The stochastic spin-up of vorticity in spontaneous tropical cyclogenesis [1]

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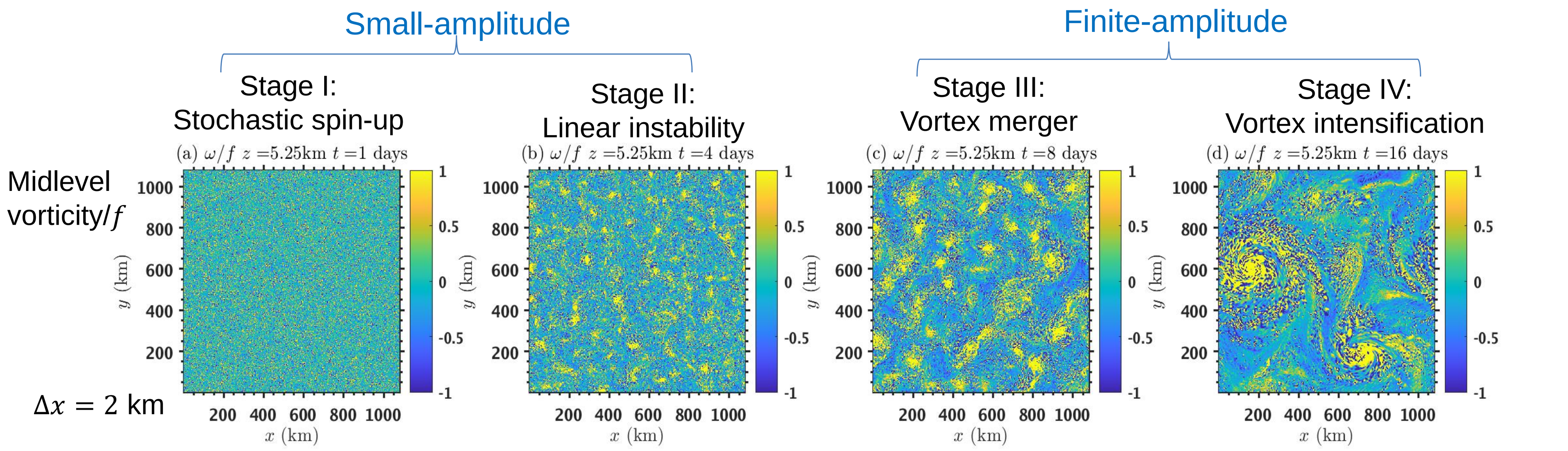
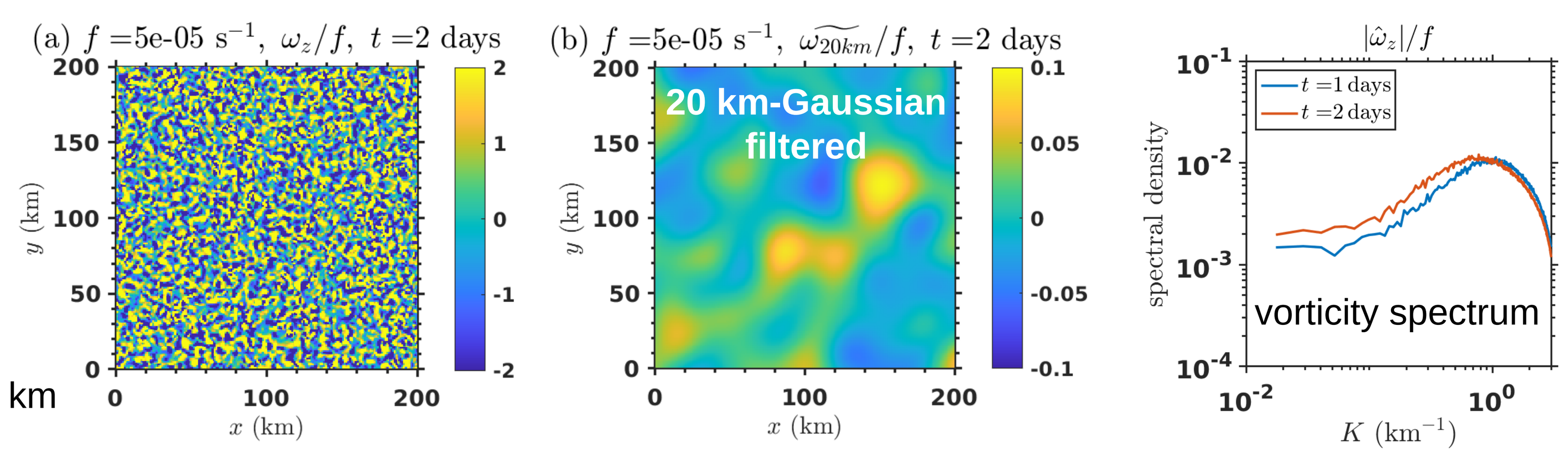
Background: spontaneous TC genesis [2]

- Doubly periodic domain, uniform SST, Bryan Cloud Model 1, SST=300 K, $f = 10^{-4}\ \mathrm{s}^{-1}$, no background shear.
- Doubling longwave radiative feedback to detect the small-amplitude instability[3].



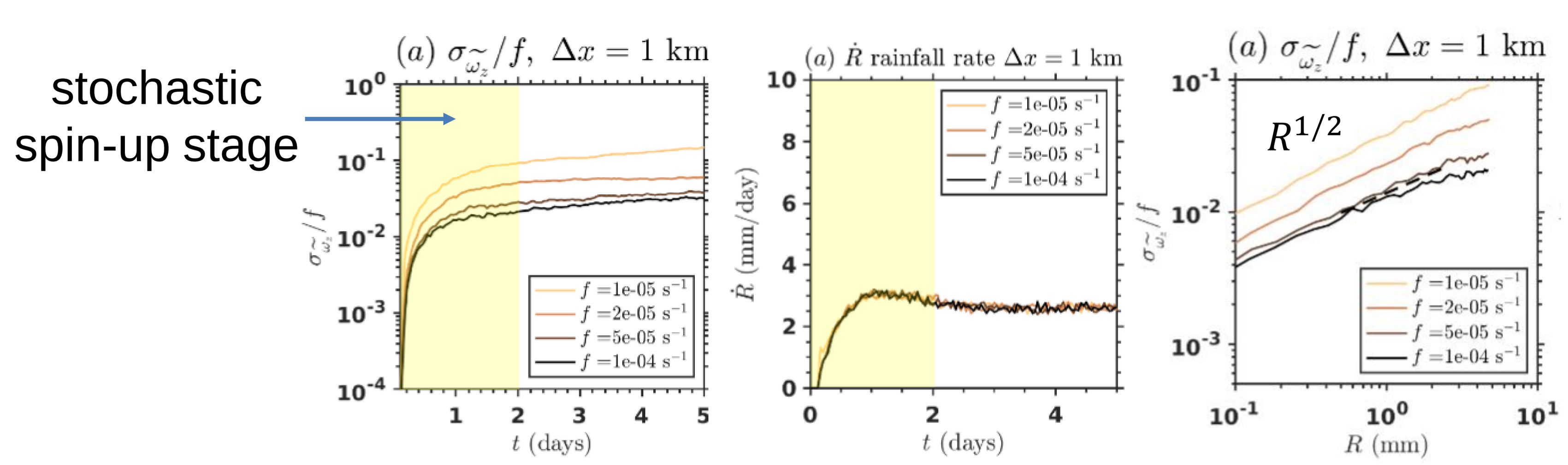
Key quantity: mesoscale midlevel vorticity

Scan to watch simulation movies



- Convection is **NOT** a small-scale noise – it projects onto larger scales [4].
- The **mesoscale vorticity** and vapor fluctuation [5] are **initial seeds** for mesoscale feedback.
- How to quantify the mesoscale imprint of the "convectively generated vorticity noise"?

Experimental result: power law



- The standard deviation of mesoscale vorticity $\widetilde{\omega_z}$, $\sigma_{\widetilde{\omega_z}}$, grows rapidly within the first two days.
- Using the accumulated rainfall rate R as a rescaled time coordinate, there is $\sigma_{\widetilde{\omega_z}} \propto R^{\frac{1}{2}}$.
- The variable R represents the accumulated convective stirring effect.

A stochastic model based on the vorticity equation

Convection: random local pulses

Background wind: random vertical shear

$$\frac{\partial \omega_z}{\partial t} = \underbrace{-f\nabla_h \cdot \mathbf{u}_h}_{f\text{ stretching}} + \underbrace{\nabla_h \cdot (w\boldsymbol{\omega}_h)}_{\text{tilting flux}} - \underbrace{\nabla_h \cdot (\mathbf{u}_h \omega_z)}_{\text{advective flux}},$$

$$w \sim \sum_{n=1}^{\infty} \exp\left(-\frac{|\mathbf{x}-\mathbf{x_n}|^2}{r_c^2}\right)\delta(t-t_n)$$

Dirac-Delta function

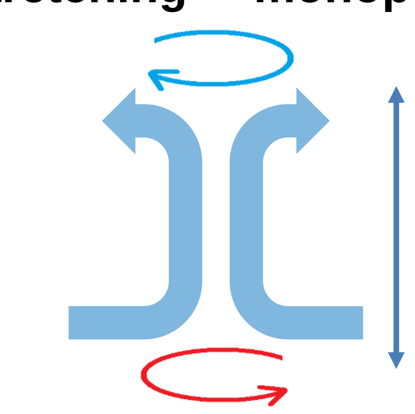
$$\boldsymbol{\omega}_\mathbf{h} \cdot \nabla_h w \sim \frac{\Delta U}{H}\mathbf{n} \cdot \nabla_h w$$

$\mathbf{x_n}$: random location vector

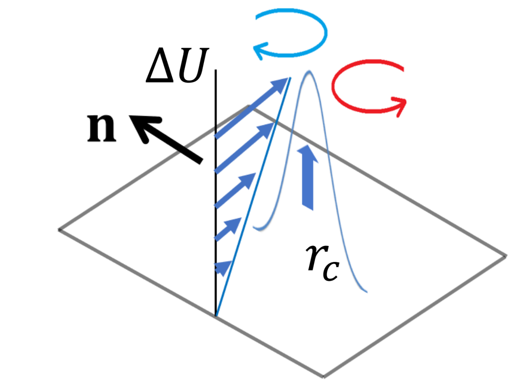
$\mathbf{n}$: a horizontal random unit vector

ΔU: magnitude of eddy vertical wind difference

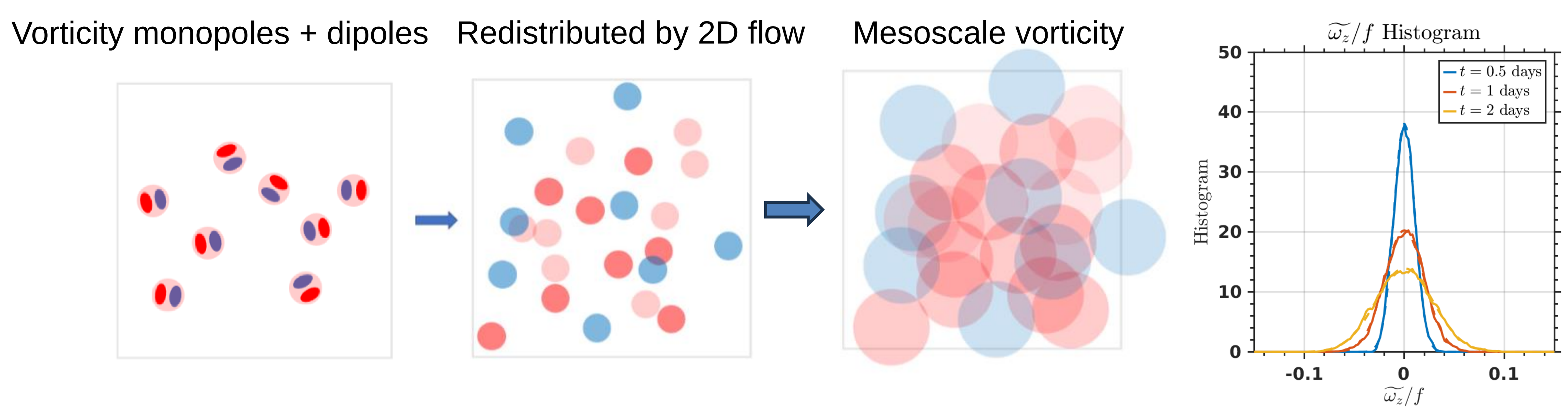
f-stretching → monopoles



Tilting of random shear → dipoles



Modeling the histogram of mesoscale vorticity



cyclones by stretching + cyclones by tilting + anticyclones by tilting:

normal distribution(+) normal distribution(+) normal distribution(-)

= normal distribution

Solution of the stochastic model

Standard deviation $\sigma_{\widetilde{\omega_z}} = (\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2)^{\frac{1}{2}} = f(\Omega t_R)^{\frac{1}{2}}(1 + \mathrm{Ro_c^2})^{1/2}$

growth timescale

Convective Rossby number: the relative importance of tilting to f-stretching

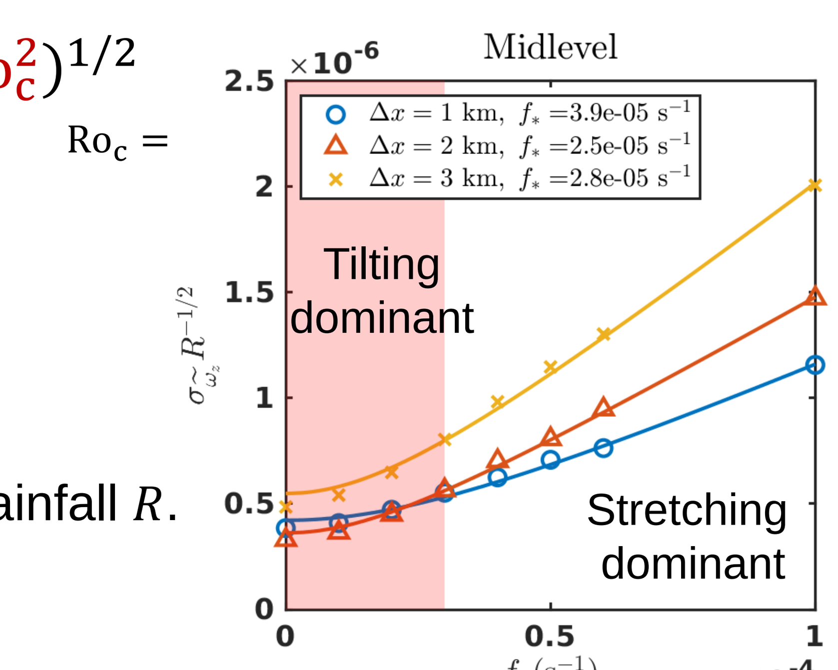
$$\mathrm{Ro_c} \equiv \frac{f_*}{f} = \left(\frac{2}{\pi}\right)^{\frac{1}{2}}\frac{\Delta U}{f r_c}$$

ΔU: eddy vertical wind difference

r_c: cloud radius

t_R: time coordinate rescaled by the domain-averaged accumulated rainfall R.

$f\downarrow$, $\mathrm{Ro_c}\uparrow$, tilting more dominant



- $\mathrm{Ro_c} \sim 1 \Leftrightarrow f_* \sim 3 \times 10^{-5}\ \mathrm{s}^{-1} \Leftrightarrow 12°\mathrm{N}$, transition between tilting & stretching regimes.
- $\sigma_{\widetilde{\omega_z}}$ is sensitive to the grid spacing Δx, which influences the simulated cloud radius r_c.
- Future work: include background vertical shear.

Welcome your feedback!
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Main References

[1] Fu H, O'Neill M. The Stochastic Spin-up of Vorticity in Spontaneous Tropical Cyclogenesis[J]. *J. Atmos. Sci.*, 2024, 81(5): 855-870.

[2] Bretherton C S, Blossey P N, Khairoutdinov M. An Energy-Balance Analysis of Deep Convective Self-Aggregation above Uniform SST[J]. *J. Atmos. Sci.*, 2005, 62(12): 4273-4292.

[3] Fu H, O'Neill M. The Small-Amplitude Dynamics of Spontaneous Tropical Cyclogenesis. Part I: Experiments with Amplified Longwave Radiative Feedback[J]. *J. Atmos. Sci.*, 2024, 81(2): 381-399.

[4] Mapes B E. Equilibrium vs. Activation Control of Large-Scale Variations of Tropical Deep Convection[M]//The Physics and Parameterization of Moist Atmospheric Convection. Dordrecht: Springer Netherlands, 1997: 321-358.

[5] Biagioli G, Tompkins A M. A Dimensionless Parameter for Predicting Convective Self-Aggregation Onset in a Stochastic Reaction-Diffusion Model of Tropical Radiative-Convective Equilibrium[J]. *J. Adv. Model. Earth Syst.*, 2023, 15(5): e2022MS003231.