1. Introduction
Rainfall estimation from polarimetric radars has been shown to be more accurate than the classical Z-R approaches. This is because polarimetric variables such as Zdr and Kdp give extra information on the size, shape, orientation and concentration of raindrops, information impossible to retrieve with reflectivity alone. Several studies (e.g. Cifelli et al. 2011, Matrosov, 2010, Ryzhkov et al. 2005, Silvestro et al. 2009, Tabary, 2007, Tabary et al., 2007, Testud et al., 2000, Ulbrich and Lee, 1999, Kabeche et al., 2011) have tried to create empirical algorithms relating rainfall (R) to radar reflectivity (Z), differential reflectivity (Zdr), and specific differential phase (Kdp) at S, C and X-band. Because of the redundancy of the three variables, the most common rainfall algorithms use only two: R(Z,Zdr), R(Zdr, Kdp). Kdp alone has also been proven to be a good rainfall estimator.

The Valparaiso University C-band Polarimetric radar has been operating since 2007. Since then the radar has observed several different types of precipitating events, from lake effect snow, to severe thunderstorms, squall lines, etc. This paper analyzes one squall line event that occurred on 29 May 2011. Three radar rainfall estimators R(Z,Zdr), R(Zdr, Kdp) and R(Kdp) are used to retrieve instantaneous rainfall rates. These results are then integrated in time to obtain cumulated rainfalls and are finally compared to ground observations from 2 stations (Aurora and DuPage, IL).

2. Method
The method used in this paper is described in Gorgucci et al., 2001. This algorithm was derived specifically for S-band, so we used a version updated for use at C-band.

The advantage of this method is that it does not assume an equilibrium raindrop shape a priori, like most other methods. Instead, it uses the capabilities of the polarimetric radar measurements to estimate a mean raindrop shape-size relationship and includes this information in the rainfall estimation algorithms. Therefore, this method can be applied for any mean raindrop shape and does not need an assumption of the shape-size model.

Briefly, the parameter β relates the raindrop axis ratio (b/a) with the equivolumetric spherical diameter (D) following the equation:

\[
\frac{b}{a} = 1.03 - \beta D
\]

so the raindrops become more oblate with increasing size. Polarimetric measurements such as Zh, Zdr and Kdp depend on the size and shape of the raindrops and therefore can be used to estimate β. For a C-band radar the relationship between β and the polarimetric variables is given by:

\[
\beta = 1.89 \cdot Z_{h}^{-0.382} 10^{0.1b_{z}} Z_{r}^{0.400}
\]

The three mentioned rainfall algorithms (R(Zh,Zdr), R(Kdp), and R(Kdp,Zdr)) algorithms have the general form:

\[
R_{\beta}(Z_{h}, Z_{dr}) = c_{1} Z_{h}^{a_{1}} 10^{0.1b_{z}} Z_{r}
\]

\[
R_{\beta}(K_{dp}) = c_{2} K_{dp}^{a_{2}}
\]

\[
R_{\beta}(K_{dp}, Z_{dr}) = c_{3} K_{dp}^{a_{3}} 10^{0.1b_{z}} Z_{r}
\]
The method allows the coefficients \((a_1, b_1, c_1), (a_2, c_2),\) and \((a_3, b_3, c_3)\) to vary with \(\beta\). The best estimates for the coefficients, found using simulations of radar measurements, are below:

\[
\begin{align*}
a_1 &= 0.89 \\
b_1 &= -0.33 \cdot \beta^{-0.74} \\
c_1 &= 5.15 \cdot 10^{-2} \cdot \beta^{-0.74} \\
a_2 &= 0.706 \cdot \beta^{-0.11} \\
c_2 &= 0.622 \cdot \beta^{-1.23} \\
a_3 &= 1.10 \cdot \beta^{0.058} \\
b_3 &= -5.99 \cdot 10^{-3} \cdot \beta^{-1.75} \\
c_3 &= 0.244 \cdot \beta^{-1.72}
\end{align*}
\]

For further details on the method see Gorgucci et al., 2001.

This method has been recently improved (Gorgucci and Baldini, 2009), being now applicable at S, C and X bands.

3. Polarimetric measurements

The three polarimetric variables used in the rainfall estimation (\(Z_h, Z_{dr}\) and \(K_{dp}\)) are shown in figure 1.

\(Z_h\) was calibrated using a technique based on the redundancy of the polarimetric variables (reference: Gourley et al, 2009). Attenuation was corrected using the differential phase shift \((\Phi_{dp})\) which is known as being proportional to the attenuation. The calculation of the attenuation coefficient was made independently for each radar pulse. We chose to do this because these type of linear systems are strongly affected by attenuation in the areas behind the convective line (with respect to the radar point of view) and significantly less affected in the other regions. Using a constant attenuation coefficient throughout the whole domain would probably be underestimating attenuation in some directions and overestimating in others. In figure 1 a one can see an area to the northwest of the radar where the signal could not recovered with attenuation correction techniques because attenuation was so strong that it completely extinguished the signal. In the trailing stratiform part, to the west of the radar, \(Z_h\) is about 5 dBZ stronger than before correction.

\(Z_{dr}\) is shown in Figure 1b. This variable is calibrated by pointing at vertical incidence and checking the offset from 0 dB. The differential attenuation correction is similar to the correction in reflectivity. The attenuation correction is responsible for the radial features in some parts mainly to the northwest of the radar, due to the variability of the attenuation coefficient from pulse to pulse. In the future we will try to address this problem, but for this study we used \(Z_{dr}\) as it is shown.

\(K_{dp}\) is, by definition, the range derivative of \(\Phi_{dp}\). Because of the noise in raw \(\Phi_{dp}\), it needs to be smoothed before computing \(K_{dp}\). We used the technique described in Hubbert and Bringi, 1995 to smooth \(\Phi_{dp}\). We then calculated \(K_{dp}\) simply by taking the difference between consecutive gates and dividing by the gate spacing. The resulting \(K_{dp}\) is shown in figure 1c.

4. Rain rate

Figure 2 shows the instantaneous rain rate in mm/h computed with the 3 algorithms at 1822 UTC. Heavy rain rates (greater than 30 mm/h) are associated with the regions of convection and weaker rain rates are seen in the rear of the system. The main difference between the algorithms are the blank areas in \(R(K_{dp},Z_{dr})\) and \(R(K_{dp})\) (no precipitation zones) that appear as very low precipitation rates in \(R(Z,Z_{dr})\). This is due to negative \(K_{dp}\) values in those specific areas, likely due to the light precipitation or even absence of
rain. The other differences are in the intensity of the rainfall, mainly in the convective areas.

5. Validation

Two airports (DuPage (KDPA) and Aurora (KARR), IL) measured precipitation accumulation several times during the duration of the event. The airports are located at 111 and 124 km range from the radar respectively. Their location can be seen in figures 1a and 2, marked with an x.

The comparison between the observed rainfall and radar estimated with the 3 algorithms is shown in figure 3. The plots show the precipitation accumulation in inches as a function of time for the specified station. The estimated radar rainfall over each station was computed by finding the closest range and azimuth and averaging the rain rates of the 9 closest gates: 3 range bins along 3 contiguous rays. The rainfall accumulations are based on the assumption that the radar derived rain rate was constant over the period between the radar scans (about 6 minutes).

It is evident that all three radar derived methods underestimate the amount of rain observed by the gauges. The amounts derived by R(Z,Zdr) are very close to the derived by R(Zdr,Kdp), while the amount calculated with R(Kdp) is a little below.

The normalized bias (NB) for each estimator and location is shown in table 1. As expected, the NB associated with R(Z,Zdr) is very similar to the R(Zdr, Kdp) bias (0.33 for KARR and 0.19 for KDPA), and the bias from R(Kdp) is slightly higher: 0.399 for KARR and 0.186 for KDPA.

<table>
<thead>
<tr>
<th></th>
<th>KARR</th>
<th>KDPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>R(Z,Zdr)</td>
<td>0.338194</td>
<td>0.172944</td>
</tr>
<tr>
<td>R(Zdr,Kdp)</td>
<td><strong>0.322148</strong></td>
<td>0.17798</td>
</tr>
<tr>
<td>R(Kdp)</td>
<td>0.398572</td>
<td>0.185598</td>
</tr>
</tbody>
</table>

6. Summary and Discussion

We used a method developed by Gorgucci et al, 2001 to calculate rainfall from radar data. This method does not need an assumption for drop size-shape relationship, which is an advantage to most other radar rainfall algorithms.

The results obtained for rain rates were integrated in time assuming a constant rain rate between radar scans. These were then compared to ground observations of rain accumulation in 2 points. The comparison shows that the radar derived rainfall from the 3 algorithms underestimate the actual rainfall. R(Z,Zdr) and R(Zdr,Kdp) are very close to each other and have a very similar NB, while R(Kdp) presents the lowest rainfalls and a higher NB. This is probably because Kdp alone does not capture the full characteristics of rain as well as the combination of (Z,Zdr) and (Zdr, Kdp).

It should be noted that the comparison is made between a rain amount that fell in a single point, and an average of 9 radar gates surrounding that station. Also, the ground observations were taken at 111 and 124 km range, so the measured value from rain gauge represents a very small domain compared to that in one radar pulse volume. So the discrepancy between radar derived rain accumulation and measured by rain gauge may not be due to the performance of the algorithm (or at least, not entirely), but to some extent to the differences in the areal sampling of the 2 instruments.
Acknowledgements:
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6. References


Figure 1  PPI at an elevation of 1.5 ° showing the three polarimetric variables used in the rainfall quantification.

a) Zh at 1836 UTC, b) Zdr at 1856 UTC, c) Kdp at 1856 UTC
Figure 2. Example of rain rate (in mm/h) for 20110529 at 1822 UTC. a) R(Z, Zdr), b) R(Zdr, Kdp), c) R(Kdp).
Figure 3. Comparison between rainfall accumulation (in inches) calculated with the described algorithms and observed in the ground. a) is for Aurora station (KARR) and b) for Du Page (KDPA).