimetric variables*

Now let's examine how to estimate spectral differential reflectivity and spectral co-polar correlation coefficient from a finite number of complex radar samples. The first step is to perform the discrete Fourier transform (DFT) of radar signals from both channels using the following equation.

$$Z_j(f) = \sum_{m=0}^{M-1} d(m) V_j(m) e^{-j2\pi m f}, \quad j = \text{H or V} \tag{1}$$

where $d(m)$ is the data window for M complex samples from either H or V channels. The second step is to estimate the auto and cross spectra using the following equations.

$$\hat{S}_j(f) = \frac{|Z_j(f)|^2}{M} - N_j, \quad j = \text{H or V} \tag{2}$$

$$\hat{S}_X(f) = \frac{|Z_H(f) Z_V^*(f)|}{M} \tag{3}$$

where $\hat{S}_j(f)$ is the estimated Doppler spectrum from either H or V channel, $\hat{S}_X(f)$ is the estimated cross spectrum, and N_j is the noise level in H or V channel and was assumed to be known. It was further assumed that the white noise from H and V are uncorrelated. In order to reduce the variance in the spectrum estimator (Doviak and Zrnić 1993), K spectra were averaged in step 3.

$$\overline{S}_i(f) = \frac{1}{K} \sum_{k=1}^{K} \hat{S}_{i,k}(f), \quad i = \text{H, V, or X} \tag{4}$$

where $\hat{S}_i^k(f)$, $i =$H, V or X is the auto or cross spectra estimated using (2) or (3). The last step is to estimate spectral differential reflectivity and spectral correlation coefficient from averaged spectra using the following equations.

$$s\hat{Z}_{dr}(f) = \frac{\overline{S}_H(f)}{\overline{S}_V(f)} \tag{5}$$

$$s\hat{\rho}(f) = \frac{|\overline{S}_X(f)|}{\sqrt{\overline{S}_H(f)\overline{S}_V(f)}} \tag{6}$$

b. *Statistical quality of spectral polarimetric variables*

The perturbation method was used in this work, where each estimator is modeled by a deterministic true value and a small and random perturbation term. For example, the spectrum estimator has the form of $\hat{S}_i = S_i + \delta\hat{S}_i$, where S_i is the true spectrum and the perturbation $\delta\hat{S}_i \ll S_i$. Note that hereafter the dependence of f for variables in spectrum domain is neglected for simplicity. The bias and variance of spectra estimated using (2) or (3) have been provided in several literatures (e.g., Bringi and Chandrasekar 2001; Doviak and Zrnić 1993). It is also shown that the bias of auto and cross spectra can be reduced by a larger number of samples. In this work, it was assumed that a sufficient number of samples are used and therefore, the spectrum estimators are unbiased. This assumption will be investigated in more detail in section c. Subsequently, the average spectra ($\overline{S}_i = S_i + \delta\overline{S}_i$) are also unbiased. Specifically, the following assumption was used for the unbiased spectrum estimator.

$$\frac{\langle \delta\hat{S}_i \rangle}{S_i} = \frac{\langle \delta\overline{S}_i \rangle}{S_i} = 0, \quad i = \text{H, V, and X} \tag{7}$$

where $\langle\cdot\rangle$ is the expected value.

The estimator of spectral differential reflectivity is represented in (8) by using the perturbation method and can be approximated by (9).

$$
\begin{aligned}
s\hat{Z}_{dr} &= sZ_{dr} + \delta s\hat{Z}_{dr} && (8)\\
&\approx \frac{S_H}{S_V} + \frac{S_H}{S_V}(\frac{\delta\overline{S}_H}{S_H} - \frac{\delta\overline{S}_V}{S_V} - \\
&\quad \frac{\delta\overline{S}_H\delta\overline{S}_V}{S_H S_V} + \frac{\delta\overline{S}_V^2}{S_V^2}) && (9)
\end{aligned}
$$

where (9) was obtained by substituting $\overline{S}_i = S_i(1 + \delta\overline{S}_i/S_i)$ into (5), taking out the term of S_H/S_V and performing binomial expansion of the denominator. Note that the terms with order higher than the second order were neglected because of $\delta\overline{S}_i/S_i \ll 1$. Subsequently, it can be shown that the bias of $s\hat{Z}_{dr}$, $\mathrm{b}(s\hat{Z}_{dr}) = \langle\delta s\hat{Z}_{dr}\rangle$ and the variance of $s\hat{Z}_{dr}$, $\mathrm{var}(s\hat{Z}_{dr}) = \langle\delta s\hat{Z}_{dr}^2\rangle$. The normalized bias and variance of spectral differential reflectivity can then be derived in the following forms after applying (7) and considering up to the second order terms.

$$
\begin{aligned}
\frac{\mathrm{b}(s\hat{Z}_{dr})}{sZ_{dr}} &= -\frac{\langle\delta\overline{S}_H\delta\overline{S}_V\rangle}{S_H S_V} + \frac{\langle\delta\overline{S}_V^2\rangle}{S_V^2} && (10)\\
\frac{\mathrm{var}(s\hat{Z}_{dr})}{sZ_{dr}^2} &= \frac{\langle\delta\overline{S}_H^2\rangle}{S_H^2} + \frac{\langle\delta\overline{S}_V^2\rangle}{S_V^2} - \\
&\quad 2\frac{\langle\delta\overline{S}_H\delta\overline{S}_V\rangle}{S_H S_V} && (11)
\end{aligned}
$$

where $sZ_{dr} \equiv \frac{S_H(f)}{S_V(f)}$ is the true and deterministic spectral differential reflectivity. The bias and variance of spectral differential reflectivity in dB scale are shown in the following equations.

$$
\begin{aligned}
\mathrm{b}(s\hat{Z}_{DR}) &= \frac{10}{K\ln 10}\{1 - s\rho^2 + \\
&\quad \frac{2sSNR_V + 1}{sSNR_V^2}\} && (12)\\
\mathrm{var}(s\hat{Z}_{DR}) &= \frac{1}{K}(\frac{10}{\ln 10})^2\{2(1 - s\rho^2) + \\
&\quad \frac{2sSNR_H + 1}{sSNR_H^2} + \\
&\quad \frac{2sSNR_V + 1}{sSNR_V^2}\} && (13)
\end{aligned}
$$

where $s\rho$ is the true and deterministic spectral co-polar correlation coefficient and $sSNR_j \equiv S_j/N_j$ is the spectral SNR in linear scale for the j channel, $j =$ H or V.

Similar procedure was applied to spectral co-polar correlation coefficient estimator and the following equations were obtained.

$$
\begin{aligned}
s\hat{\rho} &= s\rho + \delta s\hat{\rho}\\
&\approx \frac{|S_X|}{\sqrt{S_H S_V}} + \frac{|S_X|}{\sqrt{S_H S_V}}\{-\frac{1}{2}\frac{\delta\overline{S}_H}{S_H} - \frac{1}{2}\frac{\delta\overline{S}_V}{S_V} + \\
&\quad \frac{3}{8}\frac{\delta\overline{S}_H^2}{S_H^2} + \frac{3}{8}\frac{\delta\overline{S}_V^2}{S_V^2} + \frac{\delta\overline{S}_H\delta\overline{S}_V}{4S_H S_V} + \Re(\frac{\delta\overline{S}_X}{S_X})\\
&\quad -\frac{1}{2}\Re(\frac{\delta\overline{S}_X}{S_X})\frac{\delta\overline{S}_H}{S_H} - \frac{1}{2}\Re(\frac{\delta\overline{S}_X}{S_X})\frac{\delta\overline{S}_V}{S_V} + \\
&\quad \frac{1}{2}|\frac{\delta\overline{S}_X}{S_X}|^2 - \frac{1}{2}\Re(\frac{\delta\overline{S}_X}{S_X})^2\} && (14)
\end{aligned}
$$

where $\Re(\cdot)$ denoted the real part of a complex variable. The normalized bias and variance of spectral co-polar correlation coefficient can be approximated using the following equations.

$$
\begin{aligned}
\frac{\mathrm{b}(s\hat{\rho})}{s\rho} &= \frac{3\langle\delta\overline{S}_H^2\rangle}{8S_H^2} + \frac{3\langle\delta\overline{S}_V^2\rangle}{8S_V^2} + \frac{\langle\delta\hat{S}_H\delta\overline{S}_V\rangle}{4S_H S_V}\\
&\quad -\frac{1}{2}\langle\Re(\frac{\delta\overline{S}_X}{S_X})\frac{\delta\overline{S}_H}{S_H}\rangle\\
&\quad -\frac{1}{2}\langle\Re(\frac{\delta\overline{S}_X}{S_X})\frac{\delta\overline{S}_V}{S_V}\rangle + \frac{1}{2}\langle|\frac{\delta\overline{S}_X}{S_X}|^2\rangle\\
&\quad -\frac{1}{2}\langle\Re^2(\frac{\delta\overline{S}_X}{S_X})\rangle && (15)\\
\frac{\mathrm{var}(s\hat{\rho})}{s\rho^2} &= \frac{1}{4}\frac{\langle\delta\overline{S}_H^2\rangle}{S_H^2} + \frac{1}{4}\frac{\langle\delta\overline{S}_V^2\rangle}{S_V^2} + \langle\Re^2(\frac{\delta\overline{S}_X}{S_X})\rangle\\
&\quad -\langle\Re(\frac{\delta\overline{S}_X}{S_X})\frac{\delta\overline{S}_H}{S_H}\rangle - \langle\Re(\frac{\delta\overline{S}_X}{S_X})\frac{\delta\overline{S}_V}{S_V}\rangle + \\
&\quad \frac{1}{2}\frac{\langle\delta\overline{S}_H\delta\overline{S}_V\rangle}{S_H S_V} && (16)
\end{aligned}
$$

Consequently, the following bias and variance of spectral correlation coefficient can be obtained.

$$\frac{\mathrm{b}(s\hat{\rho})}{s\rho} = \frac{1}{K}\{\frac{(1-s\rho^2)^2}{4s\rho^2} + \frac{2sSNR_H+3}{8sSNR_H^2} + \frac{2sSNR_V+3}{8sSNR_V^2} + \frac{sSNR_H+sSNR_V+1}{4s\rho^2\ sSNR_H\ sSNR_V}\} \quad (17)$$

$$\frac{\mathrm{var}(s\hat{\rho})}{s\rho^2} = \frac{1}{K}\{\frac{(1-s\rho^2)^2}{2s\rho^2} + \frac{1-2sSNR_H}{4sSNR_H^2} + \frac{1-2sSNR_V}{4sSNR_V^2} + \frac{sSNR_H+sSNR_V+1}{2s\rho^2\ sSNR_H\ sSNR_V} + \} \quad (18)$$

c. *Discussions*

Before investigating the dependence of the statistical errors on different parameters, we need to determine the condition when the assumption of unbiased spectrum estimators in (7) is approximately valid. It is known that the estimated spectrum is the convolution of true spectrum and window function. The window function is defined by the Fourier transform of the lag window. In other words, the spectrum bias can be reduced by increasing the number of samples for a given spectrum and the type of lag window. Theoretically, the spectrum estimators using (2) or (3) are asymptotically unbiased (Papoulis and Pillai 2002). In this work, it was assumed that the spectrum bias can be neglected if $\delta\hat{S}_i/S_i <= 0.2$. It was further required that a sufficient number of samples are used so that the condition of unbiased estimator is met for spectrum from its peak to 30 dB down from the peak. In other words, the minimum number of samples to achieve the requirements can be determined by increasing the number of samples until the requirements are met. Fig. 1 presents the minimum number of samples as a function of normalized spectrum width for rectangular, Chebyshev and von Hann data windows. It is interesting to point out that the rectangular data window demands the largest number of samples for a given spectrum width. This is because the window function for rectangular data window has the highest sidelobes, which biases the spectrum most. On the other hand, Chebyshev and von Hann data windows produce similar results, while von Hann data window is slightly better. It can be observed that with 64 samples, the requirements for unbiased assumption is fulfilled for normalized spectrum width larger than 0.05. Hereafter, von Hann data window with 64 or more samples is considered.

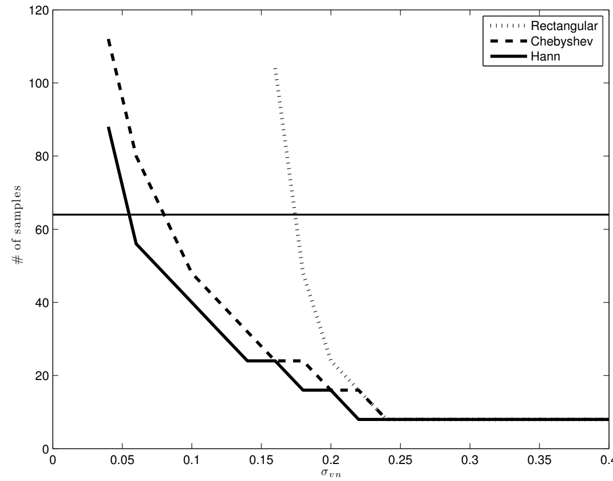

Figure 1: The minimum number of samples required to approximate the assumption of unbiased spectrum estimators is shown as a function of normalized spectrum width for three data windows. The results from rectangular, Chebyshev and von Hann data windows are denoted by dotted, dashed, and solid lines, respectively. The horizontal solid line depicts 64 samples.

It is shown that the bias and SD of the two spectral polarimetric estimators depend on the spectral SNRs, true spectral co-polar correlation coefficient, and the number of spectrum average. In this work, we assumed the noise level from the two channels are equal and known. Thus, the spectral SNR for H channel can be represented by $sSNR_H = sSNR_V\ sZ_{dr}$.

3. Verification of statistical errors using simulations

a. *Description of simulation method*

In order to further verify the statistical error of spectral polarimetric variables derived in section b, a simulation of time series signals from dual polarimetric weather radar was developed based on Zrnić (1975). In simulation, a model spectrum for V-channel ($S_V(m)$), spectral differential reflectivity ($sZ_{DR}(m)$), and spectral co-polar correlation coefficient ($s\rho(m)$) were given for M velocity bins. First of all, random fluctuations were added to the model V-channel spectrum based on the equation (7) in Zrnić (1975). This process was repeated K times for spectrum averaging as shown in the following equation.

$$Z_V(k,m) = \sqrt{-S_V(m)\ln u(k,m)}e^{j\theta(k,m)} \quad (19)$$

where $k = 1,\ 2,\ \cdots K$, $u(k,m)$ is an independent random variable uniformly distributed between 0 and 1 and $\theta(k,m)$ is also an uniform random variable but has values between $-\pi$ and π. Note that $Z_V(k,m)$ is the DFT of the time series samples for K independent spectra as defined in (1).

The next step was to generate H-channel spectra $Z_H(k,m)$ with the given spectral differential reflectivity and spectral correlation coefficient. This was done by considering $Z_V(k,m_i)$ at a velocity bin m_i as an random sequence of K samples. The following equation was used to generate the random sequence of $Z_H(k,m_i)$, which has a mean amplitude of $Z_V(k,m_i)\sqrt{sZ_{dr}(m_i)}$ and correlation coefficient of $s\rho(m_i)$ with $Z_V(k,m_i)$. This process was repeated M times for all the velocity bins.

$$\begin{aligned} Z_H(k,m) \ &= \ \sqrt{sZ_{dr}(m)}[s\rho(m)Z_V(k,m) + \\ &\quad \sqrt{1-s\rho^2(m)}R(k,m)] \end{aligned} \quad (20)$$

where $m = 1,\ 2,\ \cdots,M$, and $R(k,m)$ was generated using (19) but with independent $u(k,m)$ and $\theta(k,m)$. The time series signals for H and V channels for each k were generated by the inverse Fourier transform of (20) and (19), respectively. Finally, independent noise sequences were added to the two time series signals individually based on the desirable SNR.

b. *Simulation Results*

In this work, the spectral polarimetric variables and the derived statistical errors were demonstrated and verified using three cases. For all the three case, the V-channel Doppler spectrum was modeled by a Gaussian spectrum with mean velocity of -5 m s^{-1}and spectrum width of 4 m s^{-1}for a maximum unambiguity velocity of 25 m s^{-1}. The spectral co-polar correlation coefficient used in the model was uniform over all the velocities and has values of 0.8, 0.9, and 0.95 for Case I, II, and III, respectively. The modeled spectral differential reflectivity is a constant of 0 dB for Case I, has an exponential variation for Case II, and is a linear function for Case III. For all the three cases, the number of samples is 64 and the number of spectrum averages is 20. Moreover, the theoretical SDs for spectral differential reflectivity and spectral co-polar correlation coefficient can be calculated using (13) and (18), respectively. The modeled spectral polarimetric variables are their associated theoretical SDs are denoted by dotted, dashed, and solid lines for the three cases, respectively, in Fig. 2. For each case, 100 realizations of time series signals for both channels were generated using a constant SNR of 30 dB for V channel. Before investigating the statistical performance of the spectral polarimetric estimators, let's first verify whether the simulated time series signals can produce the desirable conventional polarimetric variables. The differential reflectivity of 0, 1.112, and 3.016 dB are expected for Case I, II, and III, respectively. The expected co-polar correlation coefficient is equal to its constant spectral correlation coefficient. At the same time, the differential reflectivity and correlation coefficient can be estimated using

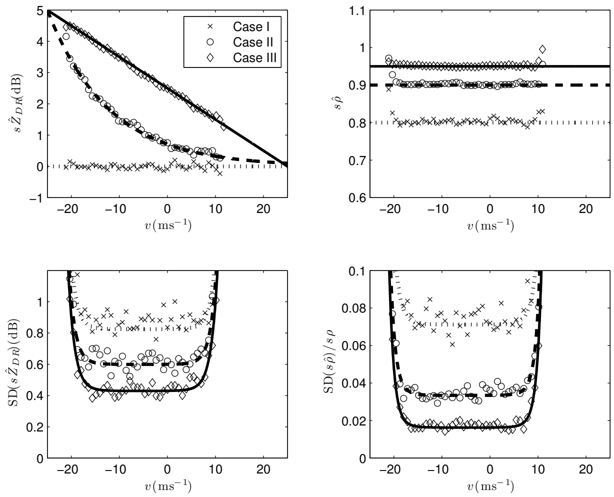

Figure 2: Statistics of the spectral differential reflectivity and spectral correlation coefficient are shown on the left and right columns, respectively, for the three cases. The mean of both estimators are shown on the top panels and the SDs are presented on the bottom panels. Both modeled spectral variables and theoretical SD for Case I, II, and III are denoted by dotted, dashed, and solid lines, respectively. The mean and SD calculated from the simulation are denoted by crosses, circles, and diamonds for the three cases.

simulated time series signals (Bringi and Chandrasekar 2001). The mean of estimated differential reflectivities obtained for the three cases is -0.007, 1.119, and 3.026 dB, and the mean of the estimated co-polar correlation coefficients is 0.799, 0.900, and 0.949. The good agreement between the expected and estimated polarimetric variables verifies the relationship and suggests the feasibility of proposed simulation.

For each realization, the spectral polarimetric variables were estimated using (5) and (6) with von Hann data window. Note that the spectral polarimetric variables are not defined for spectral SNR less than 0 dB. The mean of estimated spectral differential reflectivity and spectral correlation coefficient are presented on the top left and right panels, respectively. Additionally, the SD of the two estimators are shown on the lower two panels. It is evident that the mean of both spectral differential reflectivity and spectral correlation coefficient agree well with the model. The bias of spectral correlation coefficient becomes identifiable only toward both tails of the spectrum, where spectral SNR is evidently low. The bias of spectral differential reflectivity is also present at low spectral SNR, but it is difficult to observe due to its relatively small value for these cases. Moreover, the theoretical SDs for the two spectral polarimetric estimators are verified using simulation as shown in the bottom two panels. The SD of both estimators is not affected much by the modeled spectral differential reflectivity. This is manifested by a relatively constant values of SDs between approximately -10 m s^{-1}and 8 m s^{-1}for Case II and III, despite the model sZ_{DR} varies within that region. Moreover, the effect of modeled spectral correlation coefficient on SD of both spectral polarimetric variables is evident for a given K at large spectral SNR.

An example of estimated spectra and spectral polarimetric variables from one realization is shown in Fig. 3. The estimated average spectra for both H and V channels for the three cases are shown on the top three panels from left to right, respectively. The noise level is at approximately -50 dB. The estimated spectral differential reflectivity and spectral correlation coefficient are denoted by dashed lines in the middle and bottom panels, respectively. The model values are denoted by solid lines. Note that all the estimates associated with $sSNR \geq 0$ are denoted by gray dashed lines, which generally follow the model values with increasing fluctuations as $sSNR_V$ decreases. Thus, it is important to obtain some idea about which portion of the estimates meets the error requirements. Moreover, the minimum spectral SNR that meets the desirable bias and SD can be determined given K, sZ_{DR}, and $s\rho$. Here, $K = 20$ has been set. As discussed previously, the impact of spectral differential reflectivity on statistical error is limited. Thus, the spectral differential reflectivity of 0 dB is used for all the three cases. Additionally, the estimated spec-

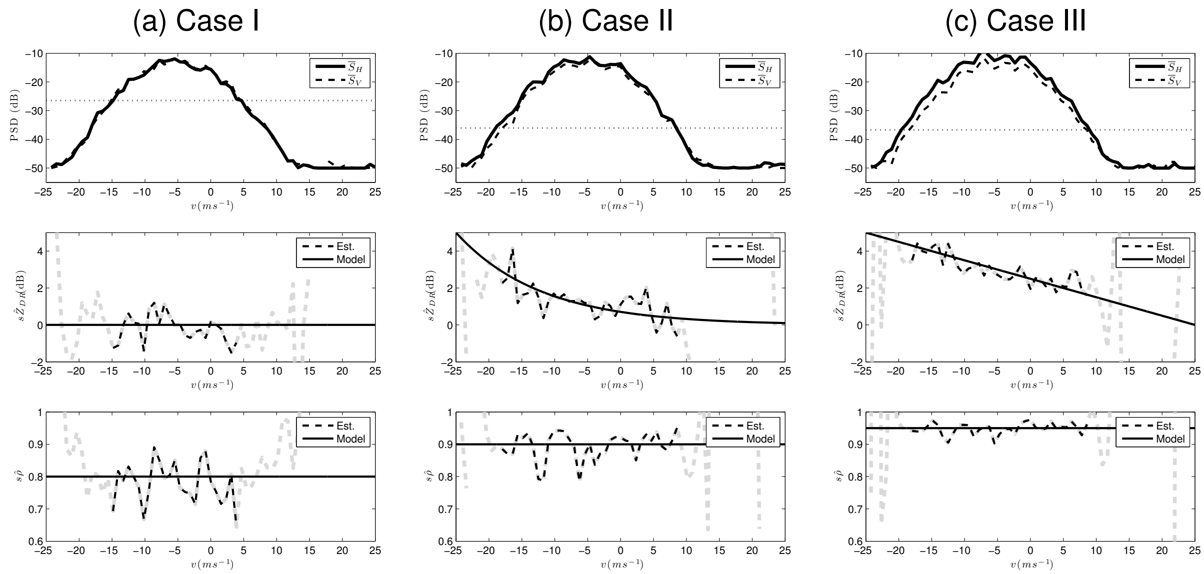

Figure 3: Examples of spectral polarimetric variables are shown for the three cases from left to right columns, respectively. The average spectra from H and V channels are denoted by solid and dashed lines, respectively, on the top row. The horizontal dotted lines depicted the spectral SNR threshold to meet required accuracy and precision. The estimated and modeled spectral differential reflectivity and spectral correlation coefficient are shown in the middle and bottom panels, respectively. The modeled values are denoted by solid line, while the estimates are depicted using dashed lines. The darker dashed lines denote the region of estimates that meet the requirement.

tral correlation coefficient averaged over the region where starts from the peak of the spectrum to 20 dB below the peak was used to approximate the true spectral co-polar correlation coefficient. As an example, we requested the maximum bias and SD of spectral differential reflectivity to be 0.08 dB and 0.85 dB, respectively. Additionally, the maximum bias of 0.3% and maximum SD of 8% were requested for the normalized spectral correlation coefficient. The resulted spectral SNR thresholds for the three cases are denoted by horizontal dotted lines on the top panels. The estimated spectral differential reflectivity and spectral correlation coefficient that meet the requirements are highlighted by black dashed lines. The adaptive threshold for spectral SNR can help to identify the region of spectral polarimetric variables with desirable data accuracy and precision.

4. Summary and conclusions

In this work, the estimators for spectral differential reflectivity and spectral cross-correlation coefficient were defined based on averaged auto and cross spectra. The bias and SD of the two spectral polarimetric variables were derived using perturbation method. These statistical errors have similar forms to those from polarimetric variables and decrease as the increasing spectral SNR. Moreover, the number of spectrum averages and spectral copolar correlation coefficient play a significant role in these errors. These derived statistics can also used to determine the minimum number of spectrum averages in order to achieve desirable requirements of the statistical errors. One of the limiting factors for the application of spectral polarimetry to operational observations could be the requirement of a relatively long time sequence ($M \times K$) to achieve reasonable frequency resolution and statistical quality.

In this work, we assumed that M is sufficiently large so spectrum estimators are unbiased. To effectively increase the number of spectrum averages, a combination of time and range averaging could be used. Another possibility is to apply overlapped data windows so that the frequency resolution is maintained, while the data quality is compromised.

These derived statistical errors were further verified using simulations, where the time series signals for both H and V channels were generated based on modeled spectral polarimetric variables and regular SNR. Three cases with different values of spectral co-polar correlation coefficient and three variations of spectral differential reflectivities were simulated. The results demonstrate that not only the model spectral differential reflectivity and spectral co-polar correlation coefficient can be reconstructed, but also the bias and SD obtained from simulations are consistent with the theoretical derivations.

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