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MULTI-LAG HYBRID CORRELATION COEFFICIENT ESTIMATOR

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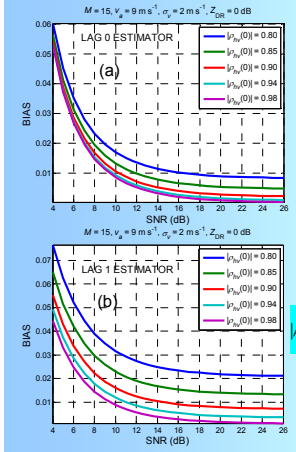
1. MOTIVATION

- The copolar correlation coefficient ($|\rho_{hv}(0)|$) is used for recognition of the types of radar echoes and in separation of returns from rain and snow.
  - The latter requires precise measurements of the correlation coefficient in areas with low and moderate signal-to-noise ratios (i.e., below 15 dB).
- These estimates are unusable when larger than one, which is common when the number of samples per dwell is small and in areas with signal-to-noise ratios lower than 15 dB.
- There are three causes to this
  - Mismeasurement of noise powers in the horizontal and vertical channels.
    - ✓ Can be mitigated using more accurate radial based noise estimator.
  - Inherent positive bias in the correlation coefficient estimator (pronounced when the number of samples per dwell is small).
    - ✓ Can be mitigated if a less biased estimator is devised.
  - Standard deviation (larger when the number of samples per dwell is small).
    - ✓ Can be mitigated by
      - increasing dwell times.
      - devising more efficient estimator.
      - averaging along range.



$$|\hat{\rho}_{hv}(0)| = \sqrt{\frac{|\hat{R}_{hv}(0)|^2}{\hat{S}_h\hat{S}_v}}$$

$$|\hat{\rho}_{hv}(0)| = \sqrt{\frac{|\hat{R}_{hv1}(T)|^2 + |\hat{R}_{hv2}(T)|^2}{2|\hat{R}_h(T)||\hat{R}_v(T)|}}$$

Figure 1. Biases of the standard estimators. (a) Lag zero, and (b) lag one estimator

2. THE NOVEL $|\rho_{hv}(0)|$ ESTIMATOR

- Based on the approximate mathematical expectations

$$\left\langle \hat{P}_h\hat{P}_v \right\rangle \approx P_hP_v + \frac{S_hS_v|\rho_{hv}(0)|^2}{M}$$
$$\left\langle |\hat{R}_{hv}(0)|^2 \right\rangle \approx S_hS_v|\rho_{hv}(0)|^2 + \frac{P_hP_v}{M-1} \qquad (1)$$

These show the expectations of $\hat{P}_h\hat{P}_v$ and $|\hat{R}_{hv}(0)|^2$ to be approximately the linear system of equations with unknowns

$$P_hP_v = S_hS_v + S_hN_v + S_vN_h + N_hN_v \text{ and } S_hS_v|\rho_{hv}(0)|^2 \qquad (2)$$

By solving system of equations (1) we create new estimates

$$\hat{E}_1 = \frac{M^2}{M^2-1}\left(\hat{P}_h\hat{P}_v - \frac{|\hat{R}_{hv}(0)|^2}{M}\right) - \hat{P}_hN_v - \hat{P}_vN_h + N_hN_v$$
$$\hat{E}_2 = \frac{M^2}{M^2-1}\left(|\hat{R}_{hv}(0)|^2 - \frac{\hat{P}_h\hat{P}_v}{M}\right) \qquad (3)$$

where

$$\left\langle \hat{E}_1 \right\rangle \approx S_hS_v \text{ and } \left\langle \hat{E}_2 \right\rangle \approx S_hS_v|\rho_{hv}(0)|^2 \qquad (4)$$

so

$$|\hat{\rho}_{hv}(0)|_{HBD} = \sqrt{\frac{\hat{E}_2}{\hat{E}_1}} \qquad (5)$$

- The estimator (5) is still biased but the bias is less sensitive to noise which allows for linear correction. The results are shown in Figure 2.

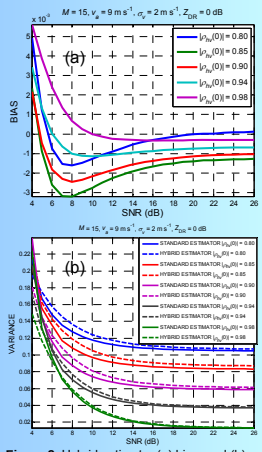
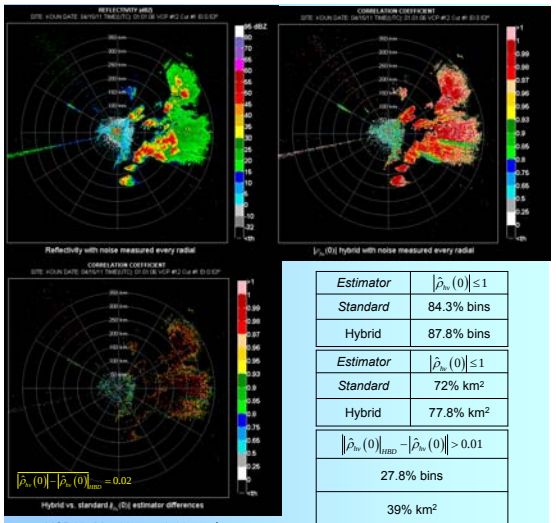


Figure 2. Hybrid estimator (a) bias, and (b) variance.

- Results show that the bias remains within the desired interval of ±0.01 for smaller values of SNR without significant increase in variance compared to the standard lag zero estimator.

3. EVALUATION ON TIME-SERIES

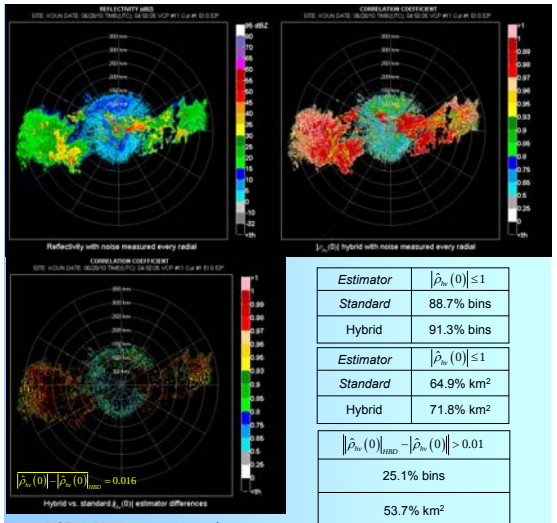


VCP12: M = 16, v_a = 9.03 m s^{-1}

Estimator	$\lvert\hat{\rho}_{hv}(0)\rvert \le 1$
Standard	84.3% bins
Hybrid	87.8% bins

Estimator	$\lvert\hat{\rho}_{hv}(0)\rvert \le 1$
Standard	72% km^2
Hybrid	77.8% km^2

$\left\lVert\hat{\rho}_{hv}(0)\right\rVert_{HBD} - \left\lvert\hat{\rho}_{hv}(0)\right\rvert > 0.01$
27.8% bins
39% km^2

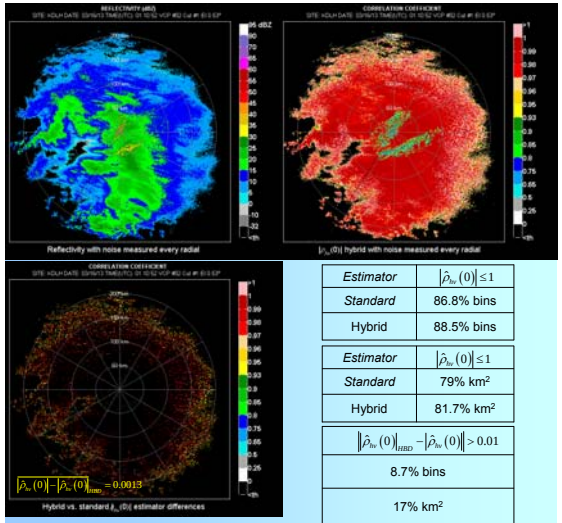


VCP11: M = 17, v_a = 8.92 m s^{-1}

Estimator	$\lvert\hat{\rho}_{hv}(0)\rvert \le 1$
Standard	88.7% bins
Hybrid	91.3% bins

Estimator	$\lvert\hat{\rho}_{hv}(0)\rvert \le 1$
Standard	64.9% km^2
Hybrid	71.8% km^2

$\left\lVert\hat{\rho}_{hv}(0)\right\rVert_{HBD} - \left\lvert\hat{\rho}_{hv}(0)\right\rvert > 0.01$
25.1% bins
53.7% km^2



VCP32: M = 64, v_a = 8.4 m s^{-1}

Estimator	$\lvert\hat{\rho}_{hv}(0)\rvert \le 1$
Standard	86.8% bins
Hybrid	88.5% bins

Estimator	$\lvert\hat{\rho}_{hv}(0)\rvert \le 1$
Standard	79% km^2
Hybrid	81.7% km^2

$\left\lVert\hat{\rho}_{hv}(0)\right\rVert_{HBD} - \left\lvert\hat{\rho}_{hv}(0)\right\rvert > 0.01$
8.7% bins
17% km^2

4. SUMMARY

- The hybrid estimator
  - ✓ exhibits improved bias compared to the standard estimators.
  - ✓ does not increase variance compared to the standard estimators.
- This results in
  - ✓ Improved accuracy of the correlation coefficient estimates.
    - May improve the function of the hydrometeor classification algorithms (HCA).
  - ✓ Moderately increased number of valid estimates.
    - To further increase the number of valid estimates range averaging in weak signal regions may be employed.

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