Geometric Interpretation of Dual-Polarization Radar Meteorological Observations

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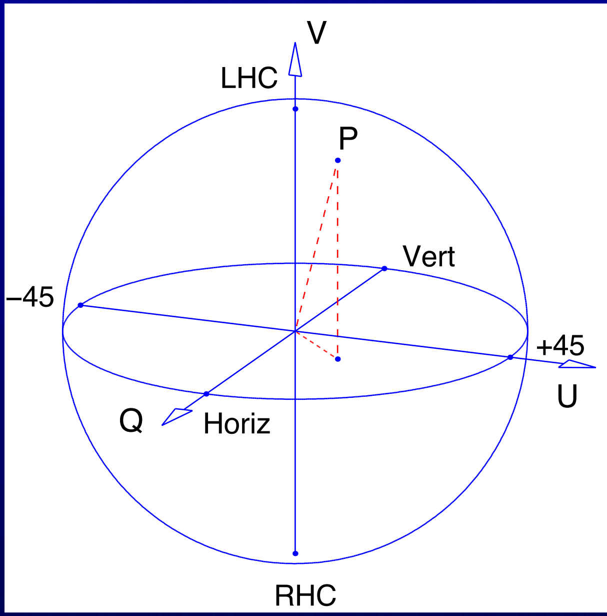
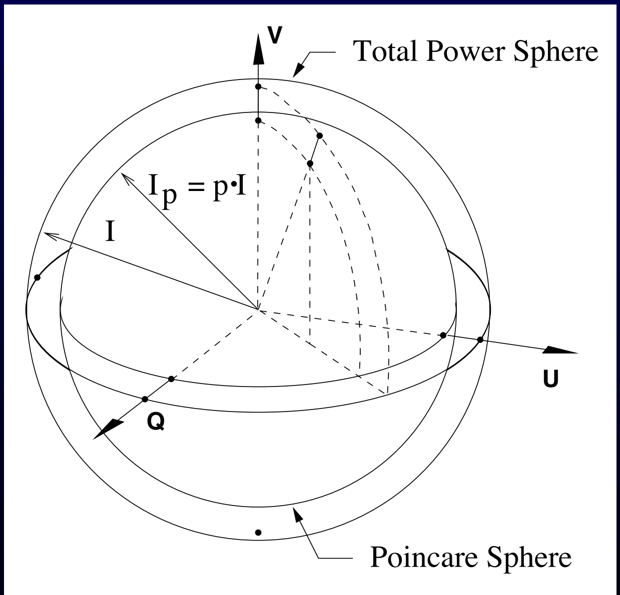
Main Points

- The polarization state of radar signals is completely specified by power values, in particular the Stokes parameters I,Q,U,V.
- Two basic classes of scatterers exist based on the symmetries of their scattering: i) oriented or alignable, and ii) randomly oriented or shaped.
- The Poincare sphere provides a valuable means for visualizing and understanding the various polarization effects and how to analyze them.
- Meteorological radar signals have an un-polarized as well as a polarized component.
- It is important to properly account for the unpolarized component in interpreting observations.

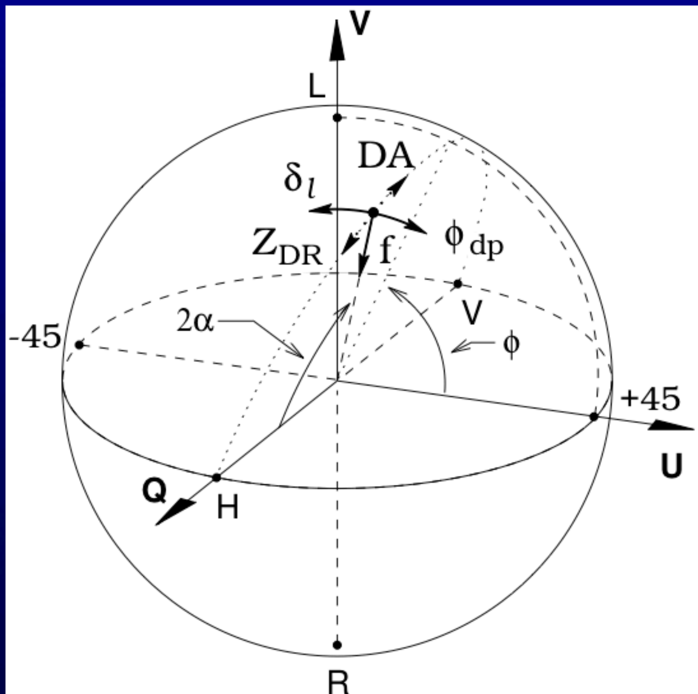
(See Conference paper 12A.5 for details)





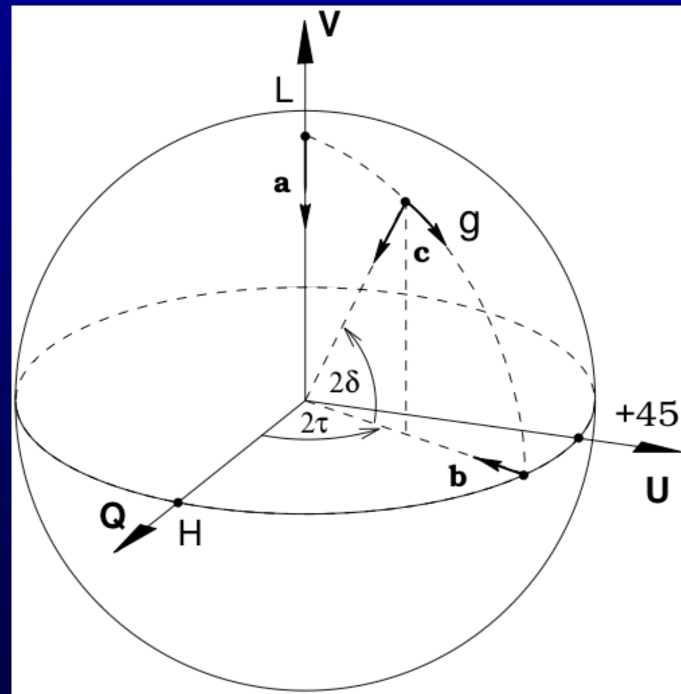
Basic Scattering Classes

Horizontally Aligned/Oriented



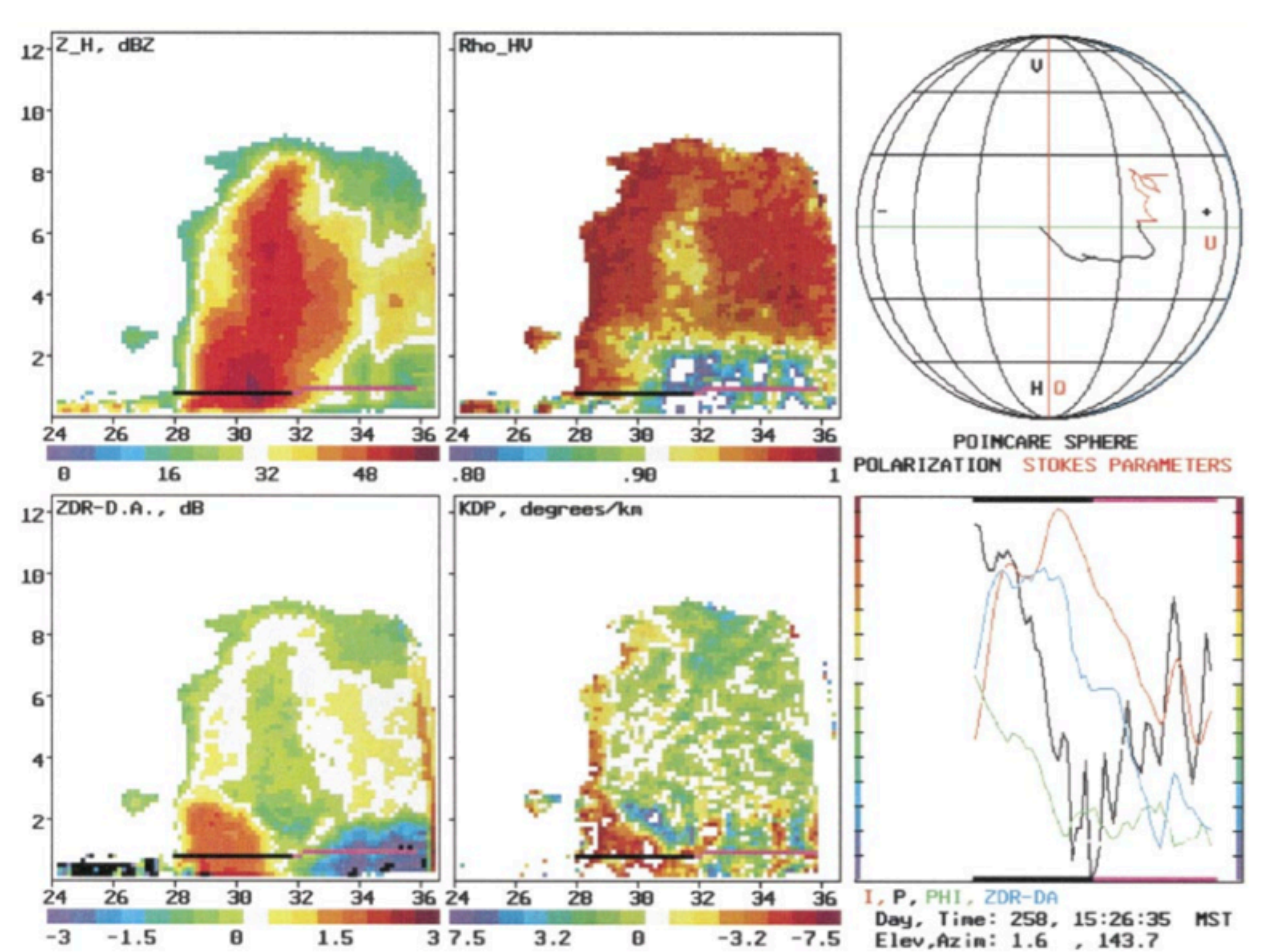
- Changes symmetric about Q (H,V) axis.
- Best described in spherical coordinate system (2α,ϕ,p).
- Effects of Zdr/DA, ϕ_dp/δ, and ρ_hv are in orthogonal directions for equal H,V powers (simultaneous transmissions).

Randomly Oriented/Shaped

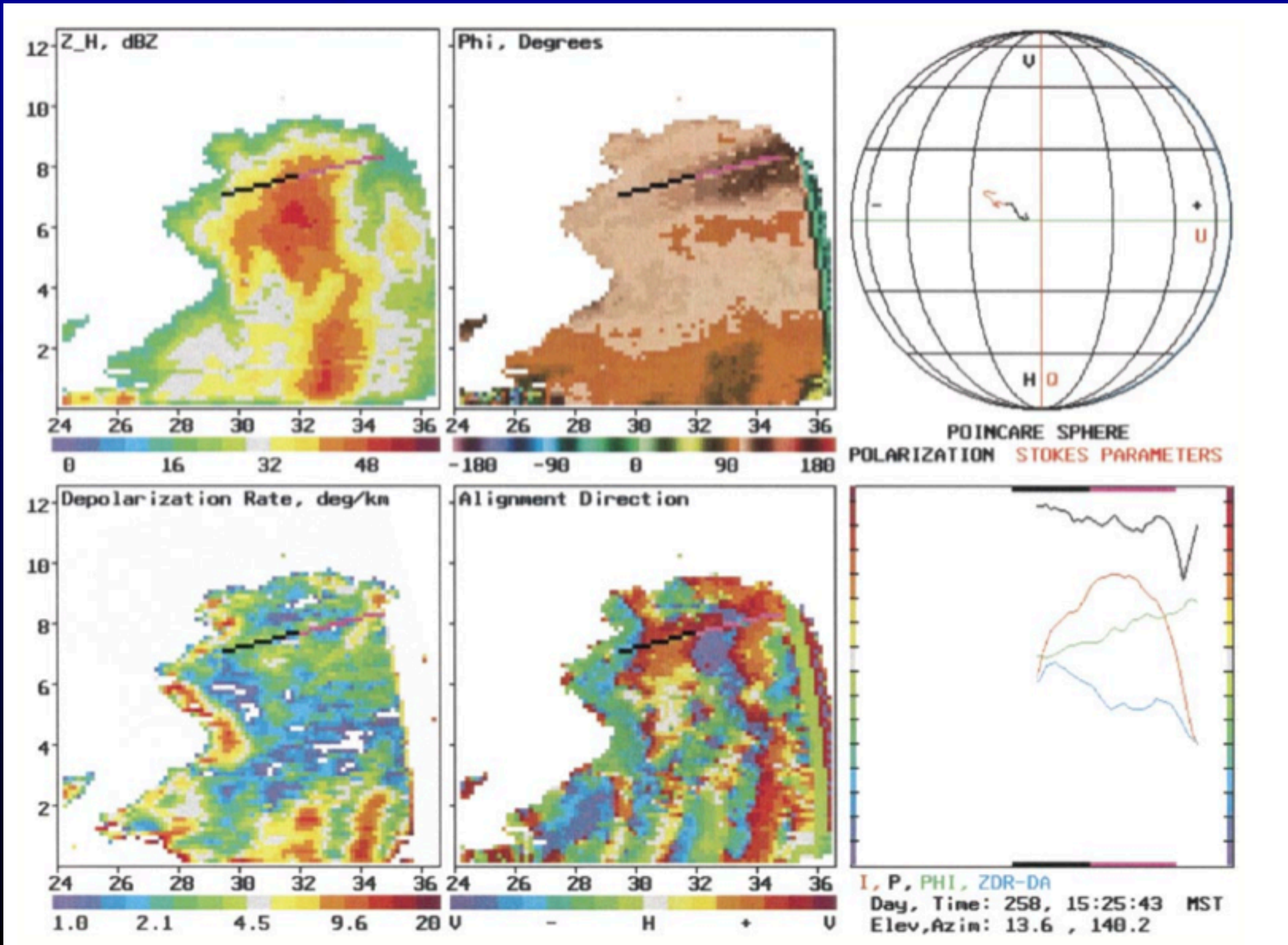


- Changes symmetric about V (L,R) axis.
- Best described in (2δ,2τ,p) coordinates.
- Affects degree of polarization p = I_p/I, and makes polarization more linear.
- Change in p twice as great for circular than for linear incident polarization.
- NEED TO KNOW transmitted polarization state to interpret data.

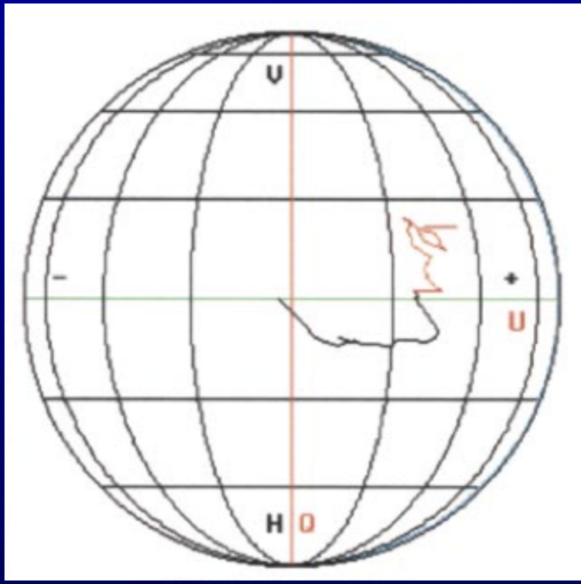
Polarization Trajectories: Mixed rain and hail



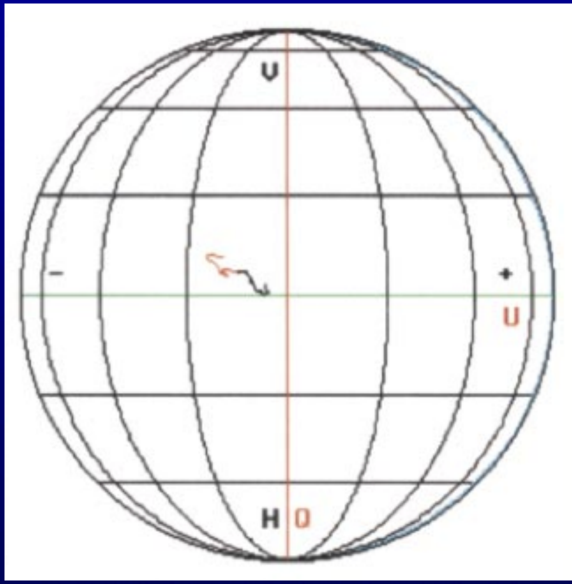
Polarization Trajectories: Electrically aligned ice particles

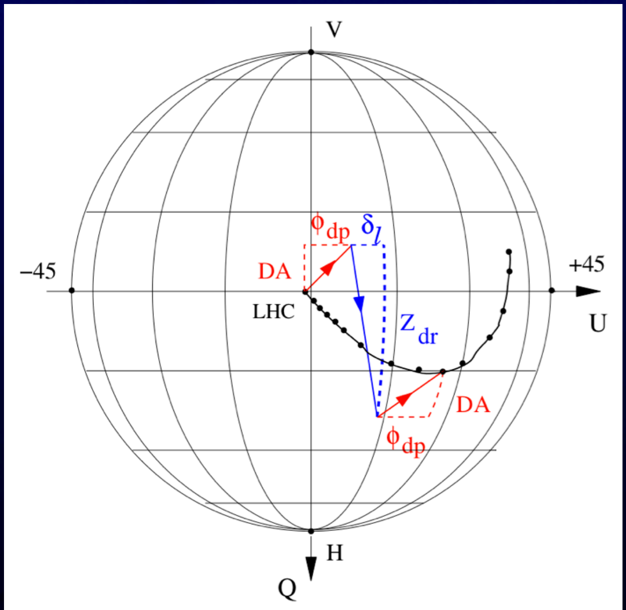


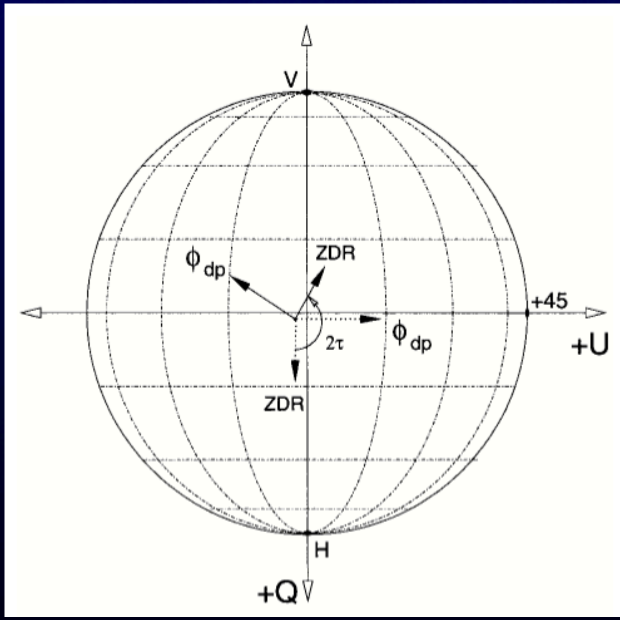
Horizontally Oriented (Rain)



Electrically Aligned (ice xtals)





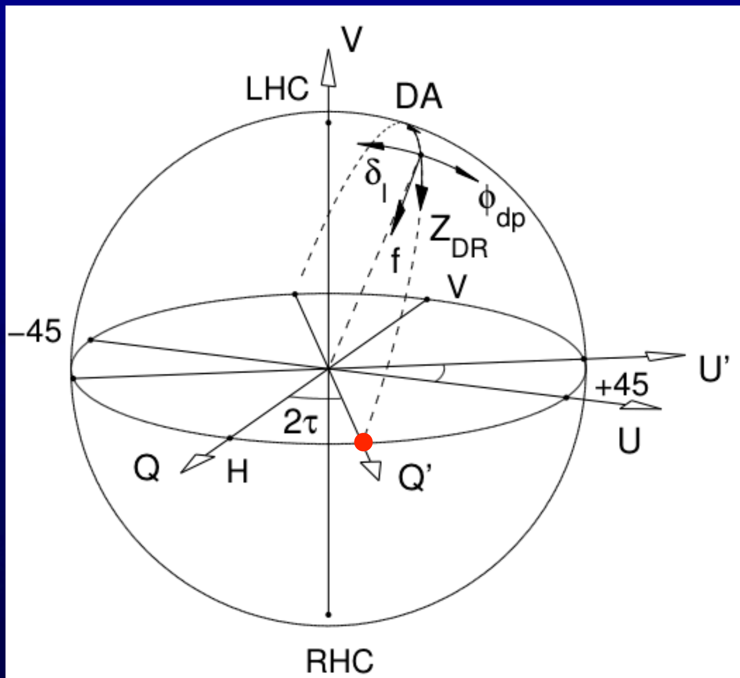


LHC transmitted polarization

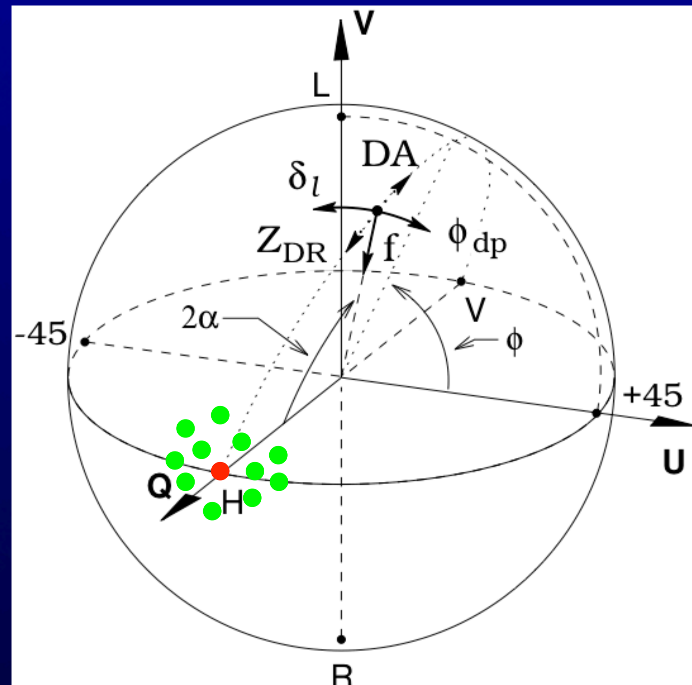
Radial changes due to unpolarized component (not shown)

Non-Horizontal Alignment/Orientation

Angle τ from Horizontal



Random orientation around H



- Determine polarization effects from coordinate rotations of basic horizontal alignment effects to new axis of symmetry.
- Superimpose effects of multiple alignment directions (green dots).
- No need to go back to scattering matrix approach - purely geometric analyses.

Effects of unpolarized component: Covariance calculations

$$\hat{E}_1 = E_1 e^{j\phi_1} + \hat{E}_{u1}(t)$$
$$\hat{E}_2 = E_2 e^{j\phi_2} + \hat{E}_{u2}(t)\,.$$

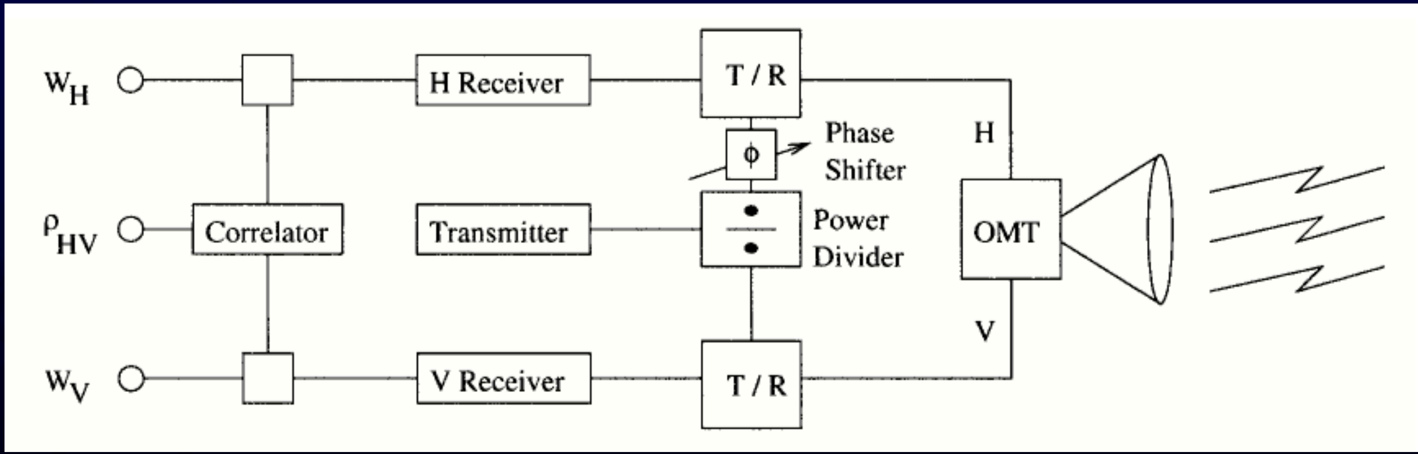
a) Reflected signals: Polarized and unpolarized components

b) Covariances and cross-covariances

$$W_1 = \langle \hat{E}_1 \hat{E}_1^* \rangle = E_1^2 + E_u^2$$
$$W_2 = \langle \hat{E}_2 \hat{E}_2^* \rangle = E_2^2 + E_u^2$$
$$W = \langle \hat{E}_1 \hat{E}_2^* \rangle = E_1 E_2 \, e^{j(\phi_1 - \phi_2)} = |W| e^{j\phi}\,.$$

c) Coherency matrix

$$J = \begin{bmatrix} W_1 & W \\ W^* & W_2 \end{bmatrix}$$



Effects of unpolarized component: Decomposition of coherency matrix into polarized and unpolarized components

$$J = \begin{bmatrix} W_1 & W \\ W^* & W_2 \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} B & D \\ D^* & C \end{bmatrix} .$$

unpolarized polarized

(1,2) = (H,V) Basis

Solve for A,B,C,D:

$$2A = (W_1 + W_2) - \sqrt{(W_1 - W_2)^2 + 4|W|^2)}$$

I

$$2B = (W_1 - W_2) + \sqrt{(W_1 - W_2)^2 + 4|W|^2)}$$

+Q

$$2C = (W_2 - W_1) + \sqrt{(W_1 - W_2)^2 + 4|W|^2)}$$

-Q

+I_p

$$D = W .$$

$$2A = \mathrm{I} - \mathrm{I}_p = \mathrm{I}(1 - p)$$

$$2B = \mathrm{I}_p + \mathrm{Q}$$

$$2C = \mathrm{I}_p - \mathrm{Q}$$

Radical is B+C = I_p:

$$B + C = \sqrt{(W_1 - W_2)^2 + 4|W|^2} = I_p .$$

Unchanged:

$$D = |W|e^{j\phi} = \rho_{HV} e^{j\phi} .$$

Effects of unpolarized component: Z_dr determination

$$J = \begin{bmatrix} W_1 & W \\ W^* & W_2 \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} B & D \\ D^* & C \end{bmatrix} .$$

$$\begin{aligned} 2A &= \mathrm{I} - \mathrm{I}_p = \mathrm{I}(1-p) \\ 2B &= \mathrm{I}_p + \mathrm{Q} \\ 2C &= \mathrm{I}_p - \mathrm{Q} \end{aligned}$$

$$D = |W| e^{j\phi} = \rho_{HV} e^{j\phi} .$$

Usual way: Incorrect/biased by unpolarized power A

$$\widehat{Z}_{\mathrm{DR}} = \frac{W_H}{W_V} = \frac{B+A}{C+A} = \frac{\mathrm{I}+\mathrm{Q}}{\mathrm{I}-\mathrm{Q}} .$$

Correct way: Involves polarized powers only

$$Z_{\mathrm{DR}} = \frac{B}{C} = \frac{\mathrm{I}_p + \mathrm{Q}}{\mathrm{I}_p - \mathrm{Q}}$$

Effects of unpolarized component:
Z_dr determination
Z_h, Z_v determination

$$J = \begin{bmatrix} W_1 & W \\ W^* & W_2 \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} + \begin{bmatrix} B & D \\ D^* & C \end{bmatrix} .$$

$$2A = \mathrm{I} - \mathrm{I}_p = \mathrm{I}(1-p)$$
$$2B = \mathrm{I}_p + \mathrm{Q}$$
$$2C = \mathrm{I}_p - \mathrm{Q}$$

$$D = |W| e^{j\phi} = \rho_{HV} e^{j\phi} .$$

Usual way: Incorrect/biased by unpolarized power A

$$\widehat{Z}_{\mathrm{DR}} = \frac{W_H}{W_V} = \frac{B+A}{C+A} = \frac{\mathrm{I}+\mathrm{Q}}{\mathrm{I}-\mathrm{Q}} .$$

Correct way: Involves polarized powers only (B, C, I_p)

$$Z_{\mathrm{DR}} = \frac{B}{C} = \frac{\mathrm{I}_p + \mathrm{Q}}{\mathrm{I}_p - \mathrm{Q}}$$

Subtract out unpolarized power

$$kZ_H = W_H - \tfrac{1}{2}(\mathrm{I}_{\mathrm{unpolarized}})$$
$$kZ_V = W_V - \tfrac{1}{2}(\mathrm{I}_{\mathrm{unpolarized}}) .$$

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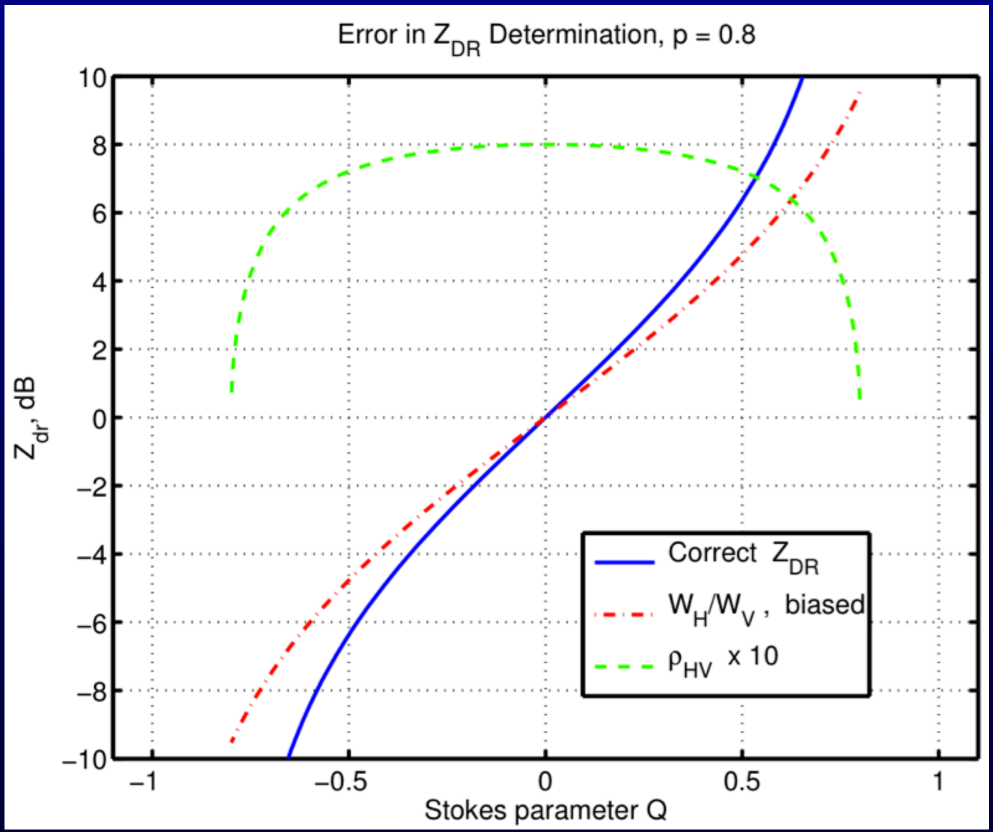
Effects of unpolarized component:

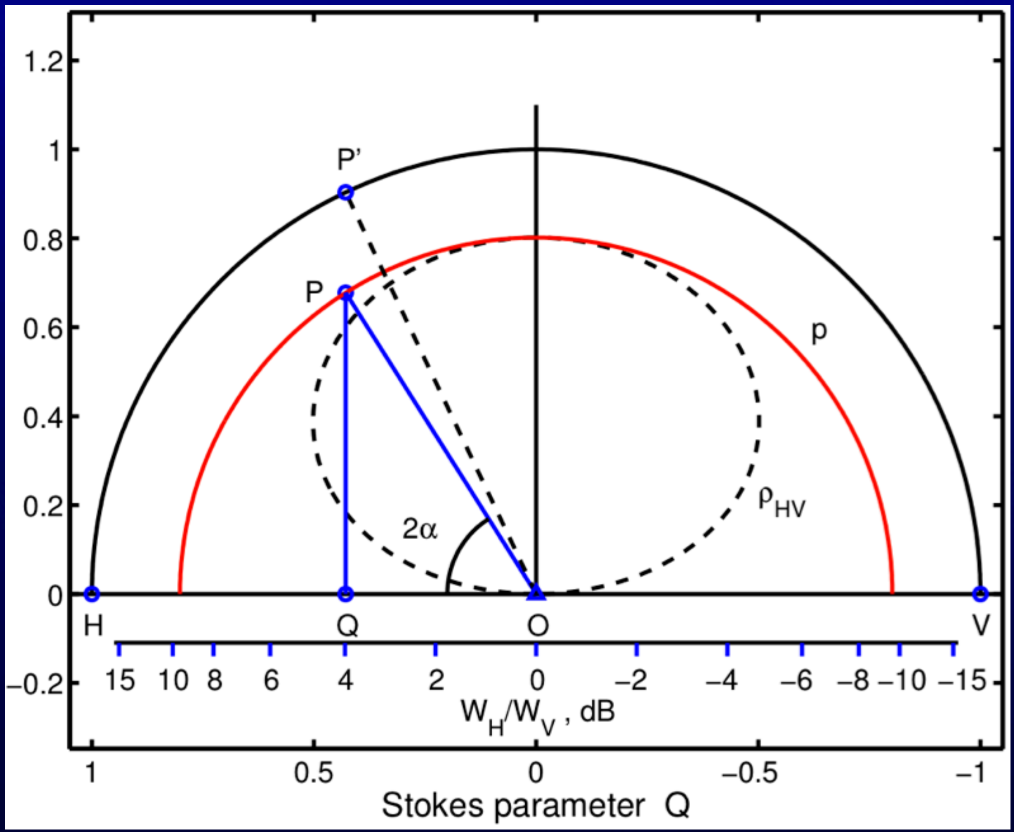
Difference between Wh/Wv & B/C



Wh/Wv bias (p = 0.9):
-0.32 dB for true Zdr = 3 dB
-0.77 dB for true Zdr = 6 dB

Zdr from B/C: No bias!

Geometric interpretation



$$Z_{DR} = \frac{1+Q/I_p}{1-Q/I_p} = \frac{1+\cos(2\alpha)}{1-\cos(2\alpha)} .$$

True Zdr: Triangle OPQ (2α)
Wh/Wv: Triangle OP'Q (x)

Summary
Data processing procedure

a) Calculate Stokes parameters:

$$
\begin{aligned}
\mathrm{I} &= W_H + W_V \\
\mathrm{Q} &= W_H - W_V \\
\mathrm{U} &= 2|W_{HV}|\cos\phi_{HV} \\
\mathrm{V} &= 2|W_{HV}|\sin\phi_{HV} \;.
\end{aligned}
$$

b) Polarized power and degree of polarization p :

$$
\begin{aligned}
I_p &= \sqrt{\mathrm{Q}^2 + \mathrm{U}^2 + \mathrm{V}^2} \\
p &= \mathrm{I}_p / \mathrm{I} \;.
\end{aligned}
$$

c) Correct calculations for Zh, Zv, and Zdr:

$$
\begin{aligned}
kZ_H &= W_H - \tfrac{1}{2}(\mathrm{I} - \mathrm{I}_p) \\
kZ_V &= W_V - \tfrac{1}{2}(\mathrm{I} - \mathrm{I}_p) \;, \\
Z_{\mathrm{DR}} &= \frac{\mathrm{I}_p + \mathrm{Q}}{\mathrm{I}_p - \mathrm{Q}} \;,
\end{aligned}
$$

d) ρ_hv and ϕ are correct as is:

$$
\begin{aligned}
\rho_{HV} &= |W_{HV}| \\
\phi &= \phi_{HV} \;.
\end{aligned}
$$

- No biases for simultaneous transmit/receive.
- Need to know transmitted polarization to best interpret data.
- Presentation will be available at http://lightning.nmt.edu/radar.

End