3D.2 A Scale-Aware Non-Quasi-Equilibrium Mass-Flux Cloud Parametrization

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1. INTRODUCTION

The representation of clouds, especially in the tropics, remains the biggest uncertainty in contemporary climate modeling. The effects and feedbacks associated with clouds are highly complex and occur on scales ranging from microphysical (e.g. 1mm) to synoptic (1000 km). General Circulation Models (GCMs) and Earth System Models (ESMs) typically require grid cells on the order of 100km wide, and therefore require so-called cloud parametrizations to account for moist processes. Members of the largest family of convective parametrizations for GCMs and ESMs, the so-called Mass-Flux Schemes, typically employ the Quasi-Equilibrium (QE) assumption introduced by Arakawa & Schubert [1]. In the QE assumption, convection near-instantaneously consumes instability to reduce the atmosphere to a stable state. However, the QE assumption has been heavily criticized. We examine some of these criticisms.

2. ILL-POSEDNESS OF PROBLEM

One important issue with the QE assumption is that it leads to a diagnostic closure which is essentially ill-posed. In other words, if $M_{B,\lambda}$ is the mass-flux at cloud base, associated with a certain cloud type, here denoted by λ, the cloud work function (CWF) for λ-type clouds is defined as

$$A_\lambda = \int_{z_B}^{z_D(\lambda)} B_\lambda \frac{M_\lambda}{M_{B,\lambda}} dz,$$

where B_λ, M_λ and $z_D(\lambda)$ are respectively the buoyancy, mass-flux, and detrainment height for λ-type clouds.

Arakawa & Schubert compute the time-derivative of the CWF, distinguish between convective terms linearly dependent (through a so-called 'cloud kernel' matrix $\mathcal{K}$) on the base mass fluxes, and large-scale forcing terms F_λ:

$$\frac{d}{dt}A_\lambda = \int \mathcal{K}(\lambda,\lambda')M_{B,\lambda'}d\lambda' + F_\lambda,$$

and set the time-derivative to zero, to obtain a Fredholm integral equation of the second kind. If a discrete instead of continuous spectrum of cloud types is employed, the cloud kernel matrix $\mathcal{K}$ discretizes to a matrix J, and we obtain

$$\underbrace{\frac{d}{dt}A_\lambda}_{\approx 0} = \sum_{\lambda'} J_{\lambda,\lambda'}M_{B,\lambda'} + F_\lambda,$$

which yields the linear algebra problem

$$M_{B,\lambda} = -(J^{-1}F)_\lambda.$$

Unfortunately, the matrix J is ill-conditioned or even singular, so that the above linear algebra problem is not solveable in practice.

A common solution to the ill-posedness of the diagnostic problem is to relax to a prognostic closure, as in Pan & Randall [4]. In Pan & Randall, evolution equations for the base mass-fluxes are systematically derived from the turbulent energy budget equations:

$$\frac{d}{dt}A_\lambda = \sum_{\lambda'} J_{\lambda,\lambda'}M_{B,\lambda'} + F_\lambda,$$
$$\frac{d}{dt}M_{\lambda,B} = \sigma_\lambda\beta_\lambda A_\lambda - \frac{1}{\tau_D}M_{B,\lambda},$$

where σ_λ is the area fraction occupied by λ-type clouds, β_λ is a dimensional constant representing buoyancy effects on the mass-flux tency, and τ_D is a kinetic energy dissipation timescale.

With these equations, Pan & Randall sidestep the issue of solving ill-posed linear algebra problem.

3. INSTABILITY AT QUASI-EQUILIBRIUM

Another issue is that the argument put forth by Arakawa & Schubert to set $\frac{d}{dt}A_\lambda \approx 0$ not only explicitly invokes a time-scale separation between convective and large-scale dynamics, but assumes that under this time-scale separation, the fast-time dynamics converge to the

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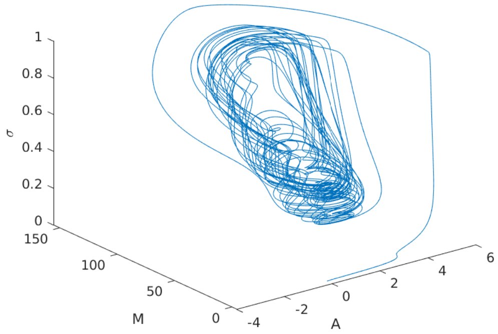

Figure 1: An example of the quasi-periodic, strange-attractor-like dynamics resulting from coupling a single cloud type to the SMCM equations for the cloud area fraction. Along the axes are the cloud CWF A, the cloud base mass flux M, and the cloud area fraction σ. For details, see [3]

equilibrium. This is not so obvious, as we will see.

Pan & Randall lend partial support to this hypoethesis. They show that when there is only a single cloud type as well as a fixed, small, fractional cloud area, the cloud mass flux exhibits damped oscillations around quasi-equilibrium. In other words, the fast convective processes indeed converge to the quasi-equilibium.

However, in Khouider & Leclerc [3], it is shown that interactions between different cloud types can lead to instability at quasi-equilibrium. The prognostic Pan-Randall equations described above, are coupled to a mean-field limit of the Stochastic Multicloud Model (SMCM) pioneered by Khouider and others (see [2]). It is discovered that even in the case of a single cloud type, variability in the cloud area fraction alone is sufficient to destabilize the quasi-equilibrium, and indeed leads to a menagerie of dynamical phenomena including chaos, hysteresis, quasi-periodicity, and strange attractors. One exhibit in this menagerie is depicted in Figure 1.

These results (i.e. that cloud-cloud effects or cloud area fraction variability are both independently sufficient to destabilize quasi-equilibrium) indicate that the quasi-equilibrium assumption must be replaced with a closure that more adequately represents the complex unstable dynamics exhibited by cloud systems.

4. GREY-ZONE RESOLUTIONS

A further issue is that the Arakawa-Schubert scheme, as a whole, predicated on a set of assumptions which fail to hold at so-called grey-zone horizontal resolutions, which we define here as those horizontal- and time-scales at which it is unclear how much of the convection is captured by the GCM (or ESM) kernel and how much by the parametrization. These can be between 10-50km in the horizontal. Indeed, they may even fail at the coarser-than-grey-zone resolutions for which the scheme was designed. These assumptions have been passed down through most of the various Arakawa-Schubert precursors, including Pan & Randall.

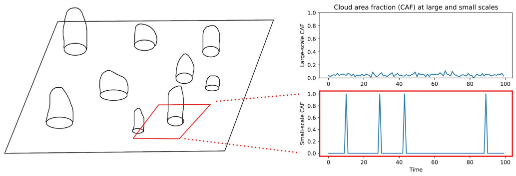

Figure 2: A cartoon of the time-dependence of the cloud area fraction on time. On the left, a cartoon of a grid cell snapshot containing cloud types of various heights and cross-sectionanl areas is depicted. On the top right and bottom right, time series for the cloud area fraction for the entire grid cell, and for a subset of the grid cell, are depicted. Over the entire area, the cloud fraction remains relatively constant, while over a smaller area, the cloud fraction is typically zero but occasionally spikes when clouds appear inside of it.

One of the most such important assumptions is that the fractional cloud area is fixed and small. However, especially at grey-zone resolutions the cloud area fraction will not in general be small and constant in time. Although its time average may in principle still be small, the cloud area fraction will intermittently saturate or partially saturate the GSM (or ESM) grid cell. A cartoon is offered in Figure 2, indicating the statistical nature of this assumption.

When clouds saturate or partially saturate the grid cell, many of the crucial assumptions underlying the Arakawa-Schubert scheme and its main descendants are no longer valid. Among those assumptions are:

1. Due to the relative size of the environmental air region compared to the convective regions, different clouds are nonadjacent and thus only entrain environmental air,

2. For the same reason, the horizontal environmental average of scalar quantities such as moisture and temperature may be approximated by the horizontal grid-cell average,

3. Steady-state equations may be taken to compute scalar quantities (such as moisture, temperature, and so on) inside the

cloud. This is supported by the conservation equation for a conserved variable ψ in a cloud of type λ:

$$\partial_t(\sigma_\lambda \psi_\lambda) + \partial_z(\psi_\lambda M_\lambda) = -\nabla \cdot (\mathbf{v}\psi_c) + S_\lambda^\psi,$$

where σ_λ, M_λ ψ_λ and S_λ^ψ are respectively the cloud area fraction, mass flux for the cloud type, horizontally-averaged value of ψ over the cloud, and cloud source term for ψ. When the cloud area fraction σ_c is fixed and small, the above equation essentially becomes a fast-slow system and the time-derivative can be neglected. The result is a steady-state plume ODE for ψ_λ.

While models have been developed which simulate the stochasticity of the cloud area fraction depicted in Figure 1, such as Khouider [2], these models do not typically address most of the underlying conceptual issues enumerated above.

We develop a cloud model from first-principles, following the framework pioneered by Arakawa-Schubert and Pan-Randall, but proceeding cautiously where the above assumptions (and others) are concerned. The resultant model is scale-aware in that it interpolates between the 'classical' regimes in which cloud area fraction is small (and all the typical Arakawa-Schubert assumptions hold) and those in which clouds completely saturate the grid cell.

References

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