126 IMPROVED VELOCITY VOLUME PROCESSING (VVP) WITH PROBABILISTIC PROPERTY OF VELOCITY MEASUREMENTS FOR A SCANNING RADAR/LIDAR

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1. INTRODUCTION

Weather radars have a big advantage in measuring precipitation due to their wide coverage, compared with other equipments such as rain gauges or disdrometers. Nowaday, many weather radars have been deployed to detect precipitation and forecast weather phenomena. In many airports, a radar has already been installed to observe intensity and velocity of precipitation for their safety operation [1]. The number of airports installing a lidar is increasing to observe wind speed in fair weather. Thus, in advanced airports, a radar and lidar are co-operated to monitor precipitation and wind in all the weather conditions.

It is essential for meteorological use to retrieve 2-D or 3-D velocities from measurements of radial velocities in a radar or lidar. Volume Velocity Processing (VVP), is a traditional and much-used method for the purpose, in which a linear projection equation connecting 2/3-D velocities to radial velocities of several directions in a defined volume is formulated on the assumption that a linear velocity field inside the volume [2]. In order to solve the linear projection equation, however, it is well known that a large volume with angular width of a few tens of degrees must be defined to accomplish estimation accurate enough for a practical use. This is because enough dependency for orthogonal directions is not extracted by radial velocities within narrow angular width. This independency corresponds to a worse-conditioned coefficient matrix of the traditional VVP formulation. Since the VVP formulation is generally solved by Least Square (LS) method, observational errors in radial velocity measurements highly contaminate LS estimation with a small defined volume. On the other hand, with a large volume, estimated velocities are spatially averaged much, by which it is difficult to detect local phenomena such as a wind shear, downburst, and tornado.

In order to estimate 2/3-D velocities accurately and stably, two approaches are proposed in this paper. One is an improved VVP formulation, where 2/3-D velocity field is connected to measured radial velocities. The improved formulation allows to calculate 2/3-D velocity field with less variables and finer resolution than the traditional VVP formulation. The other is to solve a VVP formulation by Minimum Mean Square Error (MMSE) method for considering probabilistic property of velocity measurements. These two approaches can be applied simultaneously or individually.

This paper is organized as follows. In Section 2, methodologies of the two approaches are described. Section 3 shows results of numerical simulation. Section 4 concludes this paper.

2. METHODOLOGY

2.1 IMPROVED VVP FORMULATION

In the improved formulation, every 2/3-D velocity on Cartesian grids of a desired area is defined. Expressing in 2-D for simplicity, the 2-D velocity vector on a Cartesian coordinate $\mathbf{v}_C$ is as below.

$$\mathbf{v}_C = \left[v_{Cx}^{(1)} \quad v_{Cy}^{(1)} \quad v_{Cx}^{(2)} \quad v_{Cy}^{(2)} \quad \cdots \quad v_{Cx}^{(N)} \quad v_{Cy}^{(N)} \right]^T, \quad (1)$$

where $v_{Cx}^{(n)}$ and $v_{Cy}^{(n)}$ are an x- and y-component of velocity on the *n*-th Cartesian grid, respectively. *N* is the number of Cartesian grids in a desired area. A transition matrix $\mathbf{T}$, which translates $\mathbf{v}_C$ to a 2-D velocity vector on a radar polar coordinate $\mathbf{v}_P$ is defined as

$$\mathbf{v}_P = \mathbf{T}\mathbf{v}_C , \quad (2)$$

where

$$\mathbf{v}_P = \left[v_{Px}^{(1)} \quad v_{Py}^{(1)} \quad v_{Px}^{(2)} \quad v_{Py}^{(2)} \quad \cdots \quad v_{Px}^{(M)} \quad v_{Py}^{(M)} \right]^T . \quad (3)$$

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$v_{Px}^{(m)}$ and $v_{Py}^{(m)}$ are an x- and y-component of velocity on the *m*-th radar polar grid, respectively. *M* is the number of radar polar grids in a desired area. $\mathbf{v}_P$ is connected to a radial wind vector $\mathbf{v}_r$ by a projection matrix $\mathbf{P}$ as below

$$\mathbf{v}_r = \mathbf{P}\mathbf{v}_P \,, \quad (4)$$

where

$$\mathbf{v}_r = \begin{bmatrix} v_r^{(1)} & v_r^{(2)} & \cdots & v_r^{(M)} \end{bmatrix}^T . \quad (5)$$

$v_r^{(m)}$ is a radial component of velocity on the *m*-th radar polar grid. Thus, the improved VVP formulation connecting $\mathbf{v}_C$ to $\mathbf{v}_r$ is expressed as

$$\mathbf{v}_r = \mathbf{PT}\mathbf{v}_C \equiv \mathbf{H}\mathbf{v}_C \quad (6).$$

The traditional VVP includes gradient of velocities in a calculated vector. And the calculations are individually performed between grids. The gradients of velocities are often not used as a product, which means that the traditional VVP derives unnecessary variables. Even if the gradients are used as a product, two linear approximations of adjacent grids are not connected at the border of the adjacent grids. Meanwhile, in the improved formulation, all the elements of $\mathbf{v}_C$ is useful as a product, and first order or higher order can be included in $\mathbf{T}$.

2.2 MMSE Solution for VVP

Considering additive noise, velocity measurements are expressed as

$$\mathbf{v}_r = \mathbf{H}\mathbf{v}_C + \mathbf{n} \,, \quad (7)$$

where $\mathbf{n}$ is an additive noise vector. The MMSE solution for the linear equation (7) $\hat{\mathbf{v}}_C$ is expressed as

$$\hat{\mathbf{v}}_C = \mathbf{R}_{v_C}\mathbf{H}^T\left(\mathbf{H}\mathbf{R}_{v_C}\mathbf{H}^T + \mathbf{R}_n\right)^{-1}\mathbf{v}_r \quad (8)$$

where $\mathbf{R}_{v_C}$ and $\mathbf{R}_n$ are covariance matrices of $\mathbf{v}_C$ and $\mathbf{n}$, respectively. $\mathbf{R}_{v_C}$ can be assumed by size of grid, etc.. $\mathbf{R}_n$ is well known as probabilistic property of velocity measurements as written in [3].

If $\mathbf{R}_n = \mathbf{O}$, Eq. (8) is equivalent to the least square (LS) solution. In the traditional VVP formulation, the LS solution is applied generally. It can be said that the MMSE solution is more stable than the LS solution by adding $\mathbf{R}_n$ which is a definite or semi-definite matrix. It is difficult to determine $\mathbf{R}_{v_C}$ theoretically. However, we can be input a practically proper values to $\mathbf{R}_{v_C}$ for stable solution. Since the traditional VVP formulation includes gradients of velocity, it is more difficult to input reasonable values into $\mathbf{R}_{v_C}$. For a simplified formulation of VVP, which is without the gradients, the MMSE solution can be applied. In this application, while computational cost is low, it is difficult to detect a high gradient of velocity because it is a zeroth approximation in a desired volume.

3. Numerical Simulation

Numerical simulation is carried out to show a result example of the proposed approach. In the simulation, a velocity field output from a large eddy simulation (LES) is applied as truth, then radial velocity field is calculated from it. Finally, measurement noise is added to the radial velocities. The additive noises are 0-mean Gaussian whose standard deviation is 1 m/sec for every radial velocity. It is assumed that the standard deviation is known. In Figure 1, Panel (a) shows the truth of the simulation. Panels (b), and (c) show estimated results of the traditional VVP and the proposed VVP method, respectively. From the comparison between Panels (b) and (c), the proposed VVP method accomplishes more stable solutions. The traditional VVP does not retrieve 2-D velocity field correctly because few radial velocities in small width of azimuth are available in this situation. On the other hand, the proposed VVP method retrieves a reasonable 2-D velocity field even with such a small observable area. High spatial resolution is indicated around (x, y) = (1300, 800). On the other hand, in the right side of Panel (c-1) and in the lower side of Panel (c-2), estimation is not correct. These are solved by a dual-radar or a radar network approach [4].

4. DICUSSION

An algorithm for retrieving a 2-D velocity from radial velocities by a radar or a lidar is proposed. Since VVP method has been used for this purpose. However, in the traditional VVP, both stability and high resolution are not

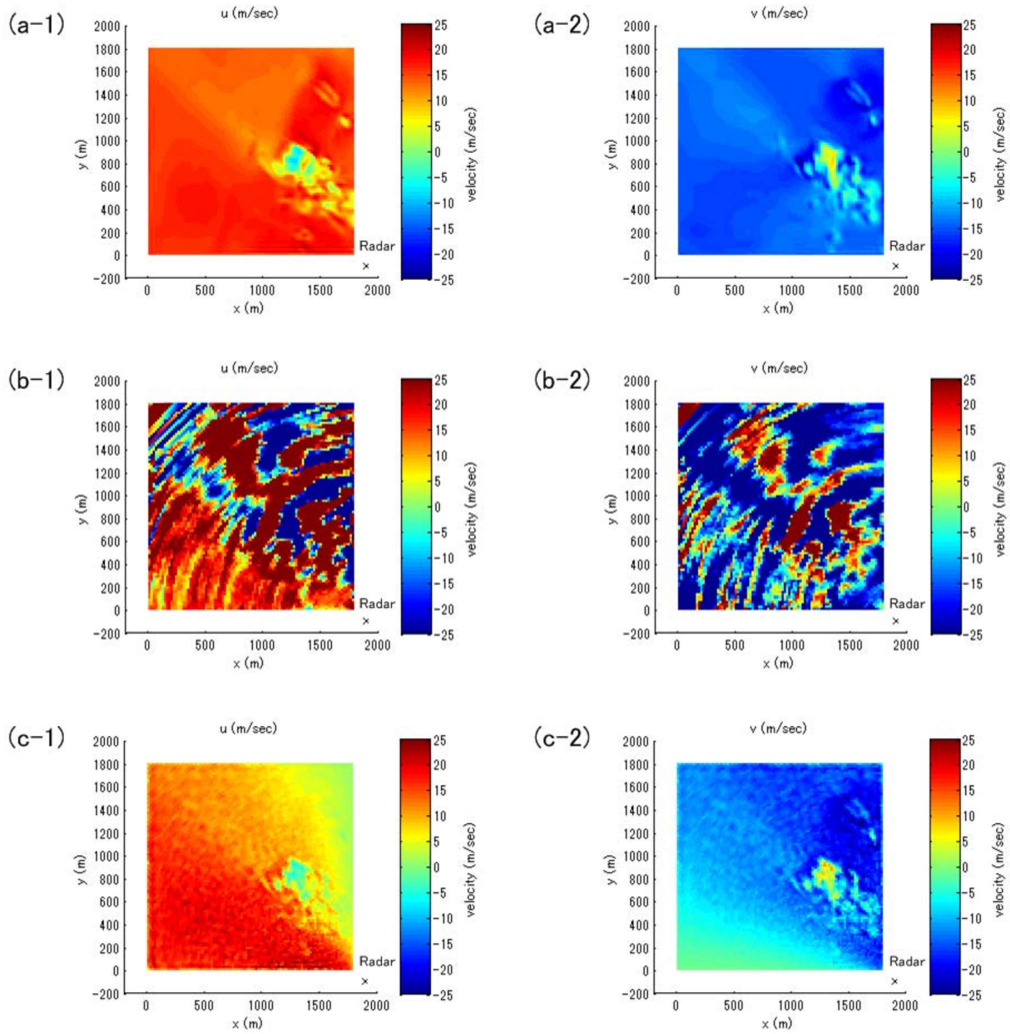

Figure 1: An Example of Simulation Result

achieved simultaneously. This paper proposes two approaches to improve VVP method. One is a new formulation connecting 2-D velocity field to measured radial velocities, which can avoid unnecessary parameters in the traditional VVP and estimate only necessary parameters efficiently. The other is the MMSE solution for inversion, which can consider probabilistic property of measurements for correct estimation. An example of simulation result indicates that the proposed improvements can retrieve correct 2-D velocities even in a situation which is severe for the traditional VVP to work properly. Since the new formulation includes 2-D velocities of all the grid, the solution can add some constraints. For example, by utilizing equation of continuity of incompressive flow, solution is possible to be more accurate.

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