Determining Geometry for Common Comparison Radar Space

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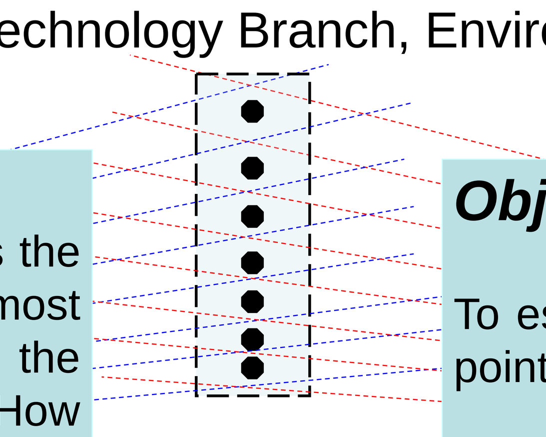
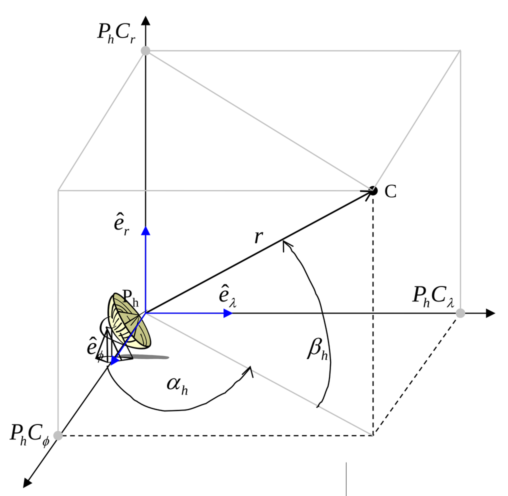


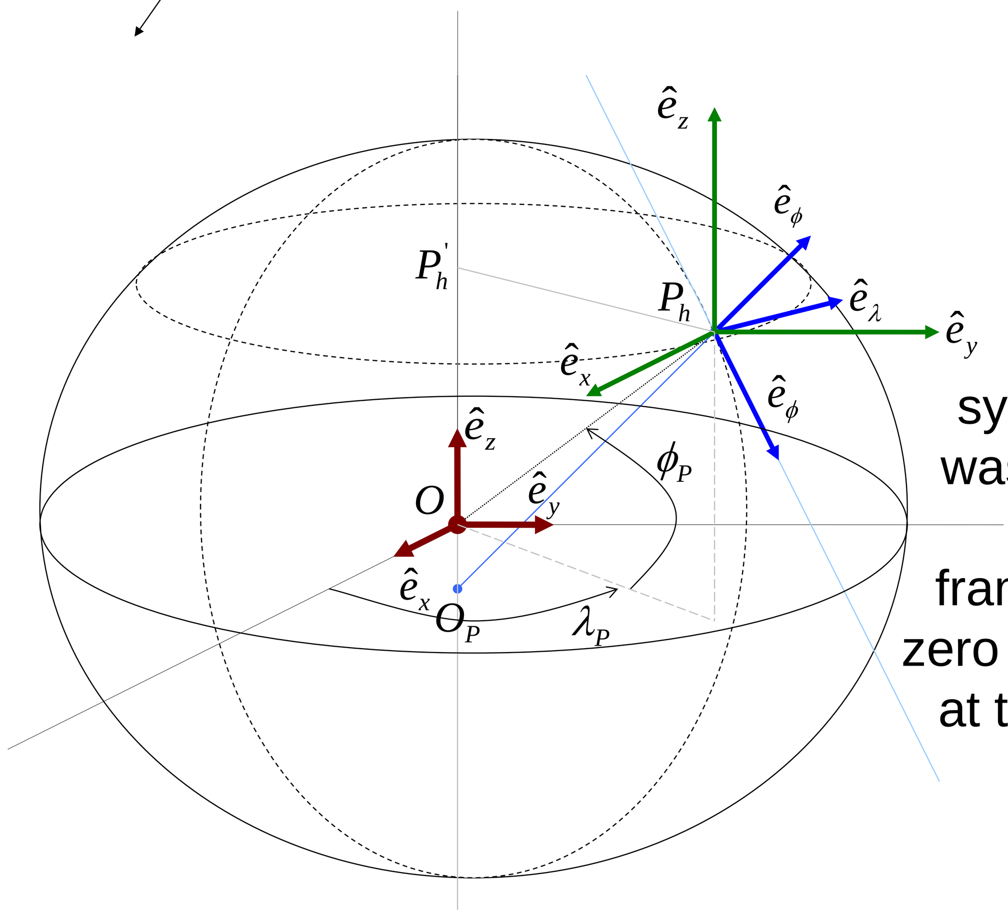
Background:
A condition for the successful inter-radar comparison between two radars is the time-space synchronization in the middle region where the comparison is most effective. In first approximation, a collection of radar beams that overlap in the middle of two radars is represented as a vertical 'wall' with fixed size. How accurate is this analogy?

Objective:
To establish the theoretical basis for the geometrical evaluation of a set of target points at equal distance from two neighbouring radars.

1. Reference systems:

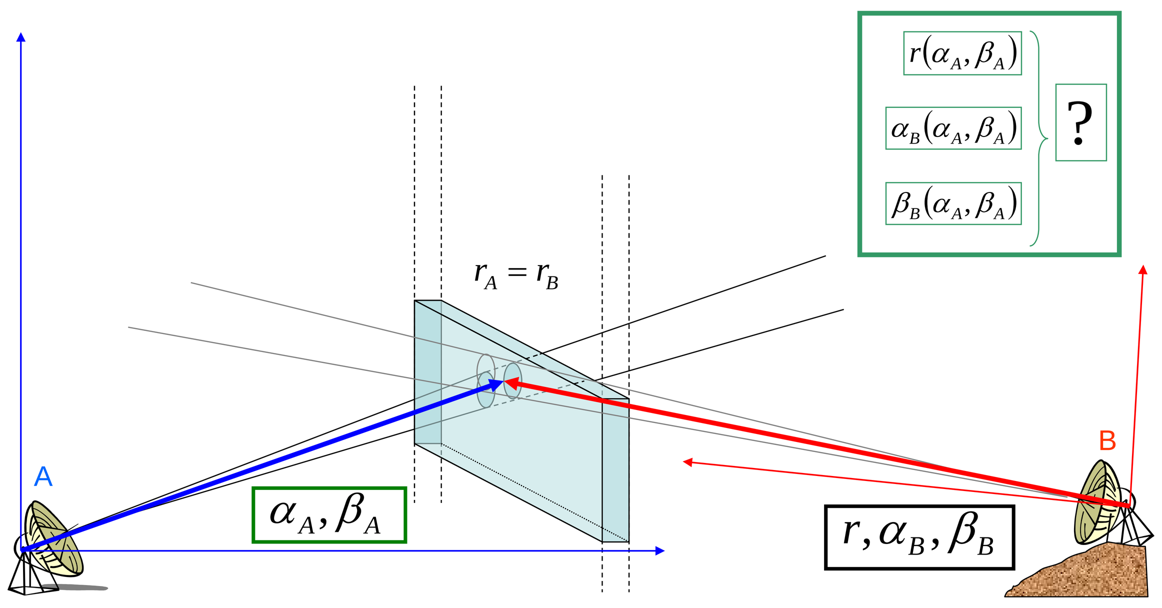


The radars locate the target volume in reference to the local spherical reference system.



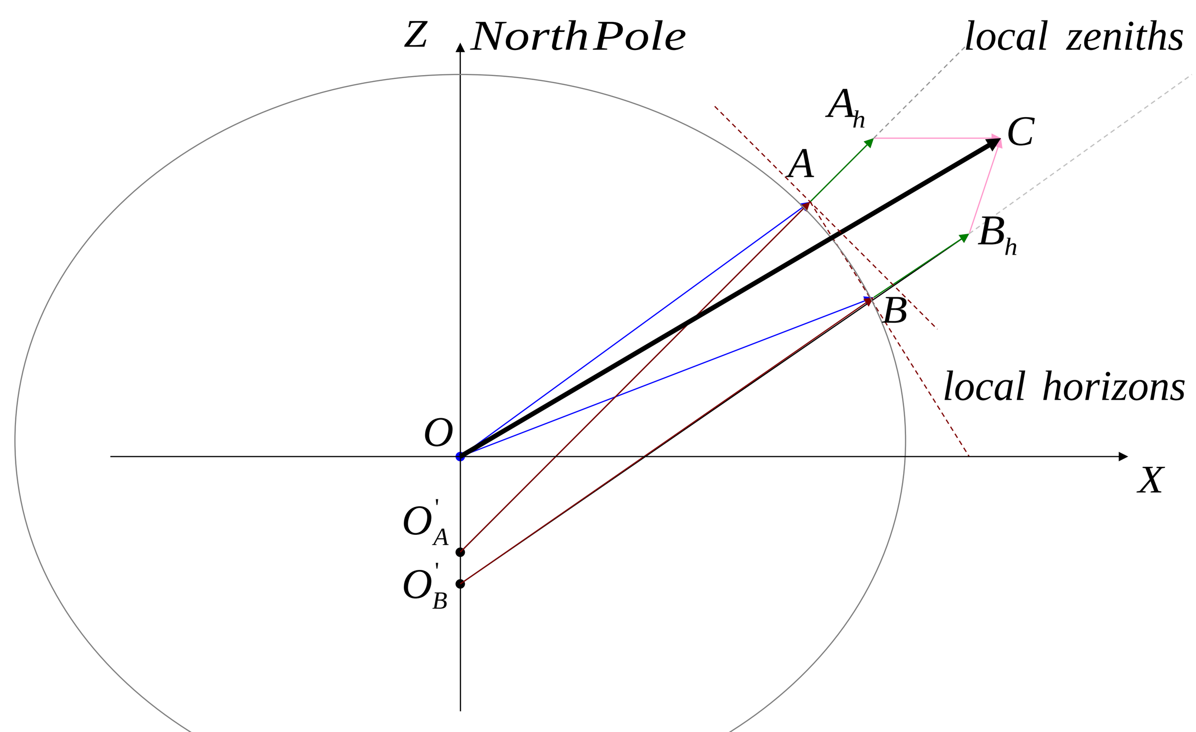
The 'fixed' geocentric coordinate system (GCS) was chosen as the prime framework with zero coordinates at the centre of the Earth.

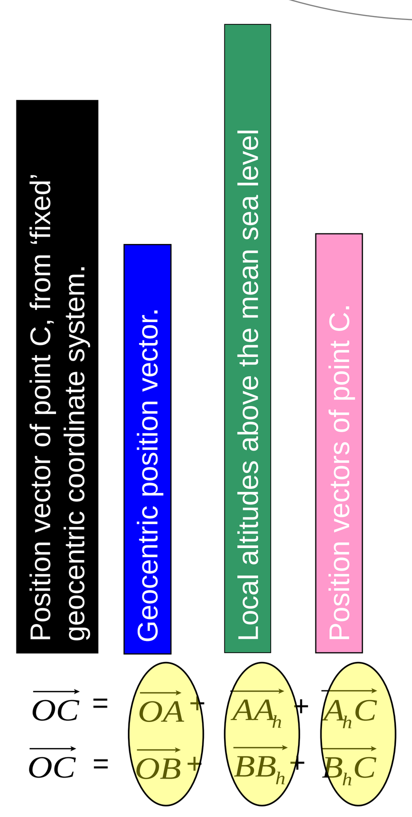
2. Common inter-radar space (CIS):



For independent angles (α_A, β_A) we need to obtain theoretical values of dependant variables (r, α_B, β_B) that determine coordinates of CIS.

3. Vectorial equations:





The position vector of a common target point of two radars relative to the GCS is the composition of the three vectors: a position vector from the geocentric origin to the surface of the oblate geoid at the geographical latitude and longitude of a radar location, an altitude vector that is normal to the oblate spheroid surface at the point of the radar location and which represents the height of the radar antenna above mean sea level, and at third vector is a local position vector of a common target point measured from the radar.

α – azimuth
β – elevation
r – radial distance
λ – longitude
ϕ – latitude
h – altitude
H – tower hight

$$\overrightarrow{OC} = \underbrace{\overrightarrow{OA} + \overrightarrow{AA_h}}_{f(\lambda_A, \phi_A, h_A, H_A)} + \overbrace{\overrightarrow{A_hC}}^{f(\alpha_A, \beta_A, r_A)} = \underbrace{\overrightarrow{OB} + \overrightarrow{BB_h}}_{f(\lambda_B, \phi_B, h_B, H_B)} + \overbrace{\overrightarrow{B_hC}}^{f(\alpha_B, \beta_B, r_B)}$$

4. CIS solution:

Since the two sets of three vectors describe the position of same target with respect to the Earth centre, equating them results in three scalar equations:

$$f_x(\alpha_B, \beta_B, r_B) = F_x(\alpha_A, \beta_A, r_A, \quad \lambda_A, \phi_A, h_A, H_A, \ \lambda_B, \phi_B, h_B, H_B) \qquad (1)$$

$$f_y(\alpha_B, \beta_B, r_B) = F_y(\alpha_A, \beta_A, r_A, \quad \lambda_A, \phi_A, h_A, H_A, \ \lambda_B, \phi_B, h_B, H_B) \qquad (2)$$

$$f_z(\alpha_B, \beta_B, r_B) = F_z(\alpha_A, \beta_A, r_A, \quad \lambda_A, \phi_A, h_A, H_A, \ \lambda_B, \phi_B, h_B, H_B) \qquad (3)$$

For a target point at equal distance from both radars:

$$r_A = r_B \qquad (4)$$

Using the four scalar equations it is possible to obtain the local azimuth and elevation from the second radar, as well as the equal distance, assuming the geographical coordinates and antenna heights are known:

$$\boxed{\alpha_B = \arccos\left[\pm\sqrt{\frac{b^2 - 2(c-a)(d-c) \pm |b|\sqrt{b^2 - 4(d-c)(d-a)}}{2\left[b^2 + (c-a)^2\right]}}\right]}$$

$$\boxed{\beta_B = \arctan\left[\frac{\cos\alpha_B (L1M3 - L3M1) - \sqrt{1-\cos^2\alpha_B}\,(J2L3 + I2M3)}{L3M2 - L2M3}\right]}$$

$$\boxed{r = \frac{OA_{hz} - OB_{hz}}{\sin\beta_B K1 - \cos\alpha_B \cos\beta_B K2 - K3}}$$

Parameters and constants:

$$a = (L1^2 - I2^2)(L3M2 - L2M3) + 2L1L2(L1M3 - L3M1) - (L3^2 - L2^2)(L1M3 - L3M1)^2$$

$$b = 2\begin{Bmatrix} -I2L1(L3M2 - L2M3) - L2[L1(J2L3 + I2M3) + I2(L1M3 - L3M1)] \\ +(L3^2 - L2^2)(L1M3 - L3M1)(J2L3 + I2M3) \end{Bmatrix}$$

$$c = 2I2L2(J2L3 + I2M3) - (L3^2 - L2^2)(J2L3 + I2M3)^2$$

$$d = (L3^2 - I2^2)(L3M2 - L2M3)$$

$$I1 = \sin\phi_B \cos\lambda_B \qquad J1 = \sin\phi_B \sin\lambda_B \qquad K1 = \sin\phi_B$$

$$I2 = \sin\lambda_B \qquad J2 = \cos\lambda_B \qquad K2 = \cos\phi_B$$

$$I3 = \cos\phi_B \cos\lambda_B \qquad J3 = \cos\phi_B \sin\lambda_B \qquad K3 = \sin\beta_A \sin\phi_A - \cos\alpha_A \cos\beta_A \cos\phi_A$$

$$I4 = \cos\alpha_A \cos\beta_A \sin\phi_A \cos\lambda_A - \sin\alpha_A \cos\beta_A \sin\lambda_A + \sin\beta_A \cos\phi_A \cos\lambda_A$$

$$J4 = \cos\alpha_A \cos\beta_A \sin\phi_A \sin\lambda_A + \sin\alpha_A \cos\beta_A \cos\lambda_A + \sin\beta_A \cos\phi_A \sin\lambda_A$$

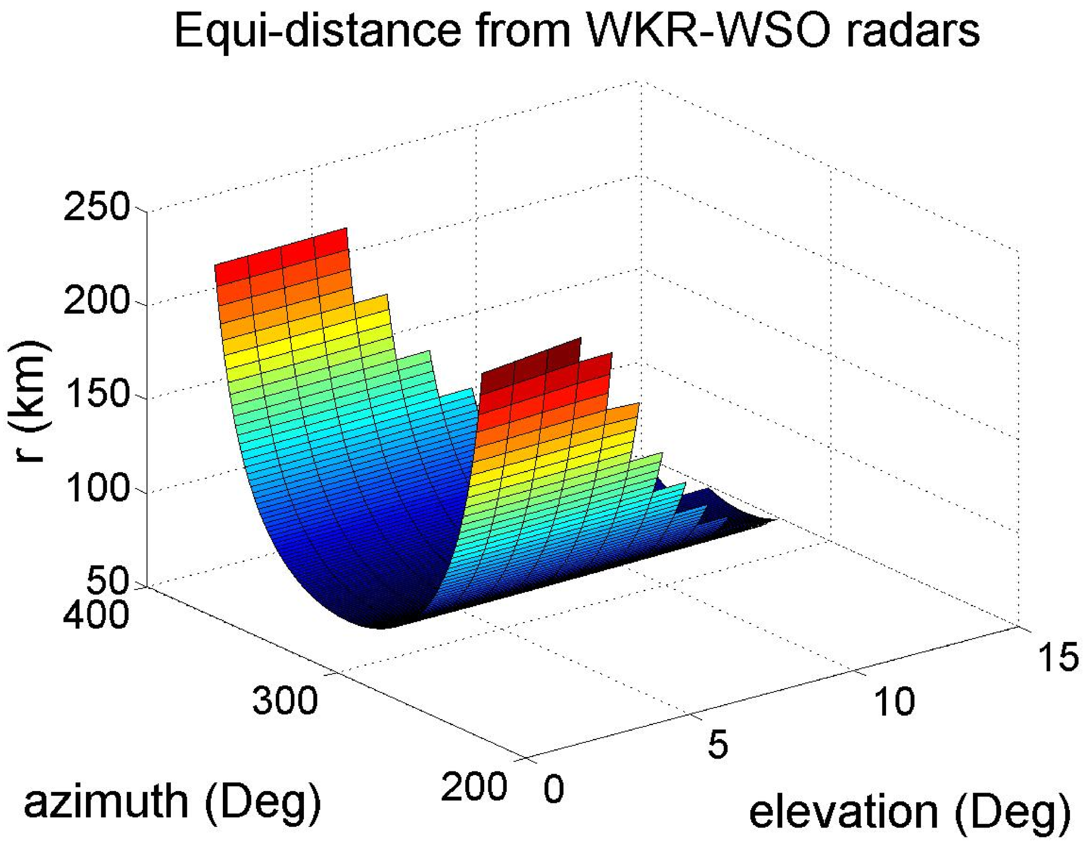
$$L1 = I1 + \frac{OA_{hx} - OB_{hx}}{OA_{hz} - OB_{hz}} K2 \qquad L2 = I3 - \frac{OA_{hx} - OB_{hx}}{OA_{hz} - OB_{hz}} K1 \qquad L3 = I4 - \frac{OA_{hx} - OB_{hx}}{OA_{hz} - OB_{hz}} K3$$

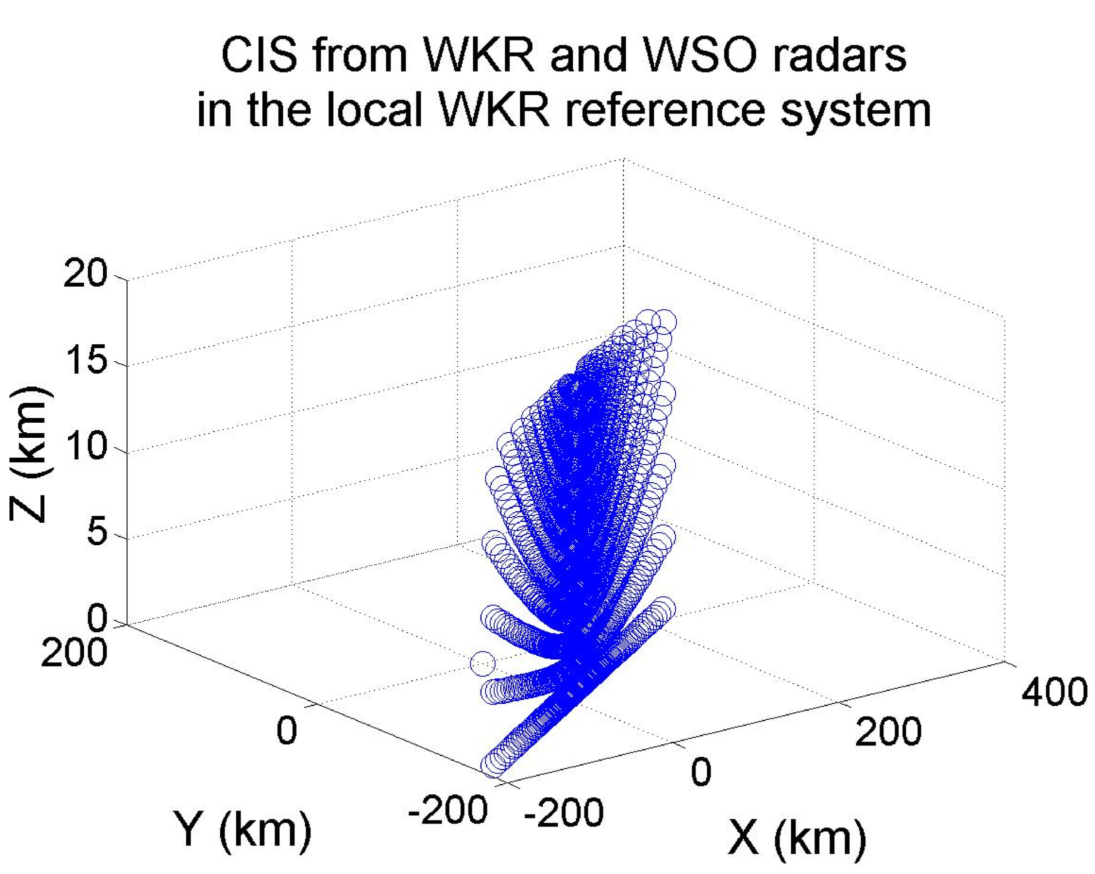
$$M1 = J1 + \frac{OA_{hy} - OB_{hy}}{OA_{hz} - OB_{hz}} K2 \qquad M2 = J3 - \frac{OA_{hy} - OB_{hy}}{OA_{hz} - OB_{hz}} K1 \qquad M3 = J4 - \frac{OA_{hy} - OB_{hy}}{OA_{hz} - OB_{hz}} K3$$

$$\overrightarrow{OA} = \frac{a}{\sqrt{1 - e^2 \sin^2\phi_A}}\left[\cos\phi_A \cos\lambda_A \hat{e}_x + \cos\phi_A \sin\lambda_A \hat{e}_y + (1 - e^2)\sin\phi_A \hat{e}_z\right] \qquad e^2 = 0.006\,694\,379\,990\,14$$

$$\overrightarrow{OB} = \frac{a}{\sqrt{1 - e^2 \sin^2\phi_B}}\left[\cos\phi_B \cos\lambda_B \hat{e}_x + \cos\phi_B \sin\lambda_B \hat{e}_y + (1 - e^2)\sin\phi_B \hat{e}_z\right] \qquad a = 6\,378\,137.0m$$

5. Graphical CIS presentation:





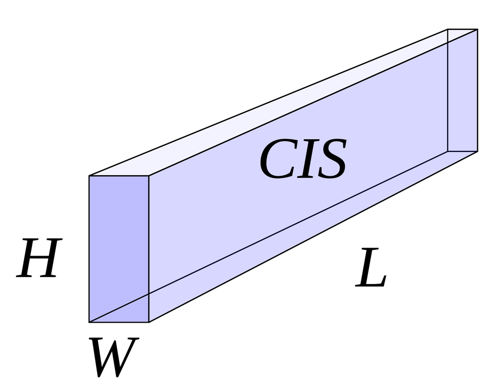
The spatial distribution of CIS points generated width 1 Deg azimuth and elevation resolution presented in local (WKR) radius-azimuth-elevation space (left) and Descartes reference system (right).

6. Analyse:

1) The first approximation:

$$(L, W, H) = const$$

very robust and inaccurate.



2) The second approximation:

$$(L, W, H) = f(\lambda_A, \phi_A, h_A, H_A, \quad \lambda_B, \phi_B, h_B, H_B)$$

much more accurate but not operationally suitable.

3) The third approximation should include the radar characteristics:

$$(L, W, H) = f\begin{pmatrix} \lambda_A, \phi_A, h_A, beamWidth_A, beamPulse_A, \\ \lambda_B, \phi_B, h_B, beamWidth_B, beamPulse_B \end{pmatrix}$$

and will be operationally applicable.

Summary:

- Converting coordinates to the geocentric coordinate system is essential for obtaining accurate common inter-radar space (CIS) coordinates
- The CIS coordinates depends on the geographical locations of a radar pair
- The CIS coordinates in the local coordinate system are not necessary in the vertical 'wall' but more often they are in the 'tilted wall'
- Obtained formulas for CIS are very accurate but not operationally suitable since they determine the geometric equidistant points not the common radar pulse volumes; next steps will be the inclusion of the technical characteristics of radars and a conversion of geometric elevation to antenna axis elevation

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