Rainfall attractors and predictability

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MOTIVATION

Predictability

Degree to which a correct prediction or forecast of a system's state can be made either qualitatively or quantitatively.

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- NWC algorithm

Approximation of the real system

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Observations

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  - •NWP model
  - •NWC algorithm
- Observations

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Predictability of the model

Inherent predictability of atmosphere

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Inherent predictability of atmosphere

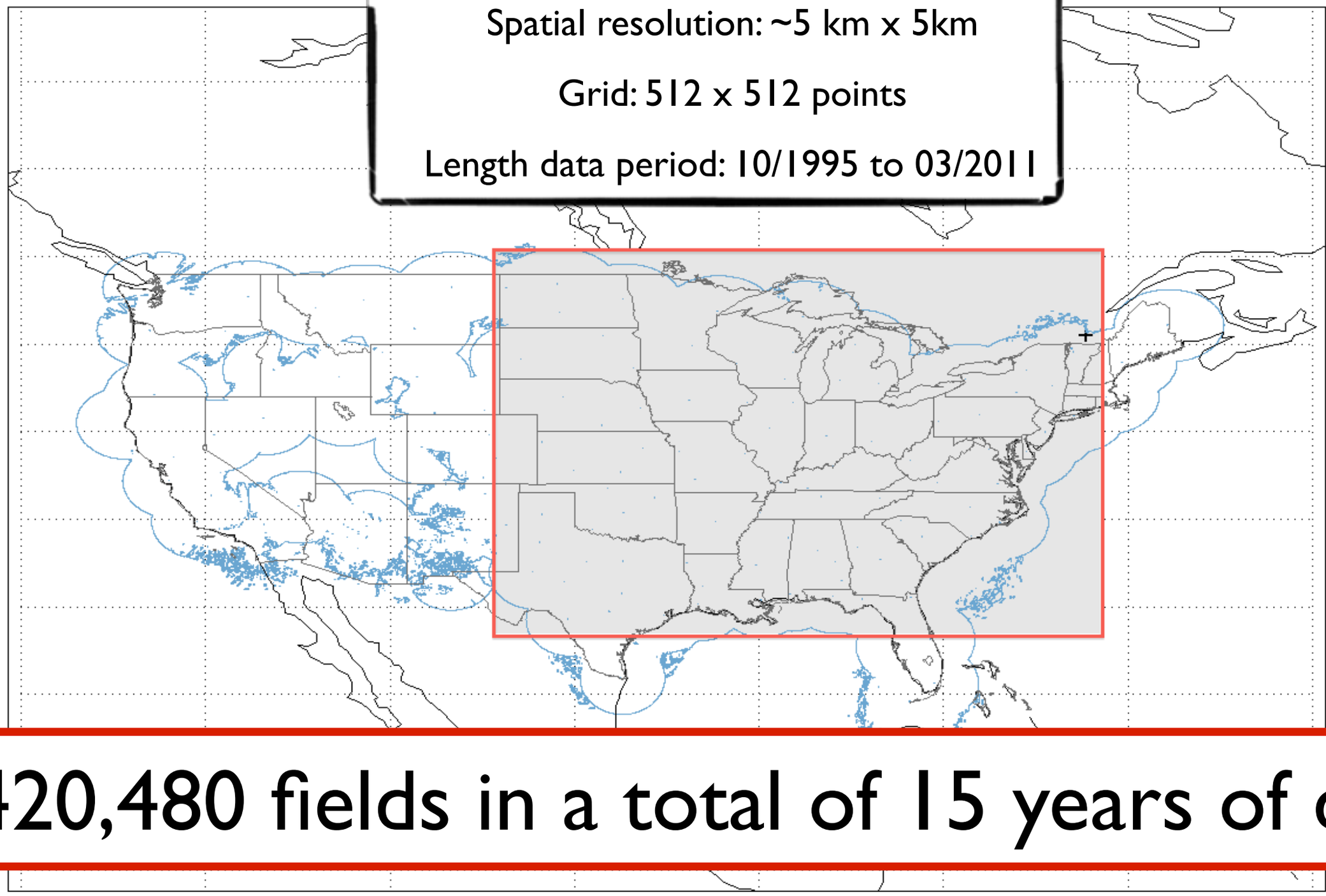
INFORMATION ABOUT DATASET

Temporal resolution : 15 minutes

Spatial resolution: ~5 km x 5km

Grid: 512 x 512 points

Length data period: 10/1995 to 03/2011



420,480 fields in a total of 15 years of data

PHASE SPACE OF RAINFALL FIELDS

The domains has 512 x 512 pixels but only 212,394 of them are rainfall data because of the radar coverage.

Phase space is a space in which all possible states of a system are represented.

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(We have only 420,480 rainfall images ~ 0.001%)

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Statistical properties of the rainfall field will be used to reduce the number of dimensions of the phase space

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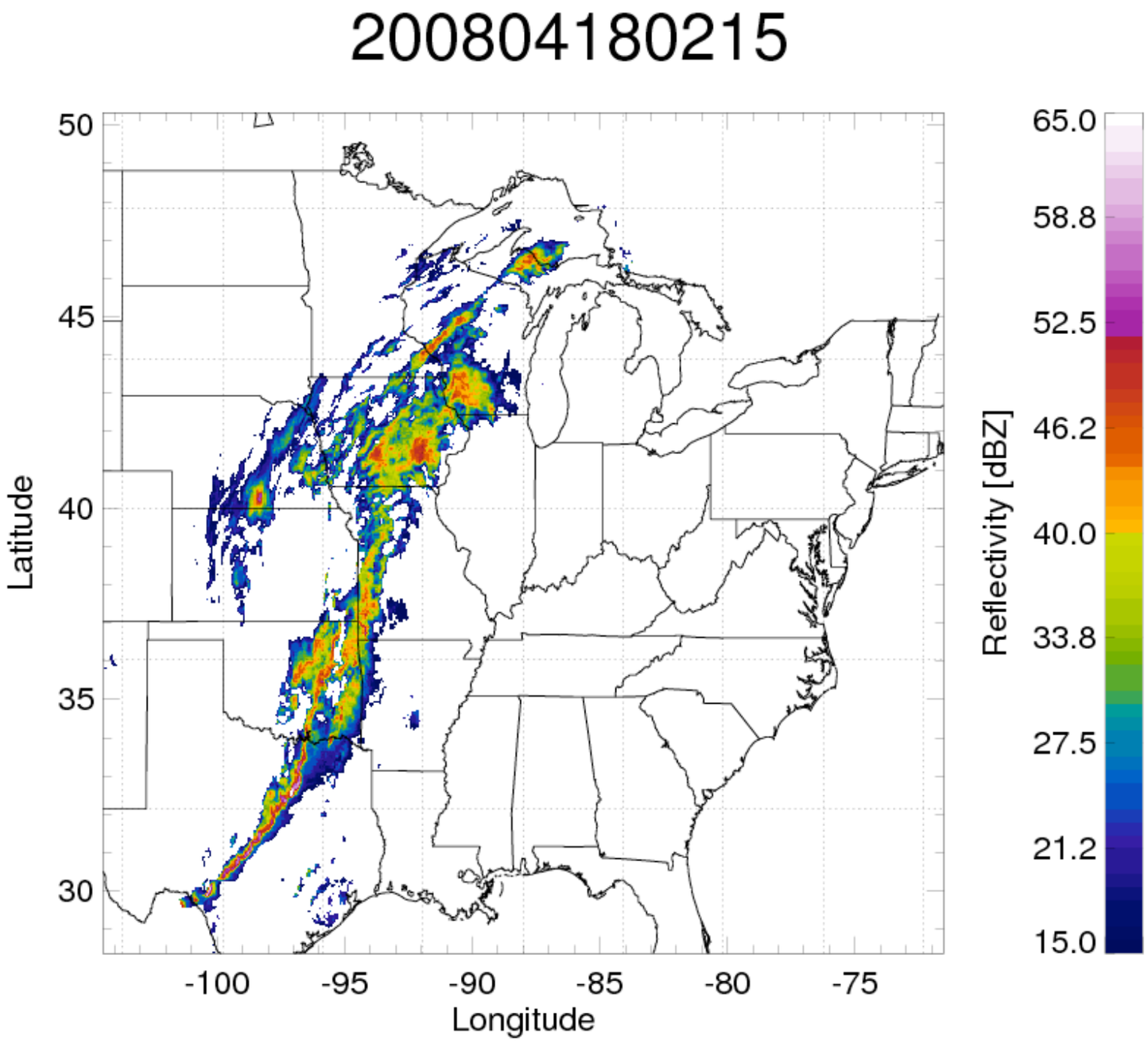
Statistical properties of rainfall field:



Marginal distr.

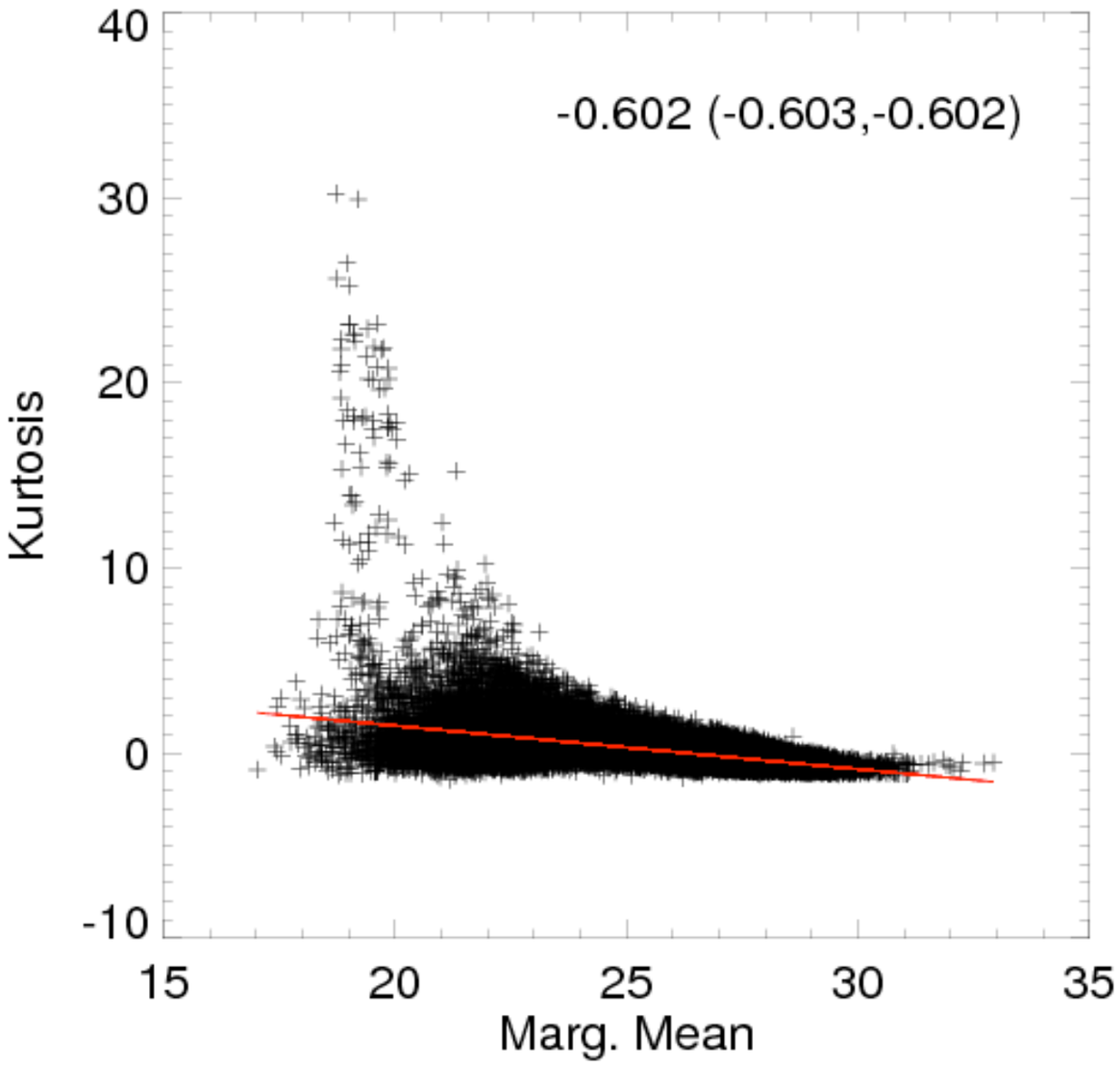
Mean	27.8
Std dev	71.7
Skewness	0.62
Kurtosis	-0.36
Area [# pix.]	22349
# cells	12
Area biggest storm	20627

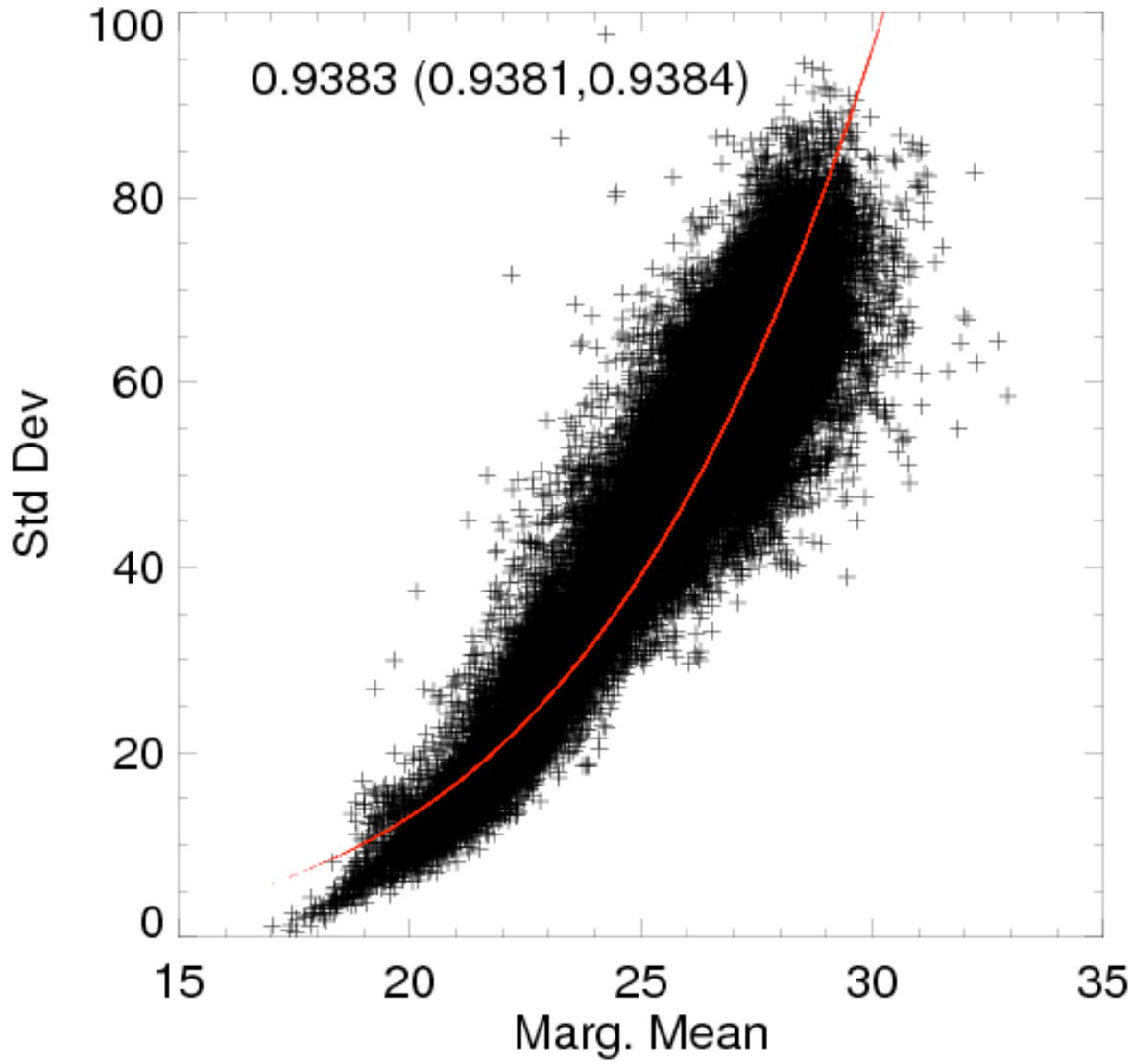
Spatial autocor.

Decor distance	120
Eccentricity	0.98
Orientation	67°
Slope PS	2.54

PHASE SPACE OF RAINFALL FIELDS

Removing of correlated statistical parameters.





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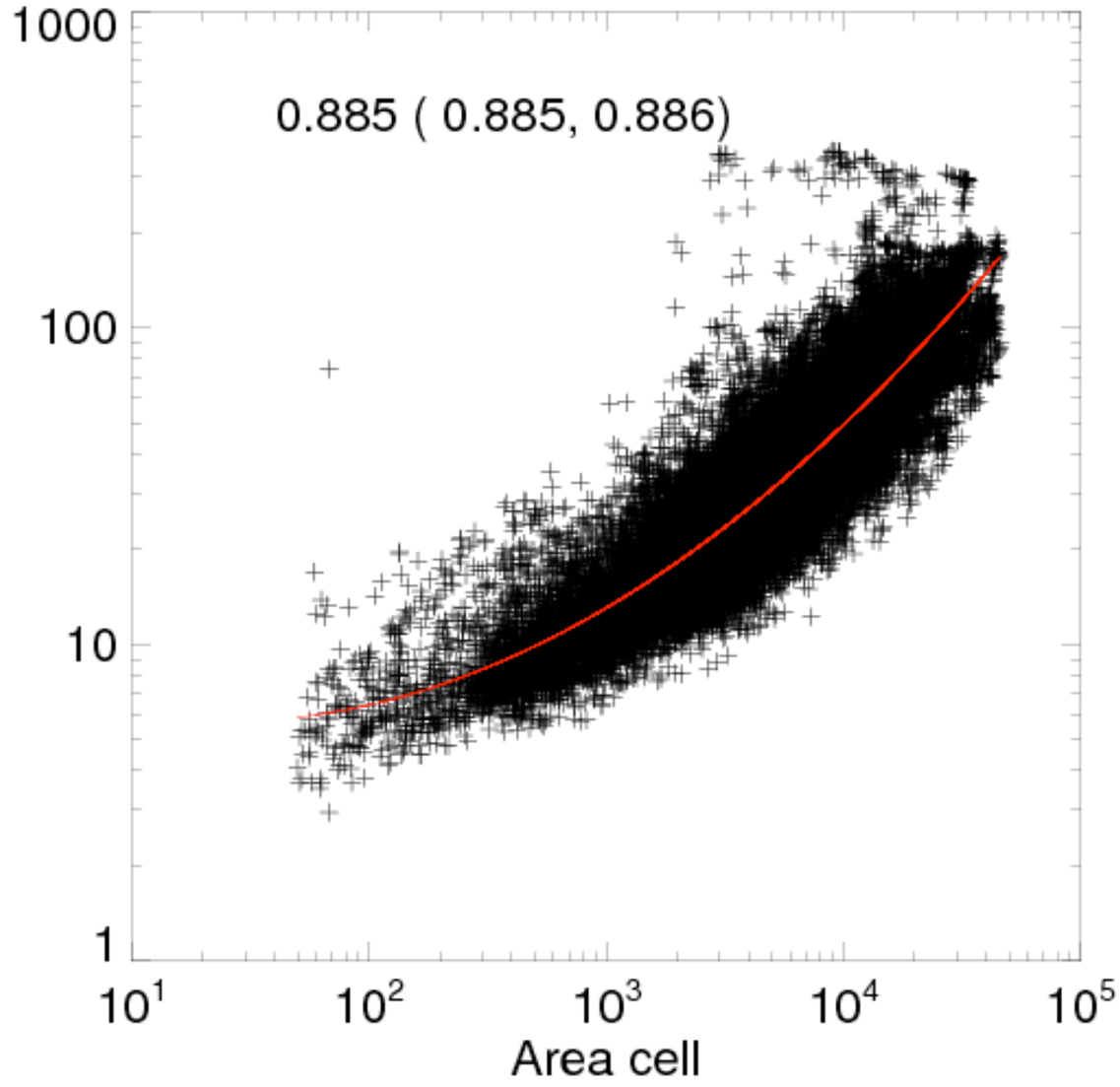
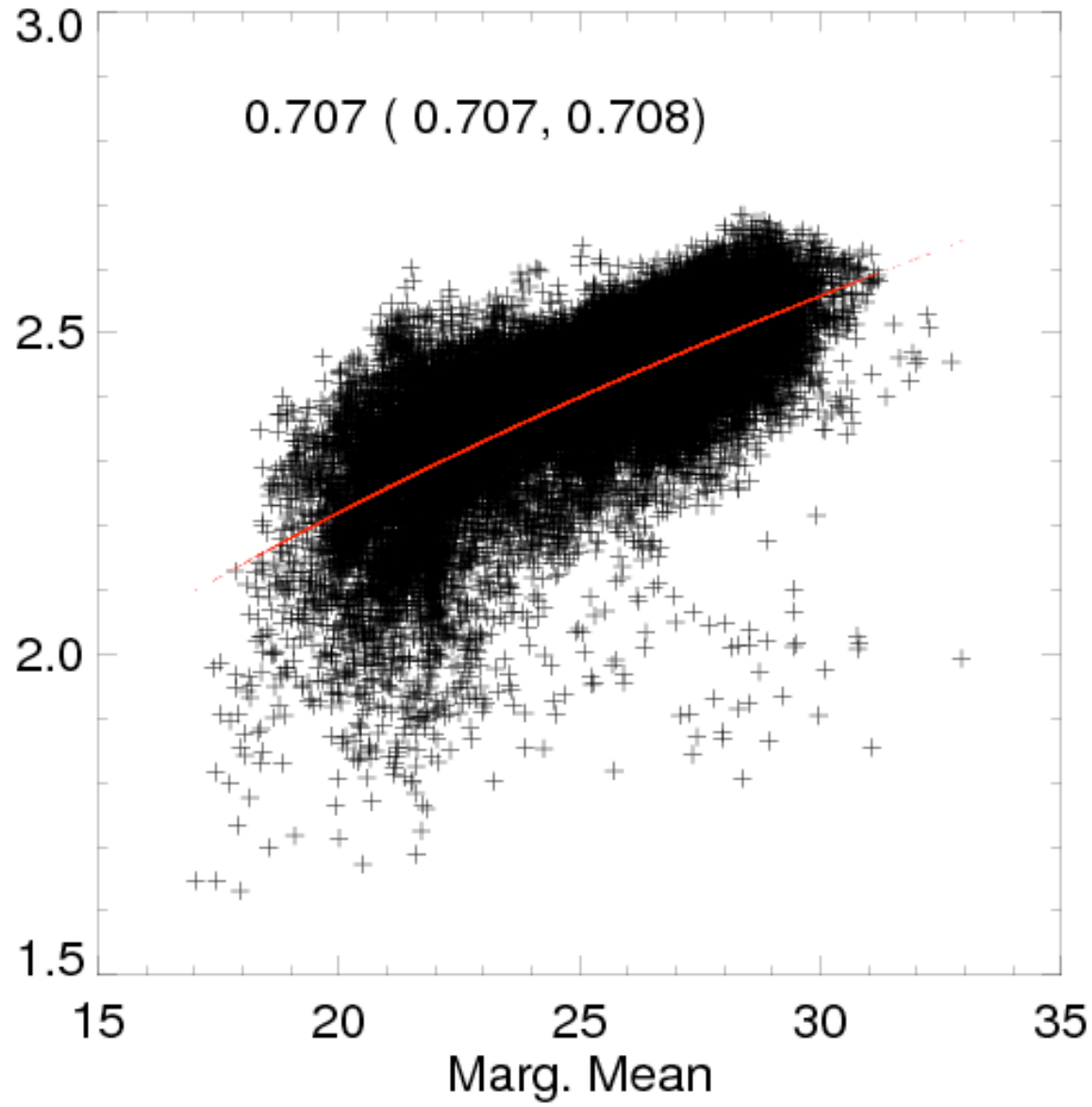
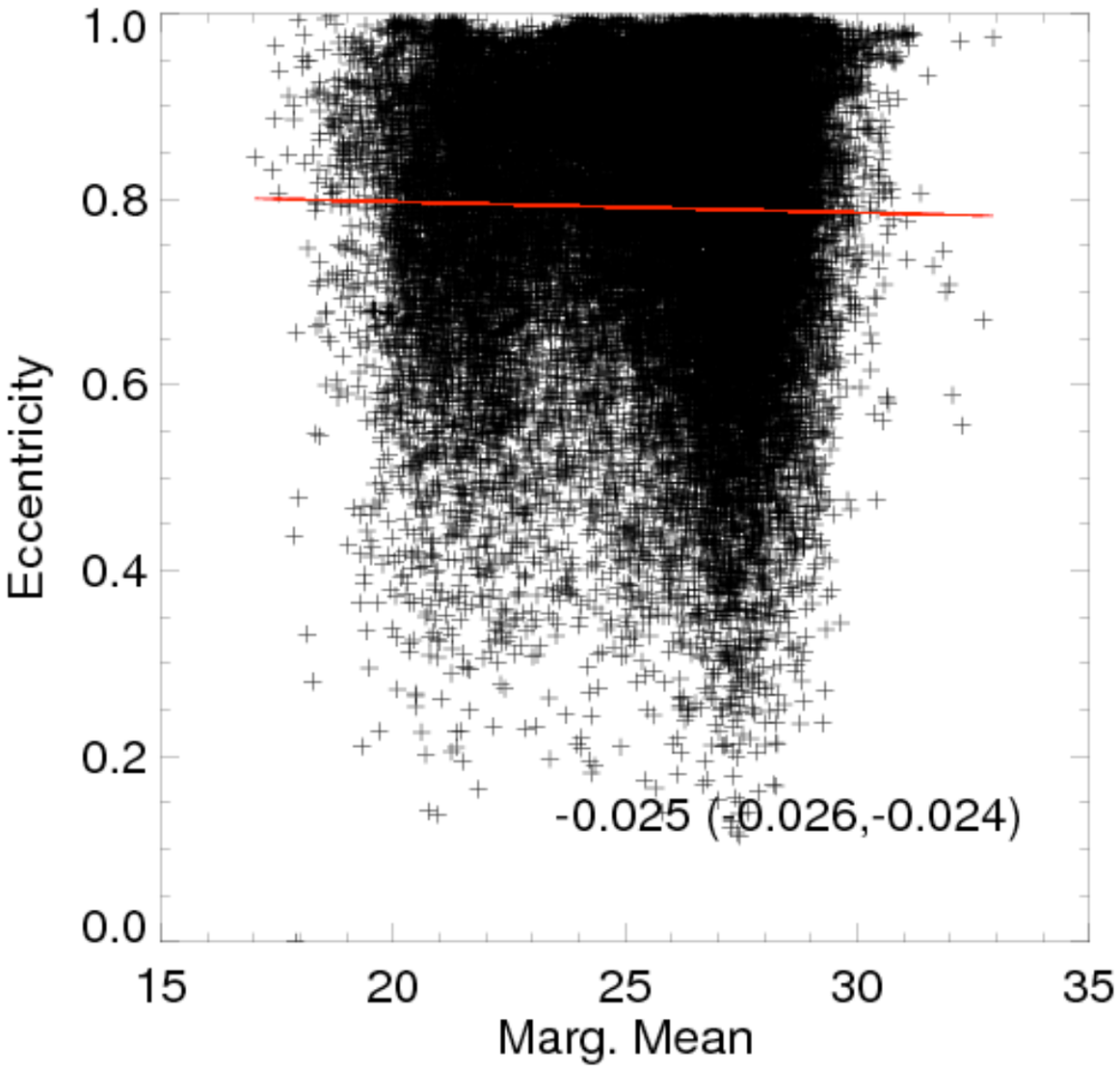
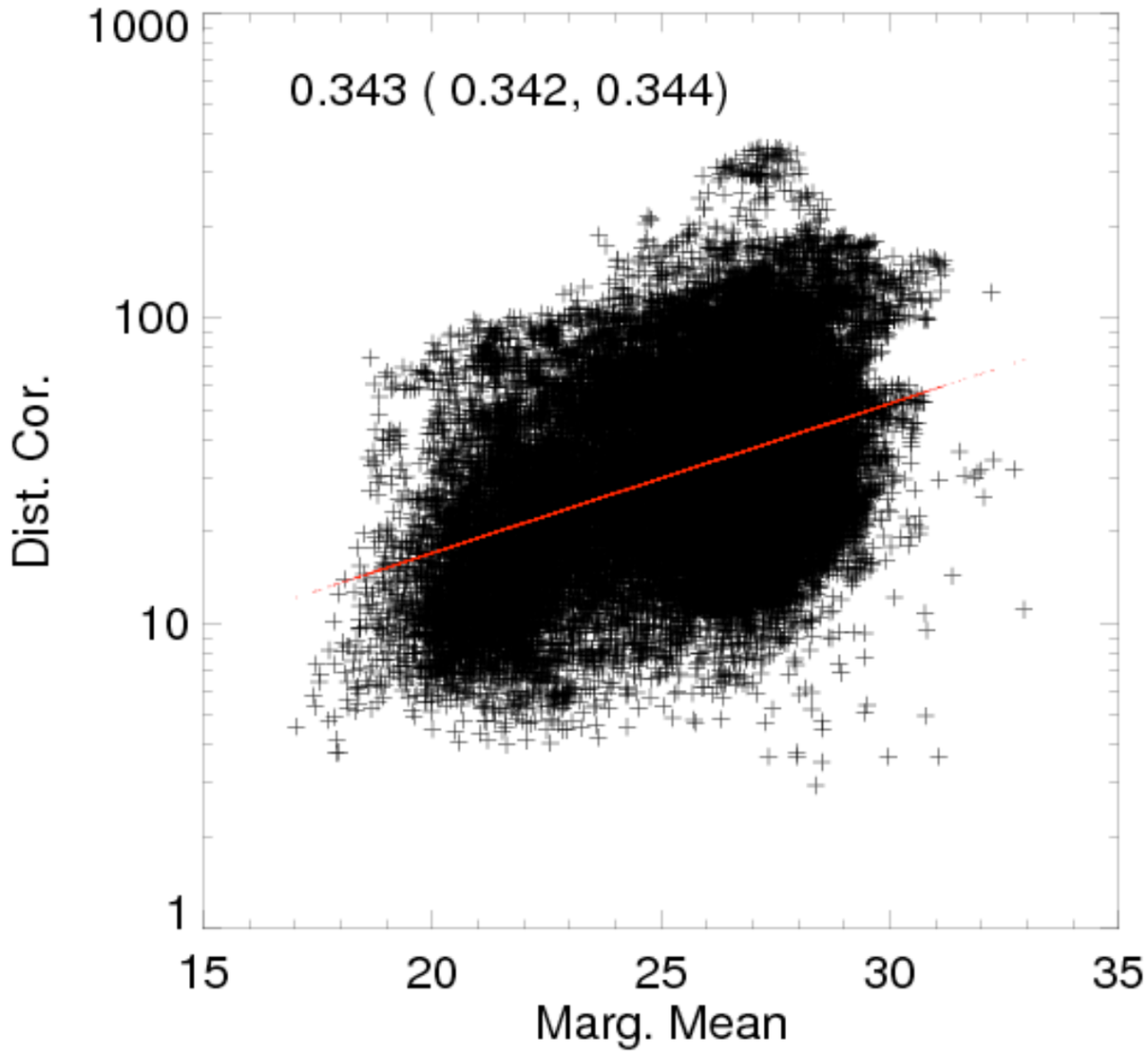




PHASE SPACE OF RAINFALL FIELDS

Keeping the uncorrelated statistical parameters.





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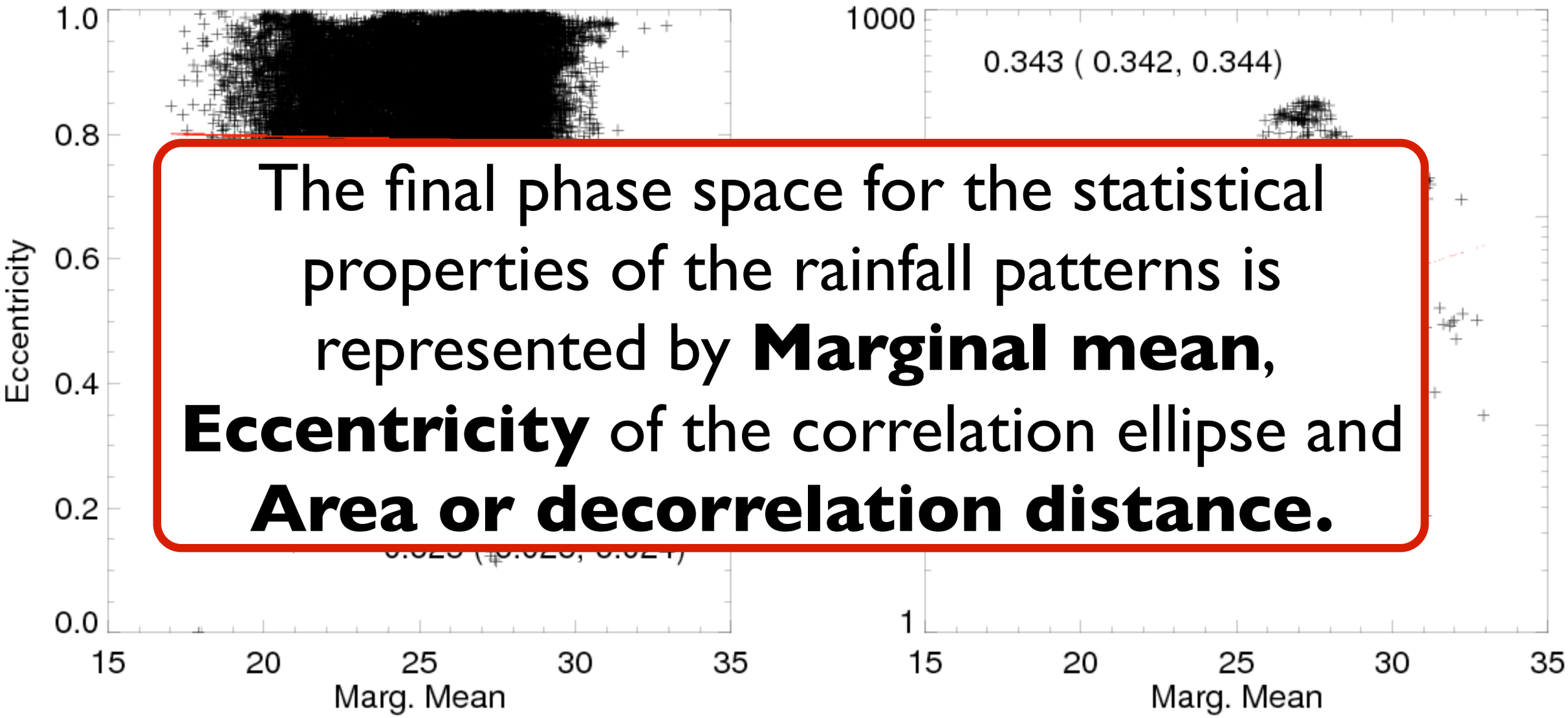
Keeping the uncorrelated statistical parameters.



The final phase space for the statistical properties of the rainfall patterns is represented by **Marginal mean**, **Eccentricity** of the correlation ellipse and **Area or decorrelation distance.**

ATTRACTOR IN PHASE SPACE

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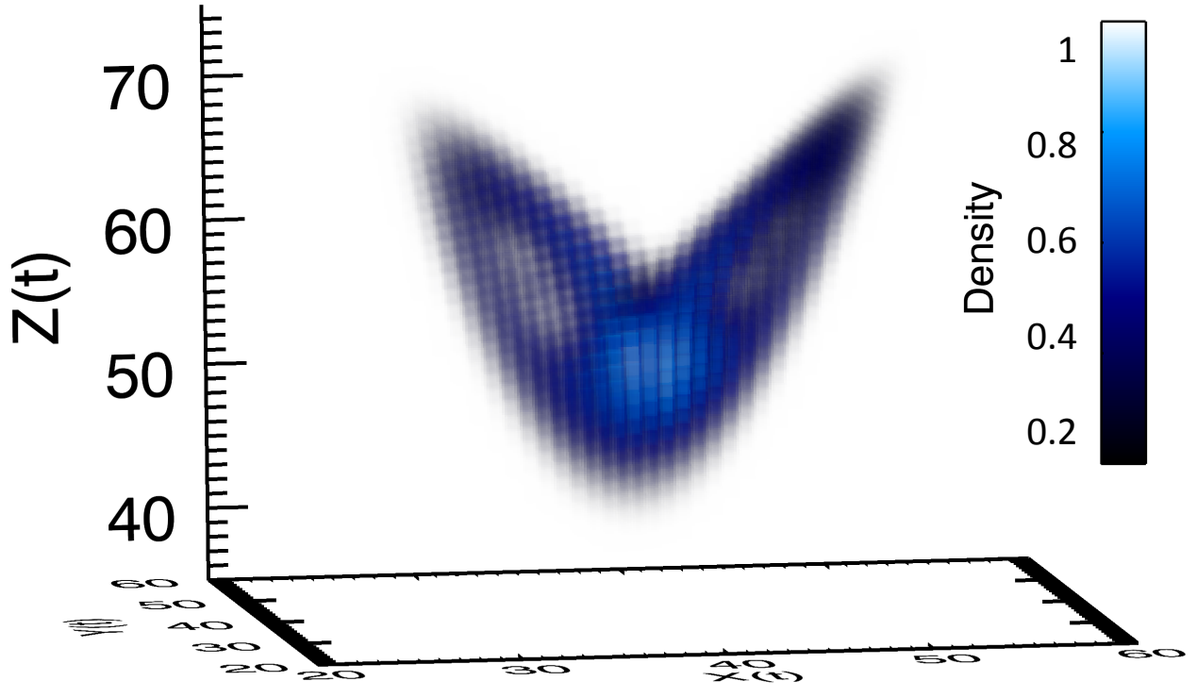
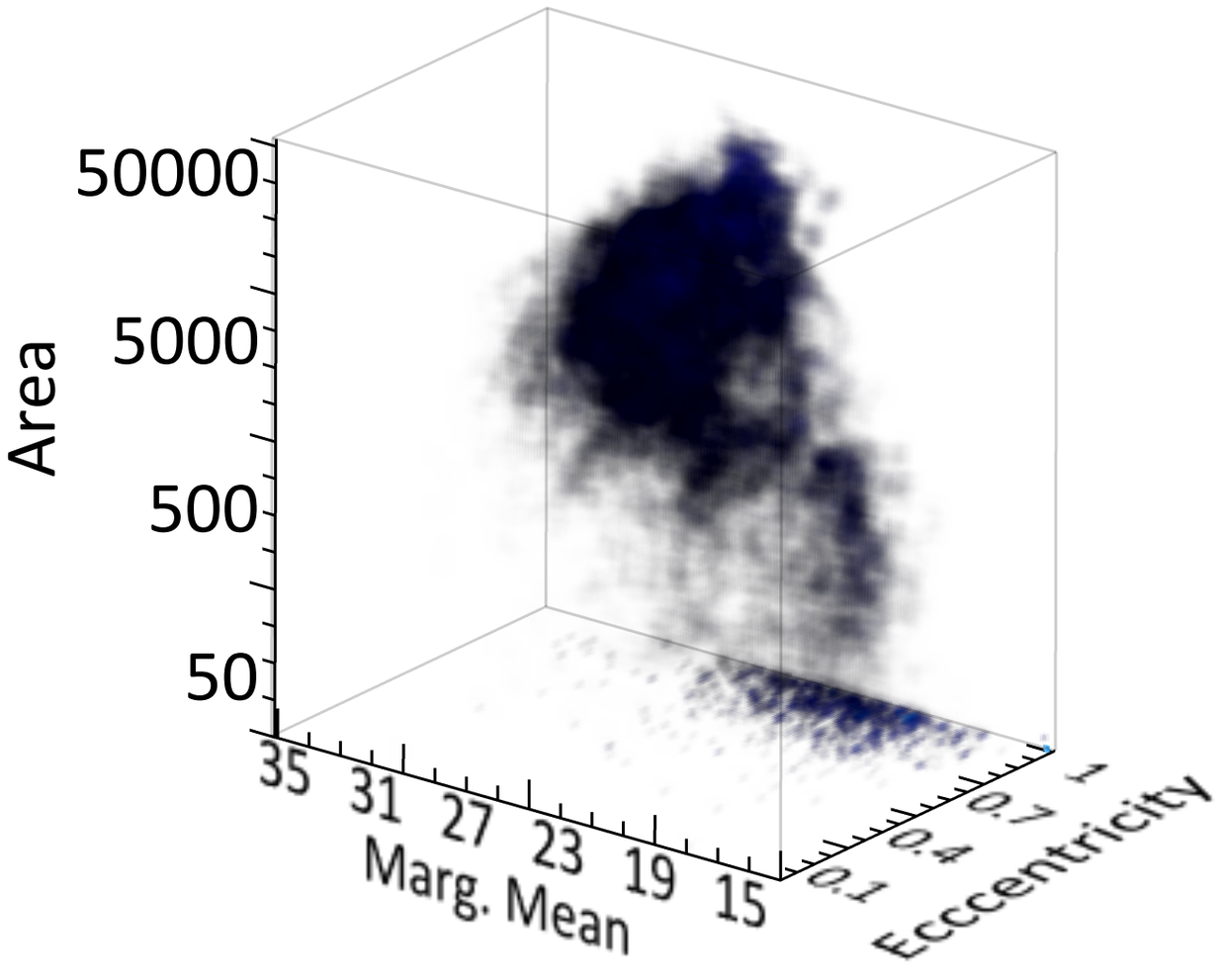
LORENZ SYSTEM

IP (x,y,x) ∈ [*-40,40 ; -40,40 ; -20,80*]

50 stochastic perturbations of I.P.

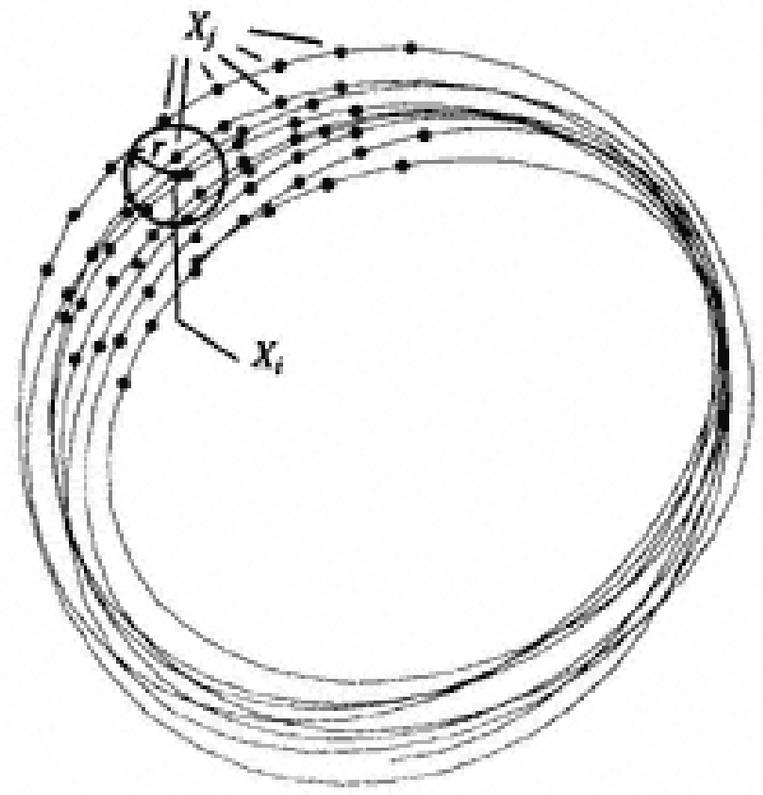


RAINFALL FIELDS

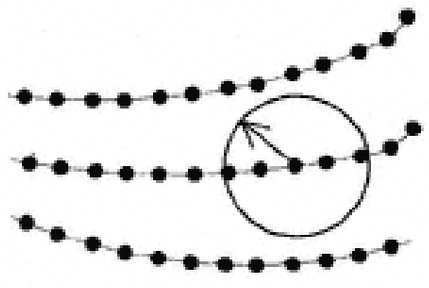


CHARAC. CHAOS: DIMENSION ESTIMATION

Correlation dimension: $$Cr = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1; j \neq i}^{N} \Theta\left(r - \left|\bar{X}_i - \bar{X}_j\right|\right)$$

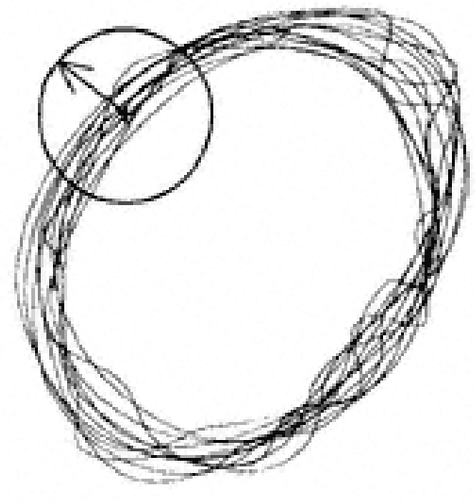


Small-scale effects



Scaling region

Large-scale effects



Addison, P. S., 1997: Fractals and Chaos: An Illustrated Course.

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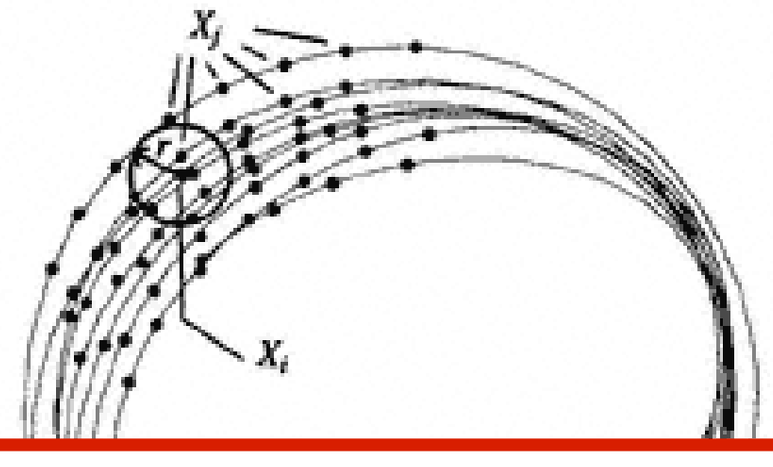
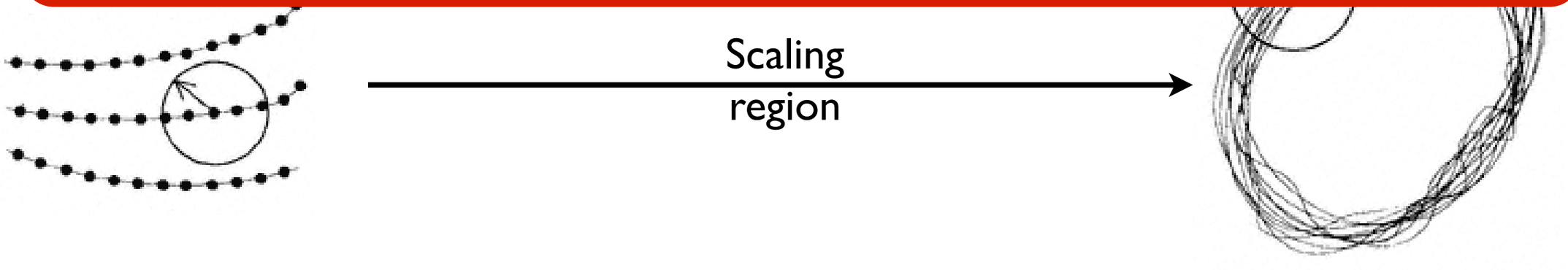


Fractal have **non-integer** dimensions.

D_c gives information about **complexity** of the system

Attractor with fractal structure is called **strange**



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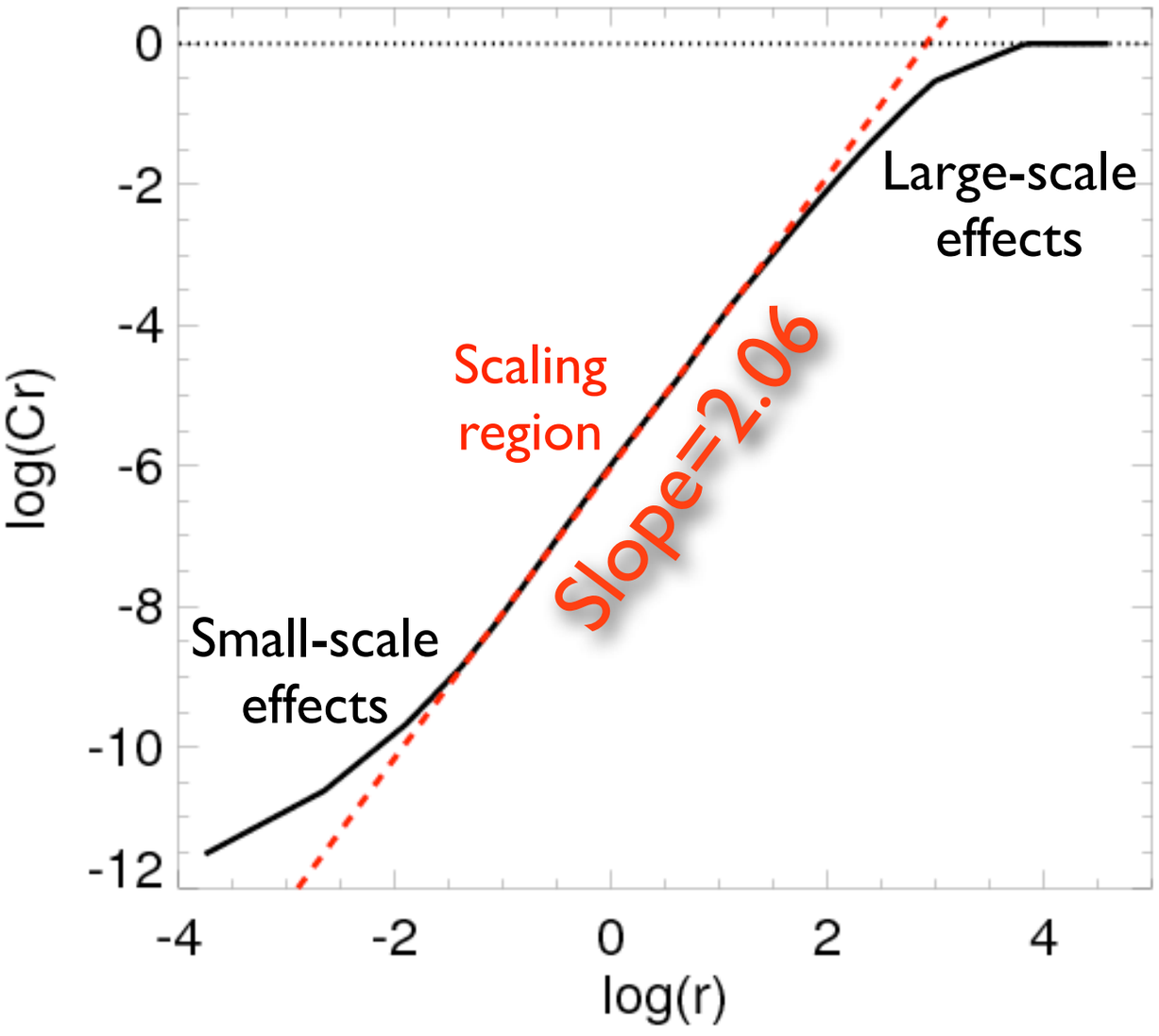

RAINFALL FIELDS

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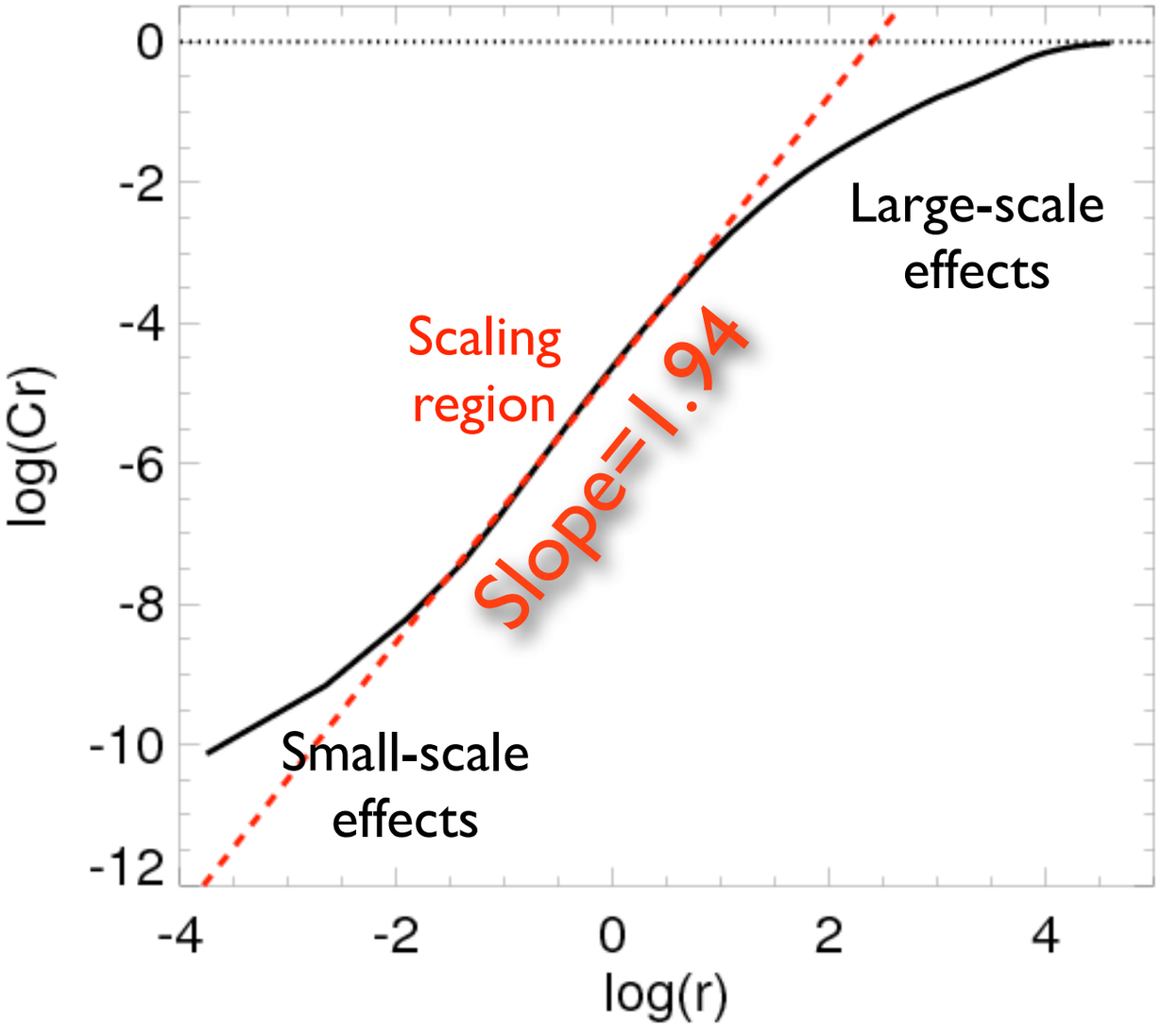

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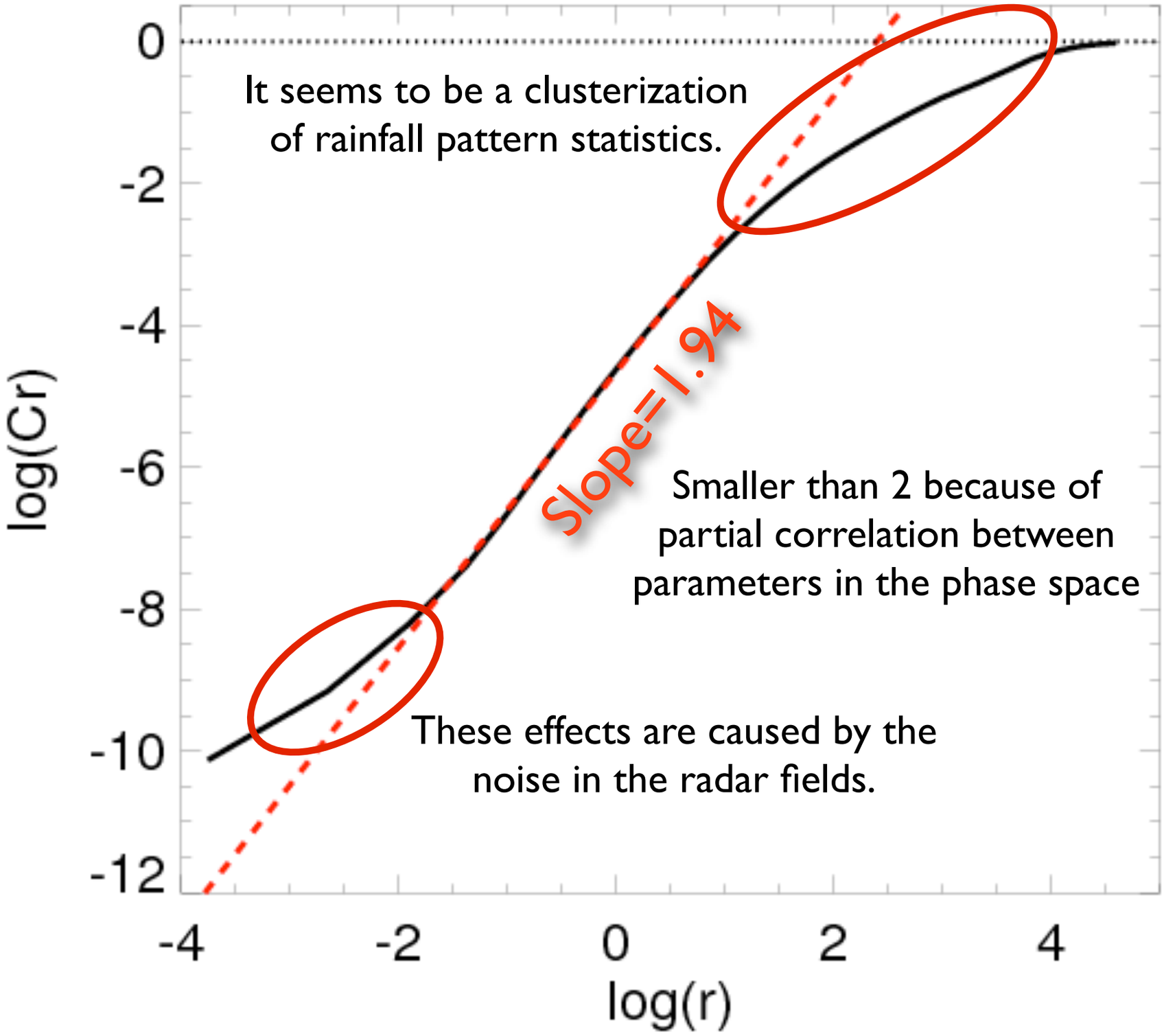
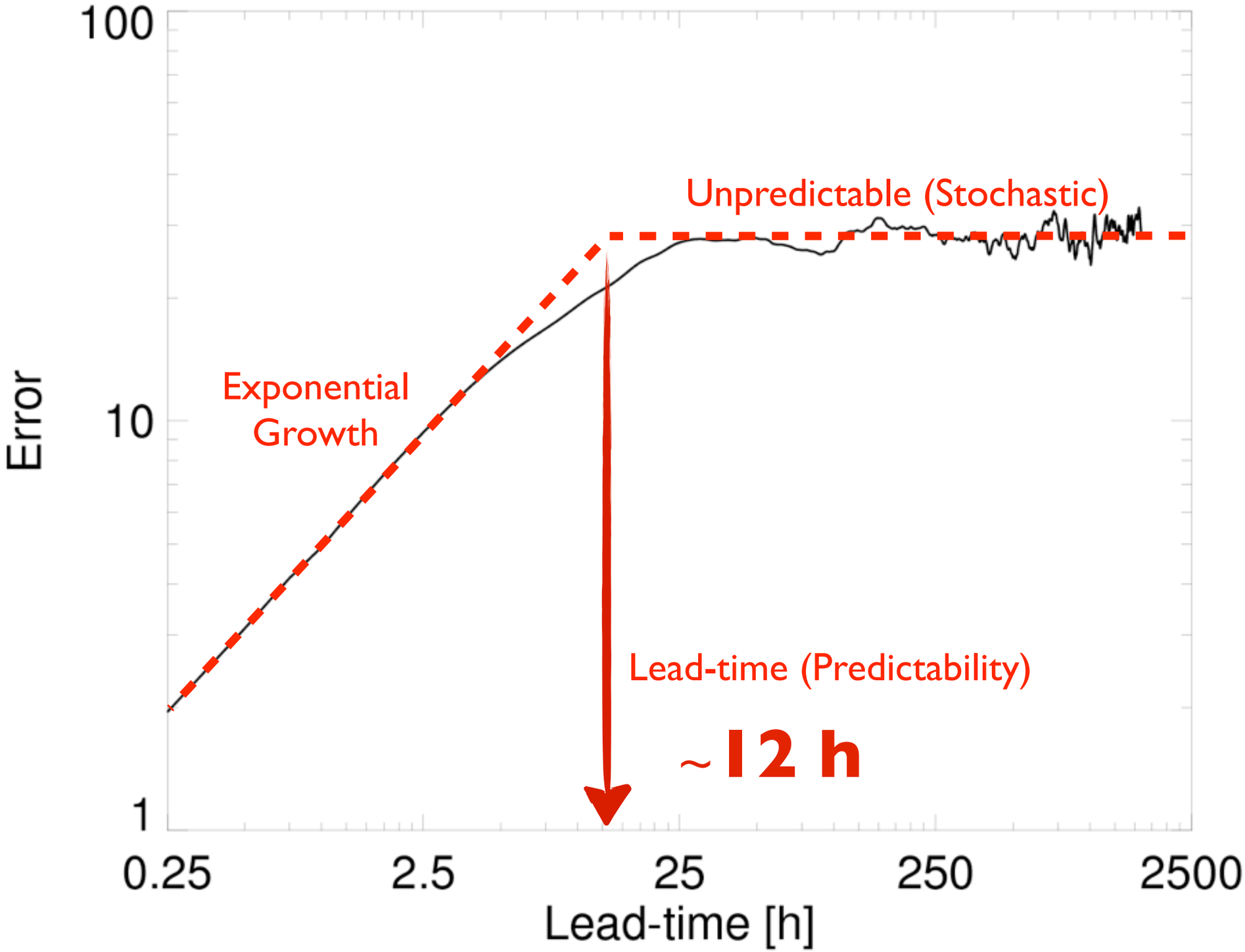
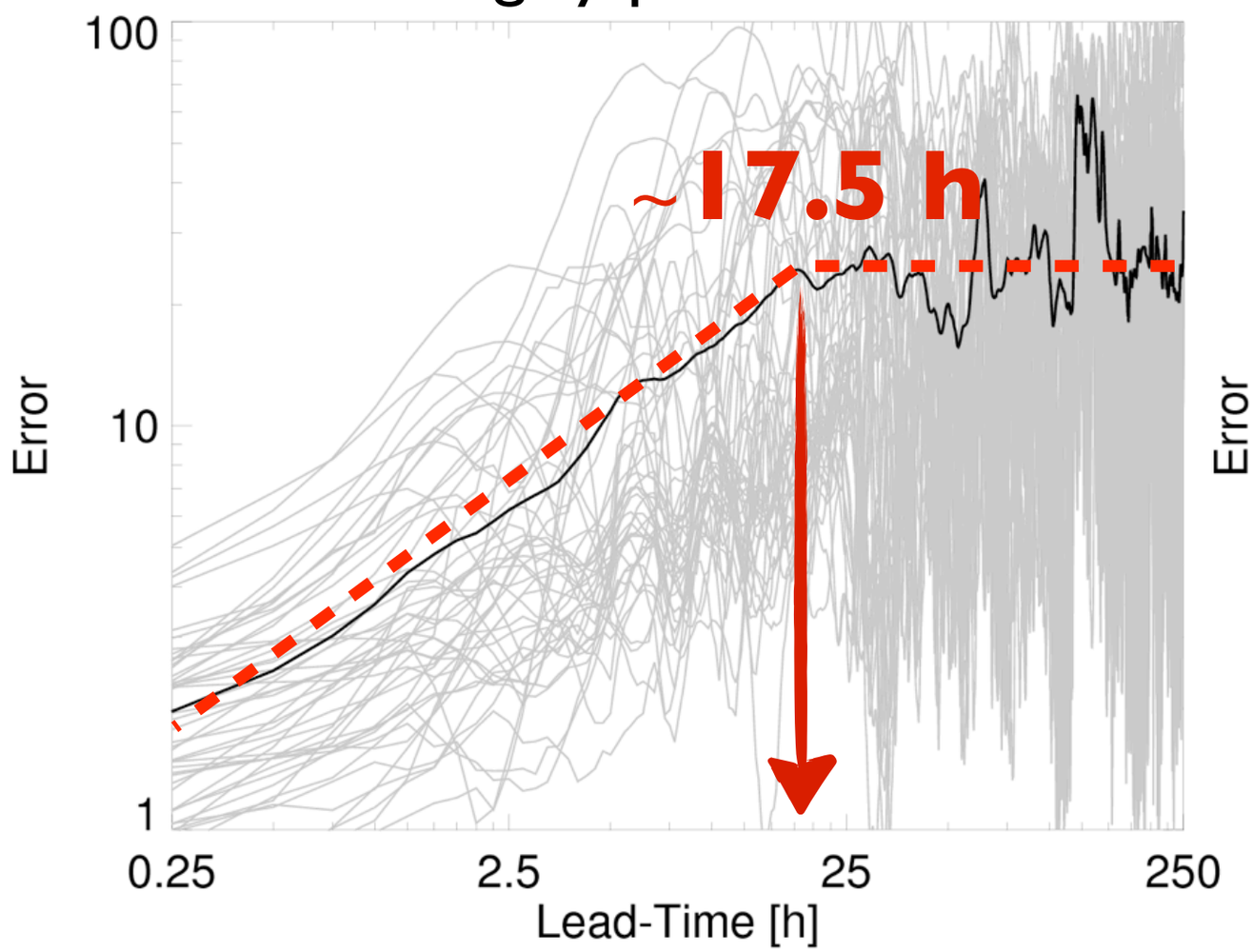
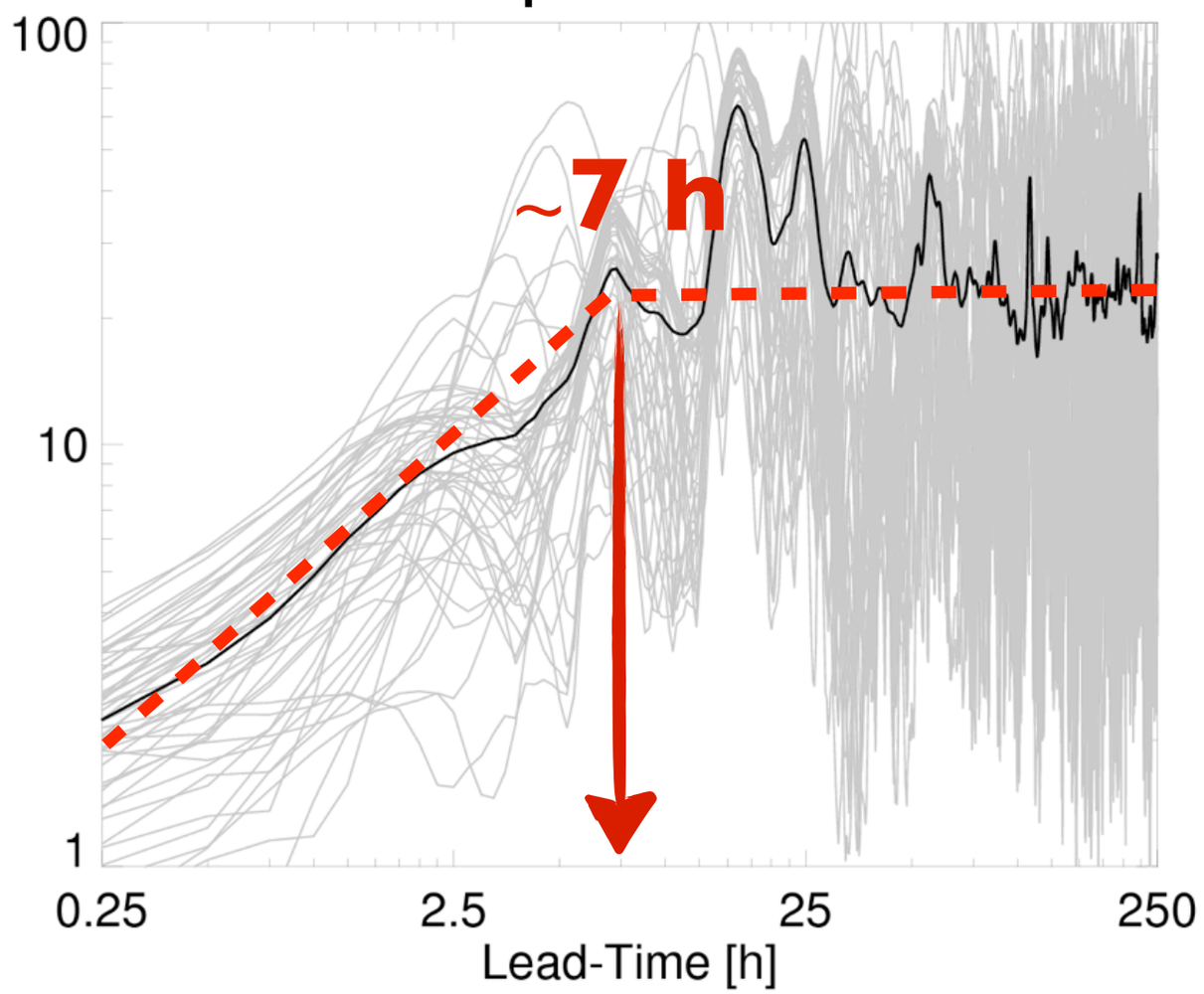


CHARAC. CHAOS: ERROR GROWTH



LOCAL PREDICTABILITY



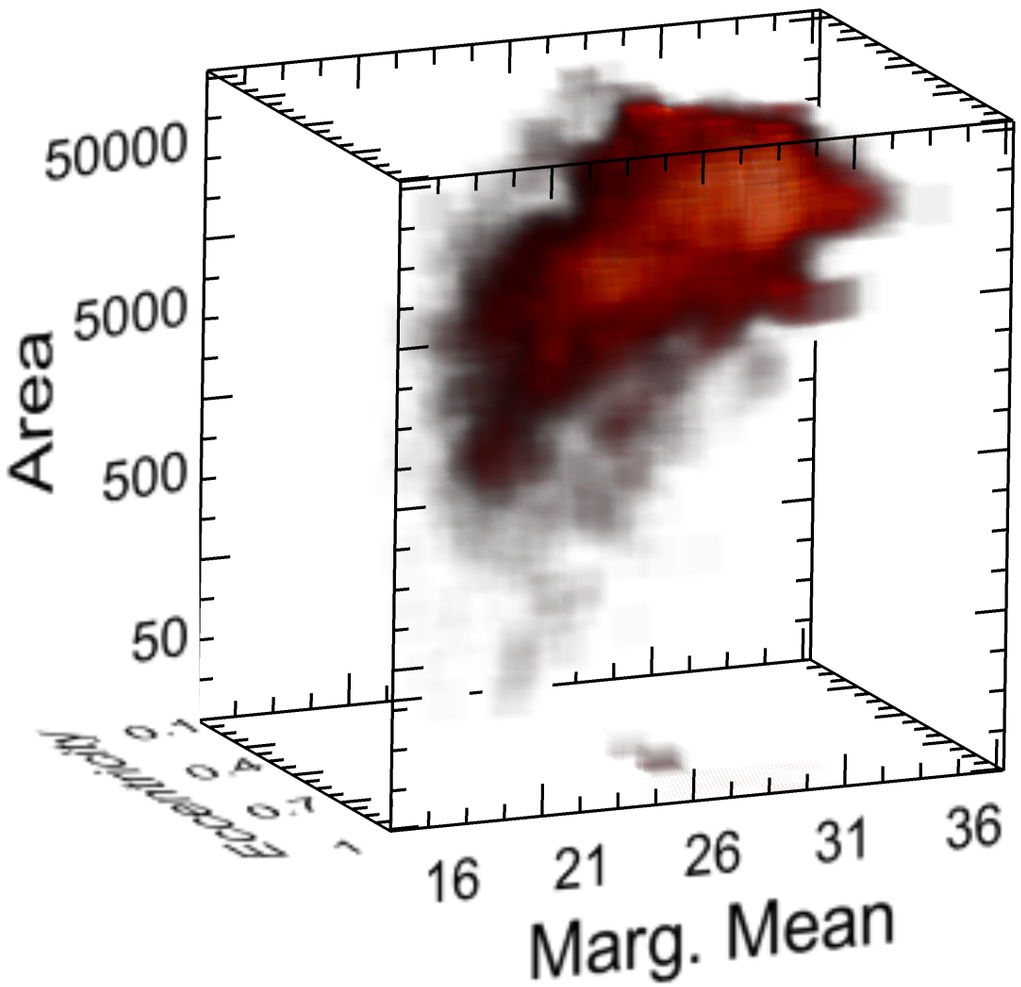


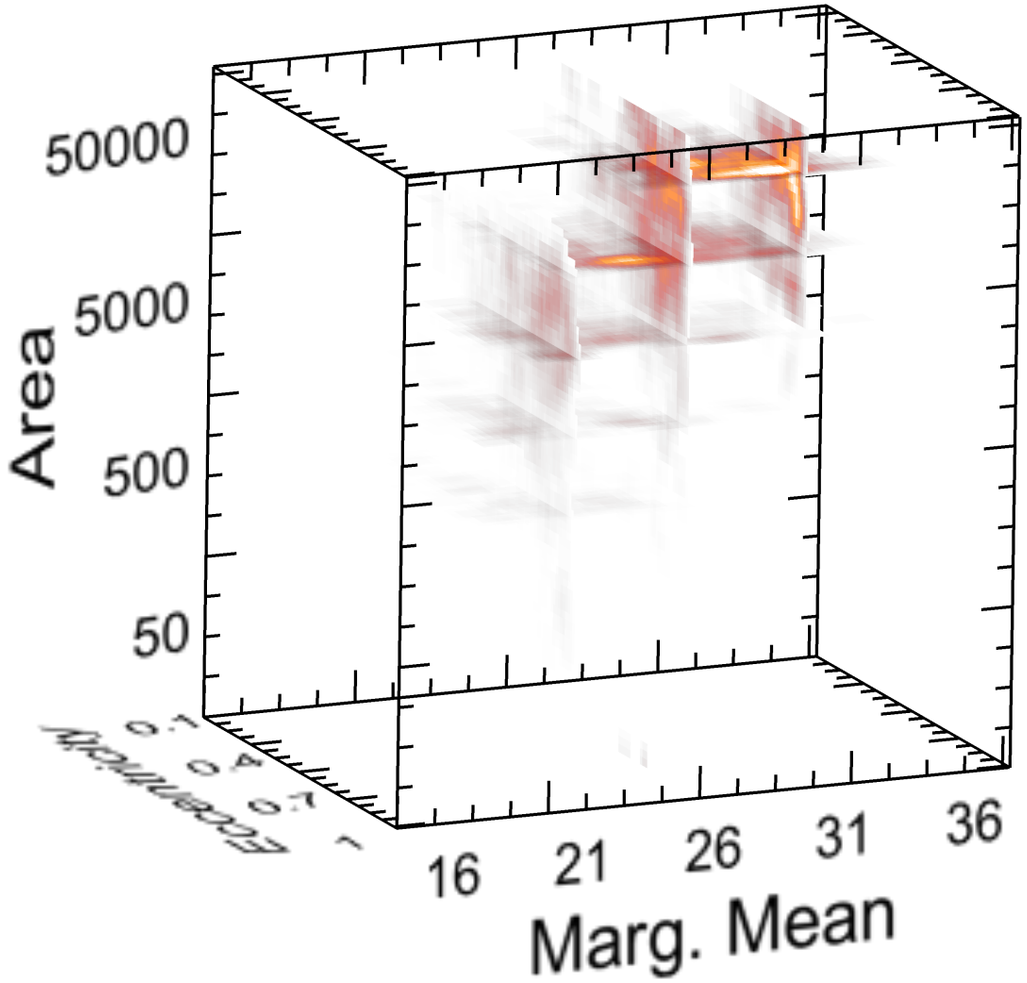
LOCAL PREDICTABILITY

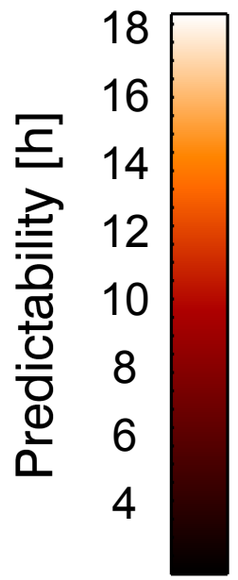
RAINFALL FIELDS

Although the boundaries are irregular, a pattern or structure can be observed in the interior of the predictability cloud.

Large mean - high eccentricity - large coverage → highly predictable

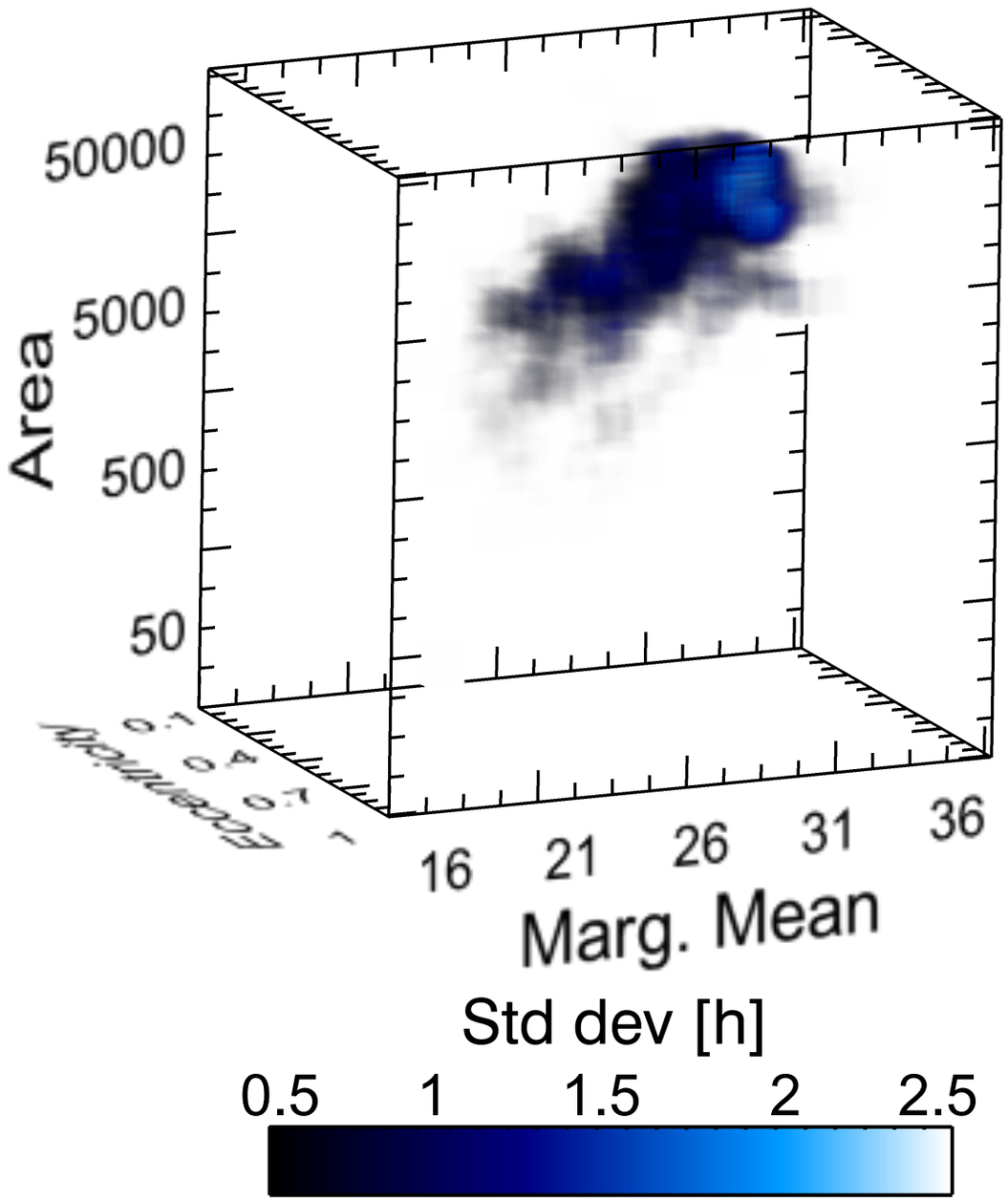




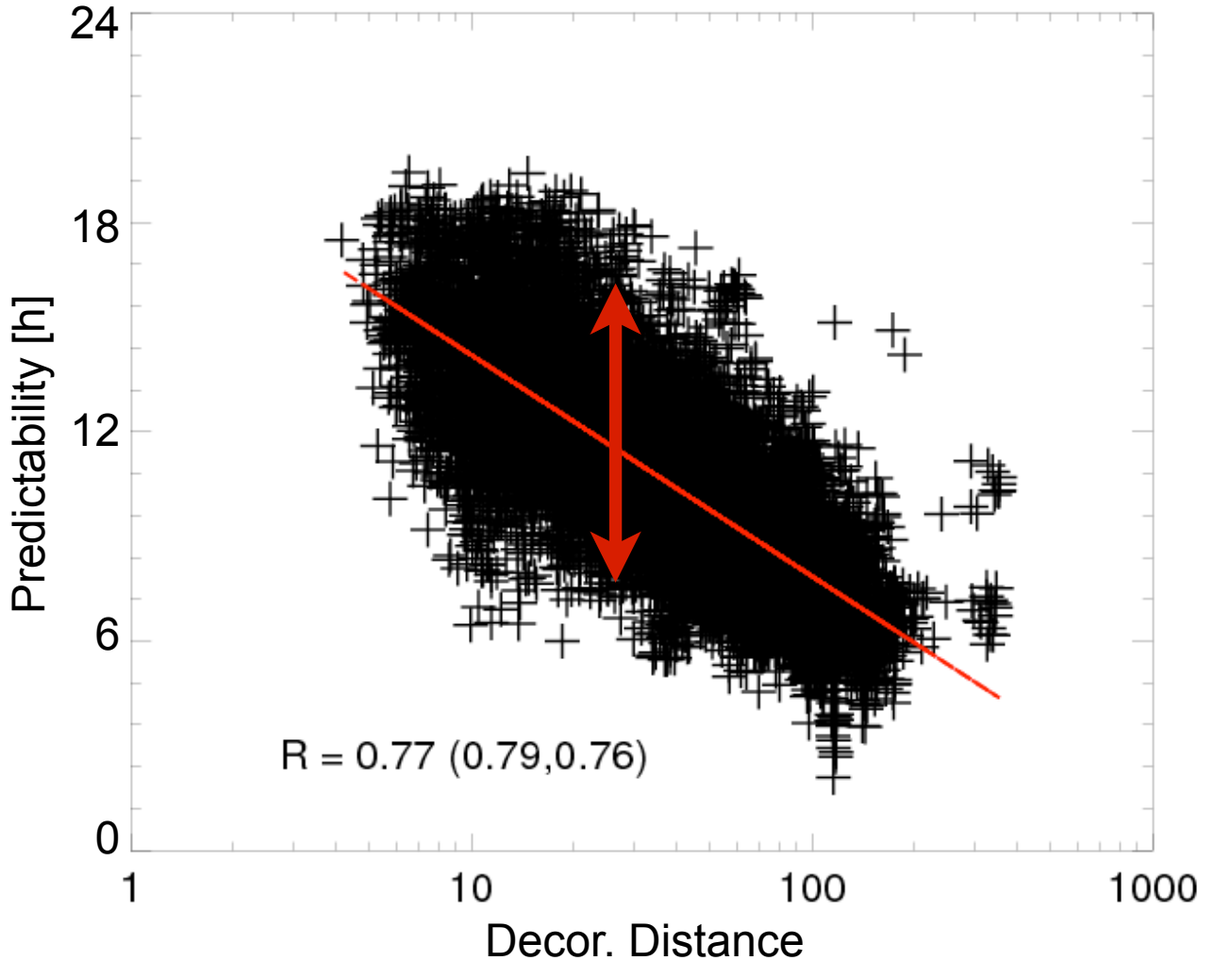


LOCAL PREDICTABILITY (VARIANCE)

Map of the standard deviation of predictability time



Correlation as a measurement of the variance explained



CONCLUSIONS

A 15 years length dataset has been used in this study:

- Three statistical properties of rainfall fields have been chosen (uncorrelated) to represent the phase space.
- Rainfall fields have a strange attractor (chaotic system) with fractal structure with correlation dimension of 1.98.
- A inherent predictability of 12 hours is obtained.
- Inherent predictability can be determined by the initial statistical properties of the rainfall field.
- ±4 hours around the prediction time explains 95% of the variance.

Acknowledgements

Thanks to my department colleagues for all their help, especially to Madalina Surcel. And to Dr. Alan Seed for his advices/comments during this conference.

Thank you for your attention.

McGill