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NUMERICAL EXPERIMENTS WITH A VARIATIONAL SCHEME FOR ATTENUATION CORRECTION FOR X-BAND RADAR

Leonid Tolstoy,
Colorado State University, Fort Collins, CO
V. N. Bringi
Colorado State University, Fort Collins, CO;
Robin J. Hogan
University of Reading, UK

1. Introduction

The variational scheme developed and evaluated by Hogan (2007) for S-band radar for estimating rain rate and detecting hail, is used here specifically for X-band application to correct the measured Z_h and Z_{dr} for attenuation due to mixed phase precipitation (rain mixed with wet ice/hail) which is an especially severe problem for lower power, short range X-band radars. While a number of algorithms for attenuation-correction (based on differential propagation phase) are available at X-band when only rain occurs along the propagation path, there is no stable algorithm as yet when rain is mixed with wet ice/hail. Hence this paper is aimed at applying the variational scheme, which in its formulation, estimates the attenuation due to both rain and hail along the propagation path. We report on several numerical experiments with the variational code to optimize the attenuation-correction in mixed phase precipitation at X-band.

The forward model was adapted from S to X-band based on scattering simulations. The measurement errors in Z_{dr} and differential propagation phase for the particular X-band radar which we used (one of the CASA IP1 radars in Oklahoma) were adapted based on reflectivity and copolar correlation coefficient along the beam. This adjusts the weights given to Z_{dr} and differential propagation phase in the cost function. We used the hail detection ratio (H_{dr}) based on (Z_h, Z_{dr}) to pre-identify hail along the path and thereby initialize the detection of hail in the variational scheme. This was followed by pre-estimating the "reflectivity weighted fraction of ice" using the "deviation from the rain line" methodology as an initialization for the "fraction of ice" in the variational scheme. The scheme then finds an optimal solution for attenuation-corrected Z_h and Z_{dr} due to rain and wet ice along the path. The preliminary improvements of the variational scheme using CASA X-band radar data in a convective storm with rain and wet hail are evaluated.

2. Variational method and forward model (FM)

In Hogan (2007), a method was described, which applies the variational approach to rainfall rate retrieval at S-band from the polarization radar variables reflectivity Z, differential reflectivity Z_{dr} and differential propagation phase Φ_{dp}. This methodology, also known as "optimal estimation theory", was used mostly in satellite retrievals, but has only recently been applied to radar applications (e.g., Austin and Stephens (2001), Löhnert et al. (2004)). This method was shown to successfully overcome problems with other techniques, which appear due to inherent measurement fluctuations or "noise" in radar variables (Z_{dr} and K_{dp}). The K_{dp}, as the range derivative of an already noisy Φ_{dp}, can become negative, which is physically impossible in rain. Furthermore, it is difficult to design conventional algorithms to make use of Z_{dr} and Φ_{dp} simultaneously in all rain/hail regimes, so the most appropriate one usually has to be chosen (Hogan (2007)).

The forward model, which is the essence of the variational method, uses the first guess of state vector consisting of the ln(*a*) for each gate of the beam, where coefficient *a* is the coefficient between reflectivity Z_h and the rainrate R:

$$Z_h = aR^b \tag{1}$$

where b is equal to 1.5.

Then these values are used as an input to the forward model to predict the observations at each gate (Z'_{dr} and Φ'_{dp}). The difference between predicted and observed variables is used to change the state vector for better fit with the observations in a least squares sense. This is done by minimization of the cost function, which was defined as

$$2J = \sum_{i=1}^{m} \frac{(Z_{dr,i} - Z'_{dr,i})^2}{\sigma^2_{Z_{dr}}} + \frac{(\phi_{dp,i} - \phi'_{dp,i})^2}{\sigma^2_{\phi_{dp}}} + \sum_{i=1}^{n} \frac{(x_i - x^a_i)^2}{\sigma^2_{x^a}} \tag{2}$$

where first two summations represent the deviation of the observations Z_{dr} and Φ_{dp} from the values predicted by the forward model Z'_{dr} and Φ'_{dp}, respectively, and the third summation represents the deviation of the elements of the state vector from some *a priori* estimate x^a (a priori *a*=200 $mm^6 m^{-3} (mm\ h^{-1})^{-1.5}$). The terms σ_{Zdr} and $\sigma_{\Phi dp}$ are the root-mean-square observational errors, and σ_x^a is the error in the *a priori* estimate, *m* is number of the input gates in the beam, *n* is a set of basis functions, typically ~m/10. This minimization process would be repeated until convergence is reached.

This method also can be used to find gates with hail and estimate the fraction of reflectivity due to the hail. If there is a hail segment in the ray, this scheme cannot find a solution for ln(*a*) that, when used in the forward model, can closely predict both Z_{dr} and Φ_{dp}, so it is done in 2 passes. The first pass is used for detection of the gates with the hail, and second is used for estimation of the fraction of the measured reflectivity due to hail *f*. The attenuation-corrected reflectivity is calculated from the observed reflectivity as:

$$Z_{h\_corrected} = 10^{0.1 A_h} Z_{h\_observed} \quad (3)$$

where A_h is the total 2-way attenuation at horizontal polarization (path integrated attenuation, PIA) in dB. A similar equation for the corrected Z_{dr} can be written involving *Ah,v*.

Using S-band data, Hogan (2007) has shown that the optimal estimation scheme produces good results for S-band, but application of it to X-band radar data (CASA radars) does not produce equally good results. The observed radar values of Z_{dr} and Φ_{dp} cannot be predicted by the FM (forward model) well enough in some cases, especially for beams which go through the storm core. The following sections of the paper describe our approach to the improvement of existing variational scheme with the goal to achieve better performance.

3. Variable observational errors in the cost function

One way to improve the performance of the optimal estimation scheme is to adjust the default errors assigned to Φ_{dp} and Z_{dr} values, which are used as an input data into the described algorithm.

For CASA radars, the root-mean-square observational error for Z_{dr} data, i.e., σ_{Zdr} has a default value of 0.5 dB. For low rain rate areas (drizzle) the values of Z_h are expected to be less than 20 dBZ, and there the observational error should be higher then this default value.

After examination of the CASA radar quality and some numerical experiments it was found that value of σ_{Zdr} in first approximation could be changed according to the empirical formula

$$\sigma_{Zdr}(Z_h) = 0.5 * \begin{Bmatrix} -0.05 Z_h(dBz) + 2, & Z_h < 20 dBz \\ 1, & Z_h > 20\ dBz \end{Bmatrix} \quad (4)$$

where 0.5 dB is the default value of the observational error. Figure 1 shows the dependency $\sigma_{Zdr}(Z_h)$.

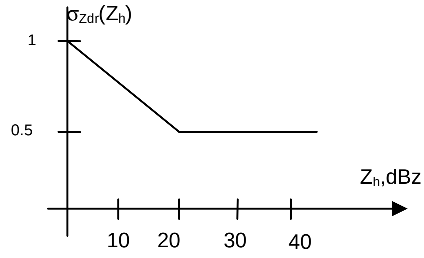

Fig.1 Empirically-based dependency of σ_{Zdr} on Z_h

It should be noted that from theory σ_{Zdr} should be dependent on the intrinsic copolar correlation coefficient and the SNR according to eq (6.115) of Bringi and Chandrasekar (2001). The empirical equation in (4) only approximates the dependence on SNR.

The default value of root-mean-square observational error ($\sigma_{\Phi dp}$) for Φ_{dp} for CASA data is 3 deg. Since $\sigma_{\Phi dp}$ is theoretically related to 1- ρ_{hv}^2 (eq. 6.143 of Bringi and Chandrasekar 2001), after examination of radar data and some numerical experiments it was found that $\sigma_{\Phi dp}$ in the first approximation could be changed according to the formula

$$\sigma_{\Phi_{dp}}(\rho_{hv}) = 3 * \begin{Bmatrix} -4.44\rho_{hv} + 5, & \rho_{hv} < 0.9 \\ 1, & \rho_{hv} \geq 0.9 \end{Bmatrix} \quad (5)$$

where 3 deg is the default value of the observational error for Φ_{dp} CASA data.

Figure 2 shows the dependency $\sigma_{\Phi dp}(\rho_{hv})$:

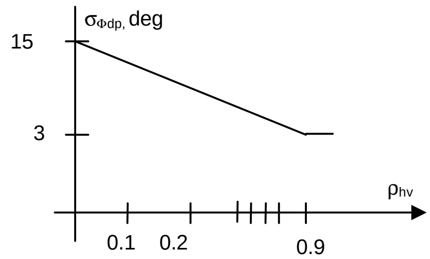

Fig 1. Empirically based dependency of $\sigma_{\Phi dp}$ on the copolar correlation coefficient, ρ_{hv}.

Note that the empirical equation in (5) only approximates the theoretical dependence on ρ_{hv}.

Sample data from the CASA radar from the event of June 10th, 2007, 22:12:57 are used to show the effect of using eqs(4,5), see Fig. 3.

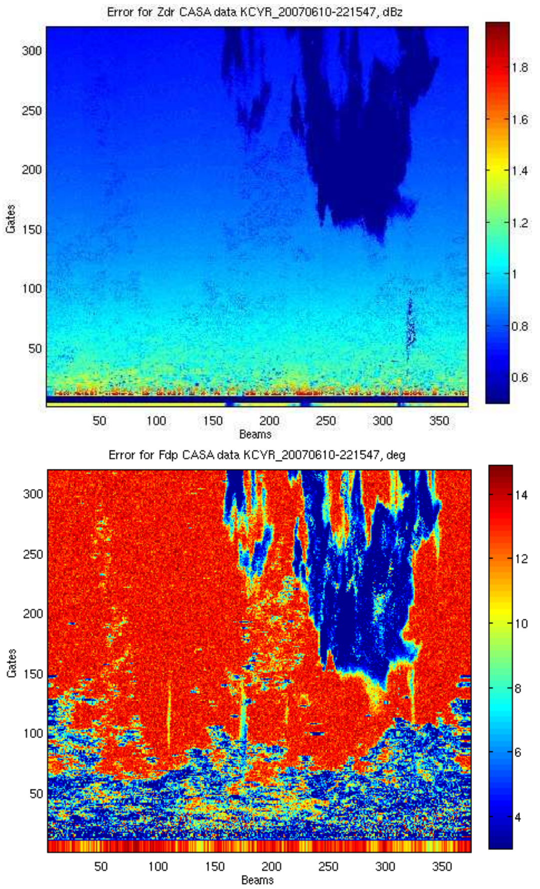

Fig 3. Observational errors σ_{Zdr} and $\sigma_{\Phi dp}$ changed according to the Z_h and ρ_{hv} values, CASA KCYR 20070610-221547.

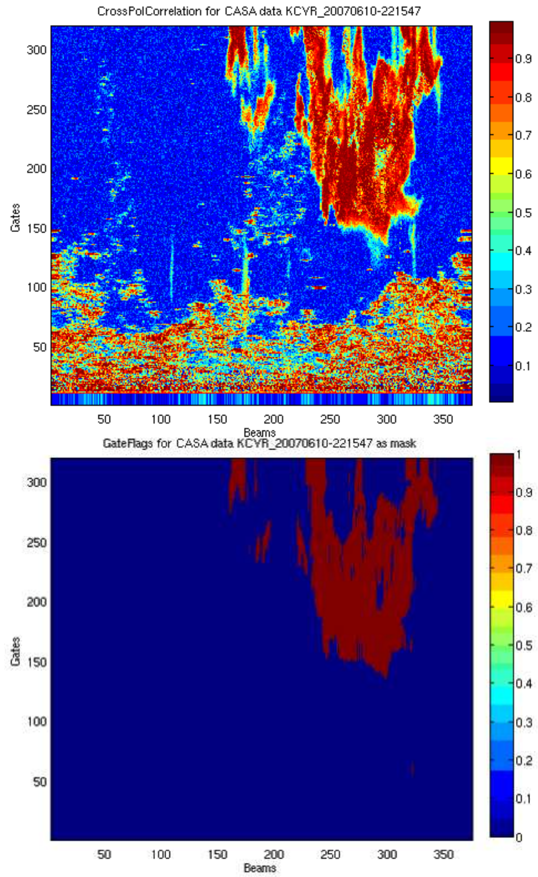

Fig 4. CrossPolCorrelation coefficient ρ_{hv}, and GateFlags used as mask for eliminating the noise, CASA KCYR 20070610-221547.

Figures 3 and 4 show that even for gates considered as “good”, i.e, with useful weather information, these observational errors can change significantly from using eqs(4,5). As the result of the above procedure of changing the default values of observational errors σ_{Zdr}, $\sigma_{\Phi dp}$ leads to the re-balancing the influence of that corresponding variable on the cost function. The overall effect is that corresponding forward-modeled range profiles (Z'_{dr} or Φ'_{dp}) tends to be close to the input variable, as in the case when only one variable (Z_{dr} or Φ_{dp}) was used in the input to the program (these variables can be used as an input to the scheme together or switched off if not available in the radar data).

This is illustrated in the following Fig. 5, which show the gate-by-gate comparison of the CASA KCYR 20070610 dataset used above, data from the 2 deg elevation angle sweep, the beam #318 which were generated using constant observational errors (left panel) and variable observational errors (right panel).

It can be seen that Φ_{dp} modeled by FM and observed Φ_{dp} are in much better agreement in the case where variable observational errors were used. In addition, the coefficient α in A_h-K_{dp} relationship

$$A_h=\alpha\, K_{dp}^{\,b} \qquad (6)$$

in this case is α= 0.15, which is reasonable in rain at X-band.

4. Estimation of reflectivity-weighted fraction of ice in a rain-hail mixture

Another way to improve the optimal estimation algorithm is related to the problem of automatic detection of wet ice and hail in the observed precipitation. In its original form, the algorithm when used with X-band data, in some cases, converges to physically unrealistic set of output variables, which are far from the input and show

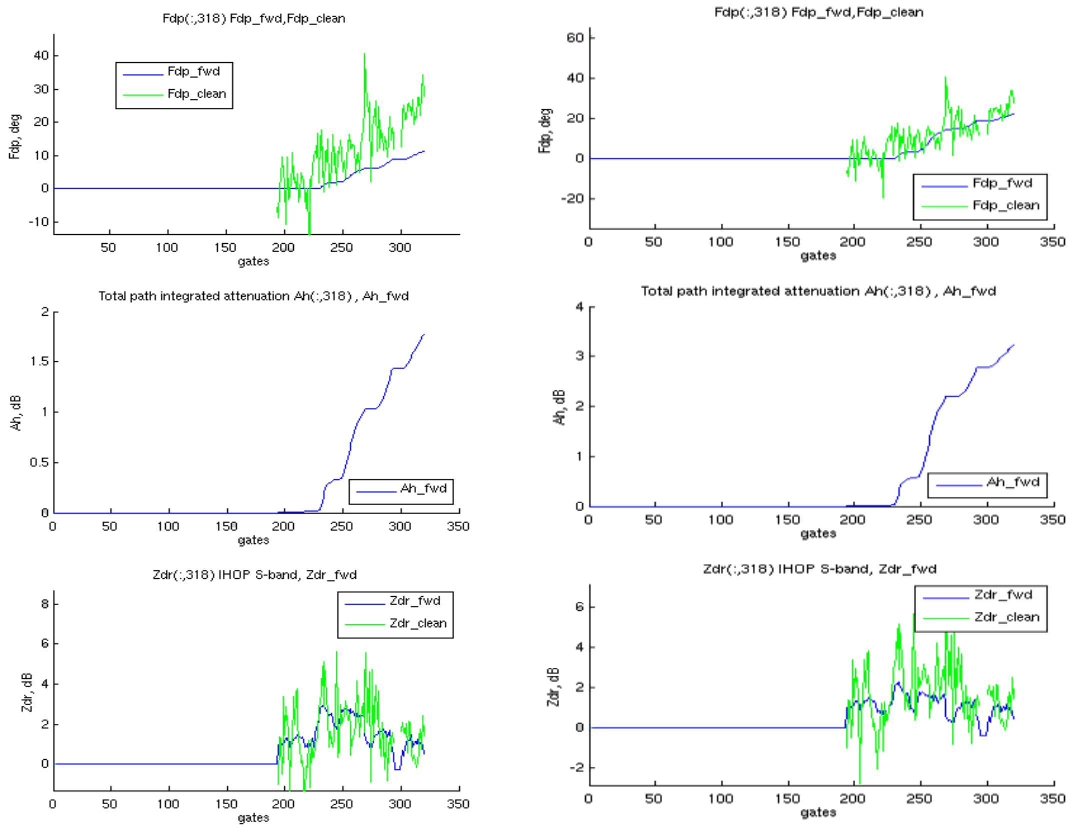

Fig 5. CASA KCYR 20070610 dataset, the 2 deg elevation angle. Gate-by-gate variables comparison for beams #318 generated using constant observational errors (left panel) and variable observational errors (right panel). Note how Φ_{dp} goes closer to the observed data, and attenuation A_h has more reasonable values in the right panel.

saturation in Φ_{dp} and A_h, where they increase up to the maximum allowed values. It was found that this situation happens mostly for beams going through the core of the storm where occurrence of the hail or wet ice is highly probable.

One can "help" the algorithm by detecting the gates with wet ice and hail and supply them to the program instead of just letting it find these gates by itself. To find the gates where the probability to find hail is high, one can use the H_{dr} concept described by Aydin et al. (1986):

$$H_{dr}=Z_h-f(Z_{dr}) \quad (7)$$

where Z_h is the measured reflectivity, and $f(Z_{dr})$ defined below:

$$f(Z_{dr})=\begin{cases} 27; & Z_{dr}\le 0 \;\; dB \\ aZ_{dr}+27; & 0<Z_{dr}\le b \\ 60; & Z_{dr}>b \end{cases} \quad (8)$$

For 3 GHz frequency (S-band) and equilibrium rain drop shape model give values a=16.5, b=2 dB.

To find the parameters of the $f(Z_{dr})$ at X-band, the variables Z_h and Z_{dr} were simulated based on the one minute drop size distribution data from pure rain from a 2D-video disdrometer installed near the CP2 radar (Brisbane, Australia). These variables were simulated for pure rain event assuming the latest information on drop axis ratios and canting angles. One can make a plot similar to the one described in Aydin et al, (1986), but this time for X-band, with the purpose to find the curve which is the rain-hail boundary. This simulated data together with the boundary line is shown in Fig. 6:

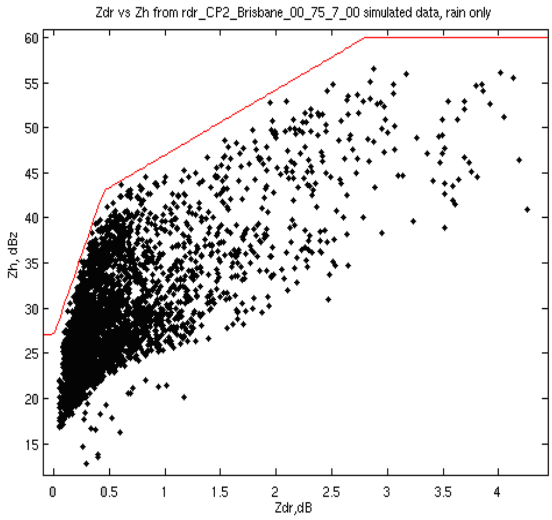

Fig.6 Z_h vs Z_{dr} scatterplot representing simulated data for rain-only case at X-band, and rain-hail boundary line designed for X-band.

Even though the data used to create this curve was collected in subtropical coastal environment different from the continental environment of Oklahoma where CASA radars are installed, the boundary line separating rain from the rain-hail mixture should be fairly robust.

For X-band the $f(Z_{dr})$ is found to have the following form:

$$f(Z_{dr}) = \begin{cases} 27; & Z_{dr} \le 0 \ \ dB \\ aZ_{dr} + 27; & 0 < Z_{dr} \le e \ \ dB \\ bZ_{dr} + c; & e < Z_{dr} \le g \ \ dB \\ 60; & Z_{dr} > g \ \ dB \end{cases} \qquad (9)$$

where a=35.56 , b=7.23, c=39.74, e=0.45 dB, g=2.8 dB.

The CASA radar data files include Z_h and Z_{dr} variables corrected for attenuation (by other algorithms), so one can use it to find H_{dr} parameter. It appears that values of H_{dr} found in this manner demonstrate spiky behavior, so one might need to apply FIR (finite impulse response) range filter (in this case FIR filter of order 20 was used) before supplying these values into the optimal estimation algorithm. Fig. 8 demonstrates H_{dr} data calculated for CASA KCYR_20070610-221547 "pure rain" case before and after smoothing by FIR filter.

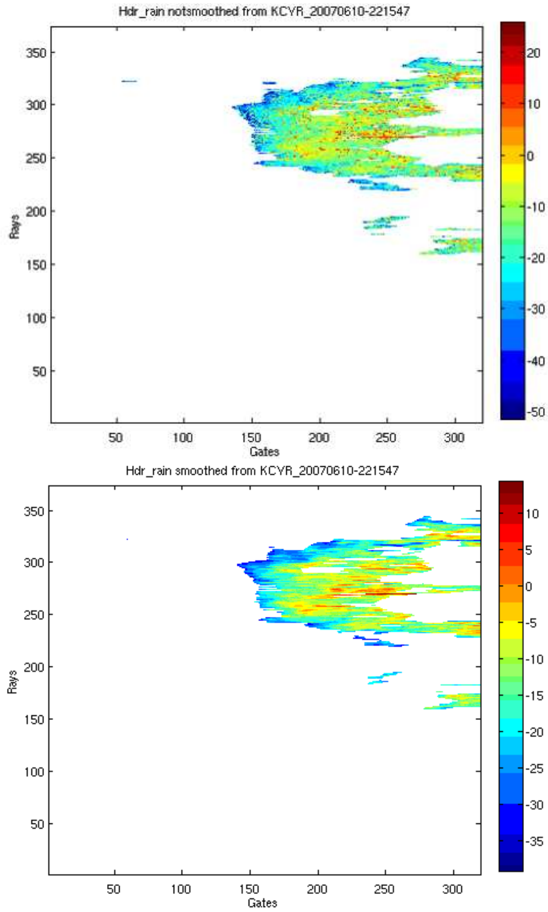

Fig. 8. H_{dr} data calculated using Z_h, Z_{dr} variables (corrected for attenuation using differential phase as a constraint) for CASA KCYR_20070610-221547. The gates with high probability of hail correspond to the H_{dr} values more than 3-5 dB.

The FM algorithm was modified to accept the H_{dr} data as an input, i.e. gates with high H_{dr} values (where H_{dr}>3 dB) were marked as having hail, and so on the second pass the FM tries to calculate the hail fraction *f* for these gates, as described in Hogan (2007).

After some experiments it was found that this method of selecting gates with high probability of hail and supplying this information to the FM does not produces sufficiently good results. The hail fraction *f* calculated by the program still demonstrates spiky, not smooth behavior.

With the purpose of further improving the algorithm in part by recognizing gates with hail, one can use the difference reflectivity factor Z_{dp} as it was proposed by Golestani et al. (1989):

$Z_{dp}=10 \log_{10}(Z_h - Z_v)$, $Z_h > Z_v$, $mm^6 \ m^{-3}$ (10)

$Z_H/Z = f_{hail} = 1 - 10^{-0.1(\Delta Z)}$ (11)

where Z_h, Z_v – reflectivity in horizontal and vertical polarizations, Z- total reflectivity, Z_H is reflectivity due to hail, ΔZ is horizontal deviation from the rain line, dB. The Z_{dp} can be used here to estimate the fraction of reflectivity due to hail in the mixed rain and hail precipitation. The simulated (based on the data from 2D video disdrometer) variables Z_h and Z_v for pure rain case at X-band were used to find a "rain line" in Z_h vs Z_{dp} space (see Fig. 9).

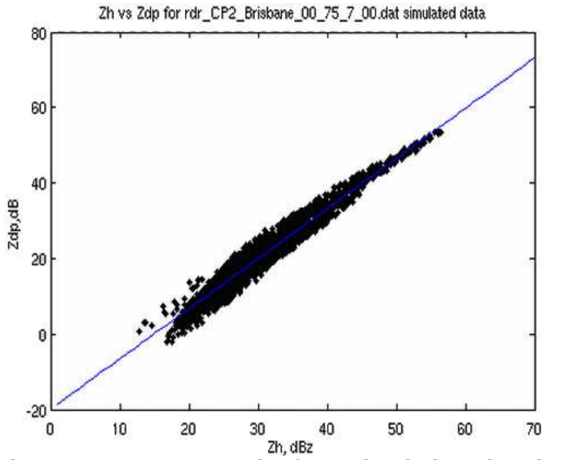

Fig.9 Z_h vs Z_{dp} scatterplot from simulations in rain, and the best fit so-called "rain line".

The equation for the "rain line" which relates Z_h and Z_{dp} is:

$$Z_{dp} = 1.327 Z_h - 19.82 \quad (12)$$

Since hail approximately gives Z_h=Z_v due to nearly spherical shapes, in a rain hail mixture, for a given Z_{dp} value, one estimates the deviation from the rain line as explained in Chapter 7 of Bringi and Chandrasekar (2001). Even though the error in this initial "guess" of f_{ice} can be quite high, we expect that the optimal scheme will take as input this first "guess" and converge to an optimal value.

As in the case of H_{dr}, we use the Z_h and Z_{dr} corrected for rain attenuation (using differential phase constraint) to calculate Z_{dp} and fraction of reflectivity due to hail f_{ice} which is used as the "first guess" for the FM algorithm. The algorithm then re-adjusts the final values of f_{ice} by minimizing the cost function. One can build the f_{ice} map for the "mixed phase precipitation" case, and compare it to the calculated and smoothed H_{dr} values for the same case, as shown in the Fig. 10:

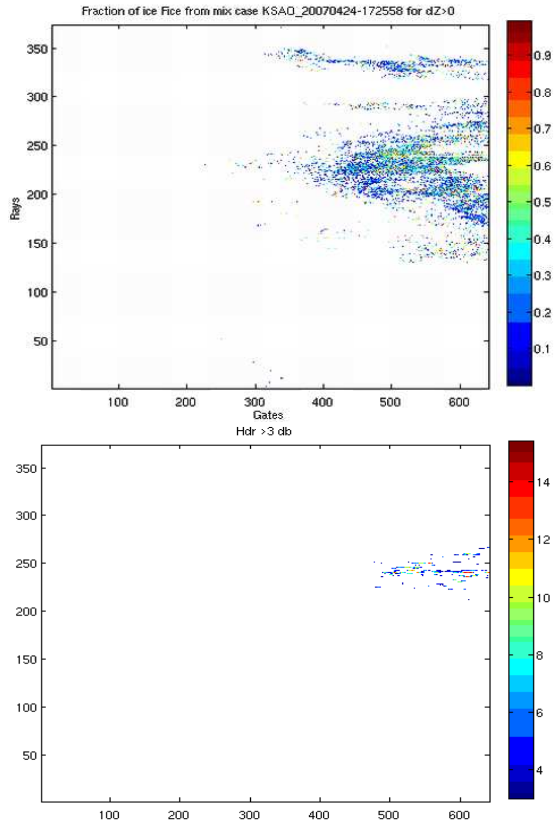

Fig. 10. Fraction of reflectivity due to ice, f_{ice}, for "mixed phase precipitation" case of April 24, 2007 (KSAO_20070424-172558.netcdf) (top panel). H_{dr} (> 3dB) is shown in bottom panel. Note that high values of H_{dr} do not always correspond to the high values of f_{ice}.

One can note that high values of H_{dr} do not always correspond to the high values of f_{ice}. The map of f_{ice} looks scattered, for the purposes of achieving better "first guess" it is desirable to use spatial averaging of the f_{ice} data, by using a 5x5 smoothing window. The same matrix can be applied to the H_{dr} data (figure 11). It can be seen that spatially smoothed H_{dr} values (top panel) are better correlated with the spatially smoothed f_{ice} values (bottom panel). But still, high f_{ice} values can be seen at gates where H_{dr} values are low. Based on this, the decision was made to supply the FM algorithm with smoothed values of the reflectivity fraction due to ice f_{ice} as a "first guess".

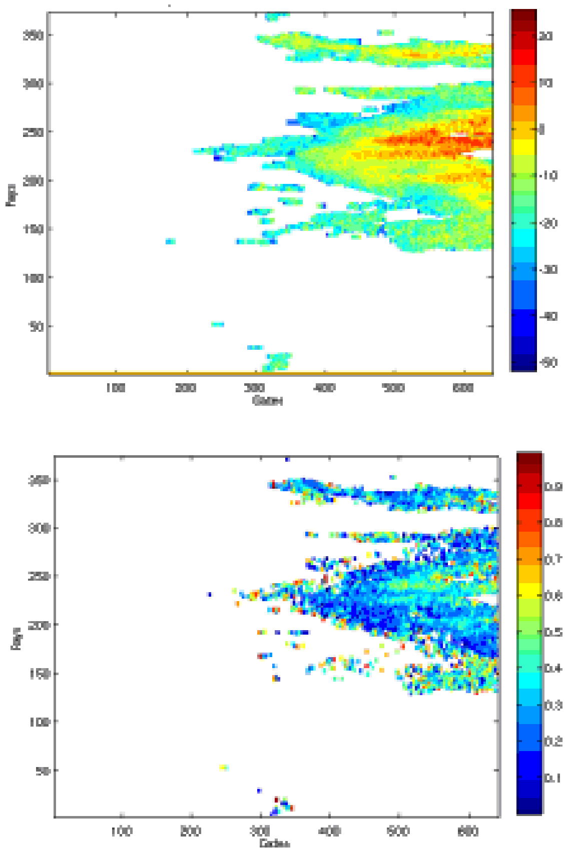

Fig.11. As in Fig. 10 except H_{dr} (in top panel) and f_{ice} in bottom panel (both after spatial smoothing).

As the result of the aforementioned modifications the FM algorithm becomes more "stable", meaning that there are less beams with saturated values of Φ_{dp}, A_h, etc, as shown on the figure 12 (see azimuth sector 280-320° at end of beams in top panel versus bottom pan el).

Figure 13 shows the comparison of the different variables for beam #200 of the "mixed precipitation" case of April 24, 2007 obtained using both original (blue lines) and modified (red lines) versions of the FM algorithm. One can see the general effect of the improvements discussed above: in the output of the modified version the differential phase Φ_{dp} can be followed more precisely, as well as Z_{dr}, and f_{ice} looks much less spiky and more smooth, and hail is found in much more gates than before.

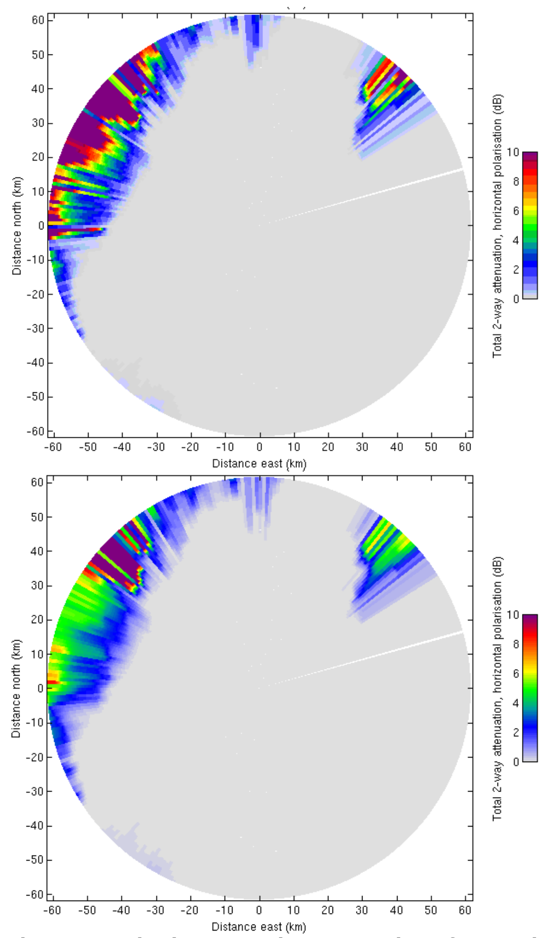

Fig.12. Path integrated attenuation for "mixed precipitation" case of April 24, 2007 (KSAO_20070424-172558.netcdf), achieved from original (top panel) and from modified FM algorithm (bottom panel).

5. Sensitivity of the variational scheme to the absolute calibration of the CASA reflectivity input variable

It was stated in Hogan (2007) that, "…We are effectively assuming that, in relative terms, the error in Z_h is much less than the errors in Z_{dr} and Φ_{dp} so that the retrieval should be forced to be exactly consistent with Z_h…" but the program might be sensitive to the absolute accuracy (absolute calibration) of the input variable Z_h. For the purpose of testing the sensitivity of the algorithm to the input Z_h variable, the data from CASA radar collected at April 24, 2007 at 17:25:58

(KSAO_20070424-172558.netcdf file) was selected. It is described on the CASA website as "significant severe wx outbreak… Large hail was prevalent along the line". It was used as an input to the OES three times: original

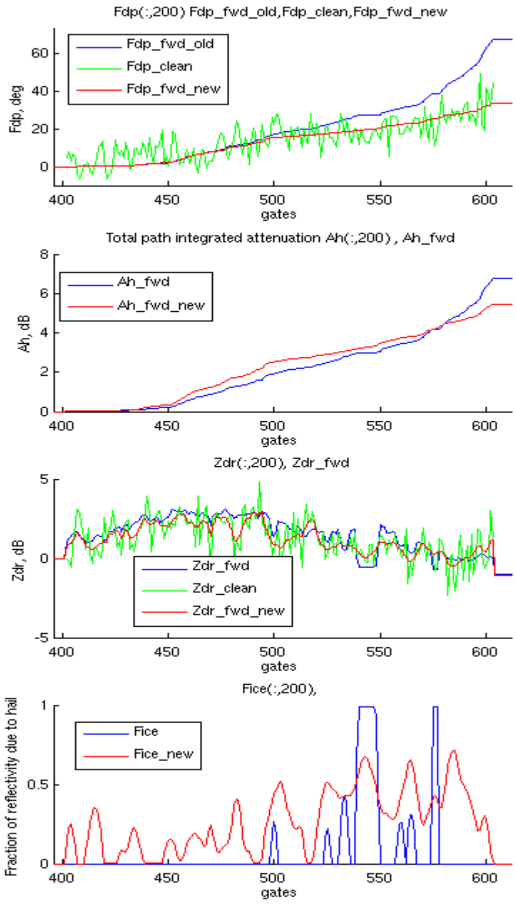

Fig.13. Differential phase Φ_{dp}, 2-way attenuation A_h, differential reflectivity and fraction of reflectivity due to ice f_{ice} for "mixed precipitation" case of April 24, 2007 (KSAO_20070424-172558.netcdf), beam #200 achieved from original (in blue) and from modified (in red) FM algorithm.

reflectivity and modified values (increased by 3 dB and decreased by 3 dB). One can compare the maximum values of the Φ_{dp} and A_h variables for these 3 sets of data for each beam of the scan (at end of the beam, where they become maximum). It can be seen from Fig. 14 that for data sets where Z_h was increased by 3 dB or even left at the original level, the values of Φ_{dp} and A_h tend to saturate for some beams, especially for ones that go through the core of the storm (beams around #230-250). For the dataset where reflectivity values were decreased by 3 dB there are no saturated Φ_{dp} and A_h data. It has to be noted that experiments were done for reflectivity decreased by 1 and 2 dB, and there were areas of saturated data in the OES output, so -3 dB seems to be the minimum for decrease in input reflectivity to avoid spuriously large Φ_{dp} values (at least for this dataset).

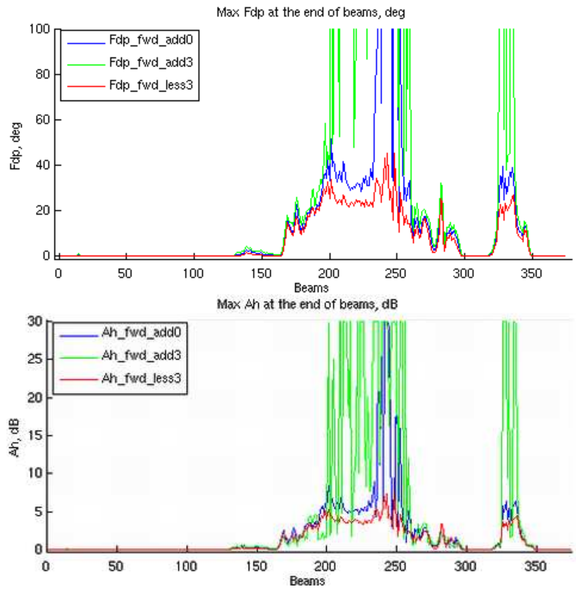

Fig.14. Maximum values of Φ_{dp} and A_h for 3 input data sets (output of optimal estimation scheme or OES). No Z_h offset (blue), Z_h+3 dB (green) and Z_h-3 dB (red).

One can compare the OES output variables for one beam (#246) which goes through the "problem zone" of the scan, which is the core of the storm, as shown on the Fig. 15 (no Z_h offset and ± 3 dB). From this figure, note that only one where reflectivity values were decreased by 3 dB (red line) can follow the input data (black line) with sufficient accuracy.

In FM, Z-R relation is used to compute rain rate, and K_{dp} is calculated and used to compute Φ_{dp} values for each gate. So if the input variable Z_h, which is assumed to be measured by CASA radars (to within an uncertainty of 1 dB) is in error (i.e., too "hot" by 3 dB) all calculated variables (like Φ_{dp}, attenuation A_h, A_v) achieve unrealistically high values, and FM cannot correct for this even after modifications introduced above.

6. Conclusions

The FM algorithm was adapted from S-band to X-band and applied to the CASA IP-1 dual-polarization radar data. The main goal was the correction of measured reflectivity in mixed phase precipitation (rain+hail) which cannot be achieved using differential phase constraints.

Several modifications to the original scheme were applied, with the general goal to increase OES stability. The principal ones being, (a) adjusting the observational errors based on Z_h and ρ_{hv}, (b) provide initial 'guess' for fraction of ice, and (c) sensitivity tests for the Z_h offset. Overall, it appears that the optimal estimation scheme can be adapted to X-band data for correction of attenuation due to mixed phase precipitation (rain mixed with wet ice/hail) provided the input data is well-calibrated (system offsets for Z_h, Z_{dr} and Φ_{dp}).

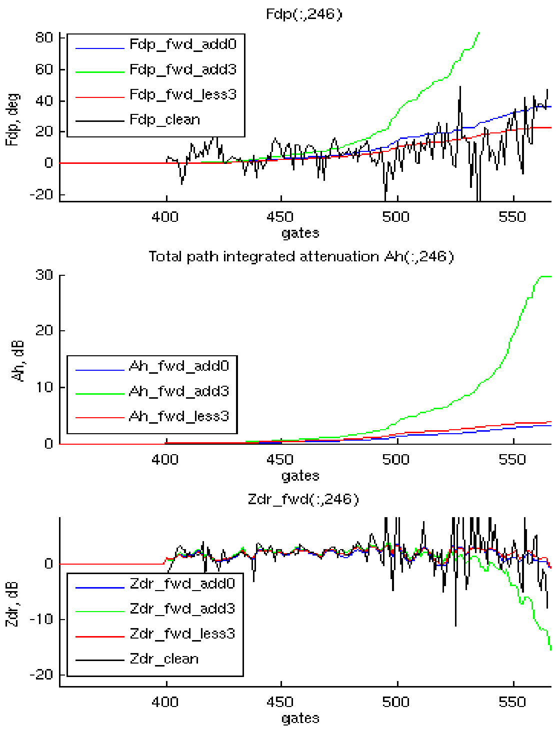

Fig.15. Three data sets. Shown are forward-modeled ("fwd") Φ_{dp}, A_h, Z_{dr} variables, compared to the input variables ("clean").

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