# ON THE UTILIZATION OF RADAR REFLECTIVITY DATA FOR ASSIMILATION INTO STORM-SCALE NWP MODELS

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## 1. INTRODUCTION

For storm-scale numerical weather prediction (NWP) models, radar data are important pieces of information in the assimilation process. The 3D-Variational (3DVAR) assimilation method, as opposed to other more complex methods, is widely used because it is computationally inexpensive and efficient. In this technique, radar reflectivity data cannot be directly assimilated but instead must be converted to hydrometeor mixing ratios. This requires a retrieval that is non-trivial and traditionally underdetermined, as many possible combinations of hydrometeors, as well as different hydrometeor size distributions, can result in the same reflectivity value, leading to a non-unique problem.

Traditional methods for retrieving the mixing ratio *q* from reflectivity *Z* are quite primitive, relying on many assumptions about the hydrometeor distributions. Because spectral bin models are computationally expensive, many storm-scale NWP models employ single-moment bulk microphysics, fixing the intercept parameter in the assumed size distributions and subsequently determining the reflectivity from mixing ratios alone. The traditional *Z-q* equations (from Gao and Stensrud, 2012) are

$$Z(q_r) = 3.63 \times 10^9 (\rho q_r)^{1.75} \quad (1)$$

$$Z(q_h) = 4.33 \times 10^{10} (\rho q_h)^{1.75} \quad (2)$$

$$Z(q_{ds}) = 9.8 \times 10^8 (\rho q_{ds})^{1.75} \quad (3)$$

$$Z(q_{ws}) = 4.26 \times 10^{11} (\rho q_{ws})^{1.75} \quad (4)$$

$$Z_{dB} = 10 \log_{10}[Z(q_r) + Z(q_h) + Z(q_{ds}) + Z(q_{ws})] \quad (5)$$

where ρ is air density and the subscripts *r*, *h*, *ds*, and *ws* refer to rain, hail, dry snow, and wet snow, respectively. More information about the derivation of these relationships can be found in the appendix of Dowell et al. (2011).

The widespread implementation of dual-polarization capability across the nationwide NEXRAD radar network has opened the door for its use in the assimilation process, particularly in real-time assimilation systems where this was previously unavailable. Dual-polarization variables provide additional insight into hydrometeor shape, habit, orientation, phase, etc. As such, they hold great potential for improving future assimilation and retrieval methods. One direct way this is possible is through the use of the hydrometeor classification algorithm (HCA), currently implemented in the NEXRAD network. Currently, before the *Z-q* relations can be applied, the hydrometeor type must be determined, and in many cases areas of mixed hydrometers are not allowed to exist. For example, in Gao and Stensrud (2012), a simple hydrometeor classification system is implemented such that

$$Z(q_r) \qquad T_b > 5°\mathrm{C} \quad (6)$$

$$Z(q_s) + Z(q_h) \qquad T_b < -5°\mathrm{C} \quad (7)$$

$$\alpha Z(q_r) + (1-\alpha)[Z(q_s) + Z(q_h)] \qquad -5° < T_b < 5°\mathrm{C} \quad (8)$$

where α varies linearly between 0 at -5°C and 1 at 5°C, and $T_b$ is the temperature from the background state. However, it is readily evident that this is an oversimplification, as in most cases this would not allow for hail at the surface or supercooled water droplets above the environmental freezing level. In both of

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these cases, the HCA could be a crucial step in determining the hydrometeor type before any retrieval relations are applied.

The possibility of the HCA including a rain/hail mixture category, as well as being able to distinguish hail sizes within such volumes, is currently being investigated. In the future, use of such a classification in an assimilation process would require a technique to determine the $q_r$ and $q_h$ separately, an impossible task for reflectivity alone.

## 2. METHODOLOGY

In order to investigate the complex relationship between reflectivity and hydrometeor mixing ratios, the melting hail model with spectral microphysics and coupled radar operator described by Ryzhkov et al. (2013a,b) was used. In these simulations, the freezing level was at 4 km with a moist adiabatic lapse rate from the surface to the freezing level. No updraft was included.

Hail and graupel distributions were prescribed at the freezing level according to previously found empirical relationships, taking care to account for the wide range of possible hail-size distributions. The hailstones were then allowed to melt within the layer below 4 km and the evolution of the size distribution of the partially melted hailstones and raindrops resulting from complete melting down to the surface is governed by the model described in Ryzhkov et al. (2013a). It is assumed that both hail and graupel aloft follow an exponential distribution described by

$$N(D) = N_0 e^{-\Lambda D} \tag{9}$$

In all cases, the graupel distribution was characterized by $N_{0g} = 8000$ m$^{-3}$ mm$^{-1}$ and $\Lambda_g = 1.6$ mm$^{-1}$, creating a Marshall-Palmer-like rainfall distribution at the surface. Three hail size categories were created: small hail ($D_{max} \leq 25$ mm), large hail ($25 < D_{max} \leq 50$ mm), and giant hail ($D_{max} > 50$ mm).

Ulbrich and Atlas (1982) found that a relationship exists between the slope parameter, $\Lambda_h$, and the maximum hail diameter, $D_{max}$, reporting that 90% of their distributions had $\Lambda_h D_{max} \geq 5$ and 66% had $\Lambda_h D_{max} \geq 7$. This was a part of the National Hail Research Experiment in 1976. Cheng and English (1985) corroborated these results in Alberta, Canada, finding that 87% of their distributions had $\Lambda_h D_{max} \geq 5$, 61% had $\Lambda_h D_{max} \geq 7$, and 42% had $\Lambda_h D_{max} \geq 8$. Following these results, we allowed our prescribed distributions to have $\Lambda_h D_{max} \geq n$, where $n$ ranged from 5 to 11 and $D_{max}$ varied every 5 mm.

The intercept parameter, $N_0$, was found following Cheng and English (1985), who expanded upon a previous data set to find that

$$N_{0h} = 100\Lambda^{4.11} \tag{10}$$

which characterized the entire double-moment distribution via a single parameter. However, Federer and Waldvogel (1975) found

$$N_{0h} = 758\Lambda^{4.18} \tag{11}$$

with Cheng and English (1985) concluding that the leading coefficient varies amongst different storms and is related to storm thermodynamics. As such, our intercept parameters were given by

$$N_{0h} = A\Lambda^{4.11} \tag{12}$$

where A varied from 50 to 800 in intervals of 50. A total of 1,952 unique hail distributions were modeled. The distributions are summarized in Table 1.

For the purpose of determining rain mixing ratios and their associated reflectivity, bins that contained less than 0.01 g m$^{-3}$ of ice water content were considered ice-free, extracted, and converted to mixing ratios. In this case, only small hail distributions were included as none of the large or giant hail distributions had melted completely by the ground.

In addition to these model runs, a dataset of disdrometer observations was used to both corroborate the model results and fill in missing data areas. This disdrometer was located in Norman, OK and took routine observations for 7 years, resulting in over 47,000 drop size distribution (DSD) observations.

| Size | $\Lambda$ [mm$^{-1}$] | $\Delta\Lambda$ [mm$^{-1}$] | $D_{max}$ [mm] | $\Delta D_{max}$ [mm] | A ($N_{0h}=A\Lambda^{4.11}$) | $\Delta$A |
|---|---|---|---|---|---|---|
| Small | 0.25 – 1.10 | 0.05 | 10 – 25 | 5 | 50 – 800 | 50 |
| Large | 0.10 – 0.30 | 0.05 | 30 – 50 | 5 | 50 – 800 | 50 |
| Giant | 0.05 – 0.25 | 0.05 | 55 - 75 | 5 | 50 - 800 | 50 |

*Table 1: Summary of modeled hail size distributions.*

## 3. RESULTS AND DISCUSSION

The disdrometer data and model results are plotted in Fig. 1 along with the $Z_R$-$q_r$ equation currently used for radar data assimilation. For lighter rain (dBZ < 40), the median values from the disdrometer observations agree fairly well with the $q_r(Z_R)$ equation (not shown). However, above 40 dBZ the two curves begin to diverge. This difference becomes quite substantial past 55 dBZ, with errors of a factor of two in the mixing ratio retrievals possible. The model simulations, in which the rain mixing ratio is a result of melting graupel and hail, provide a natural continuation of the median disdrometer measurements where observations become sparse. This result suggests that while the current $q_r(Z_R)$ relation is fairly accurate for light and moderate rain, a separate parameterization will be required for heavy rain and should be investigated.

Another assimilation procedure that was investigated was the ability to determine both $q_r$ and $q_h$ using $K_{DP}$ to isolate the rain component. Since $K_{DP}$ is zero for isotropic scatterers, hail that is approximately spheroidal would have a $K_{DP}$ near zero. A $K_{DP}$-$Z_R$ relationship could then be applied by which the contribution of hail to the total radar reflectivity factor is estimated as the difference $Z_H$=$Z_{TOT}$-$Z_R$ (Balakrishnan and Zrnic, 1990a). The $q_h$ can then be computed from the $q_h(Z_h)$ relation. Hence, both $q_r$ and $q_h$ could be retrieved. To investigate the feasibility of this, individual model runs were examined. For this example, a hail distribution described by $\Lambda$ = 0.42 mm$^{-1}$, $N_0$ = 400$\Lambda^{4.11}$, and $D_{max}$ = 24 mm was examined. The graupel distribution used was as described above.

As seen in Fig. 2, the liquid water content (LWC) increases monotonically toward the ground as melting occurs, and the $K_{DP}$ profile follows a similar profile, as is expected. Indeed, the $K_{DP}$-LWC relationship is quite well established, as seen in Fig. 3, and is primarily independent of the initial hail size distribution. The $K_{DP}$-$Z_R$ relation used for the middle panel in Fig. 2 (from Doviak and Zrnic, 1993) was:

$$Z_R = 65{,}800(KDP)^{1.386} \quad [mm^6\, m^{-3}] \tag{13}$$

However, when this calculated $Z_R$ is subtracted from the total reflectivity, the resultant $Z_H$ profile is non-monotonic implying that the university $q_h(Z_H)$ relation similar to the one in (2) does not exist. At the surface, where the ice water content (IWC) is 0.04 g m$^{-3}$, the $Z_H$ is approximately 56 dBZ. If this were accepted as valid and applied to the assimilation equation for hail, the use of Eq. (2) would result in a hail mixing ratio of 1.36 g m$^{-3}$, over an order of magnitude off. With the

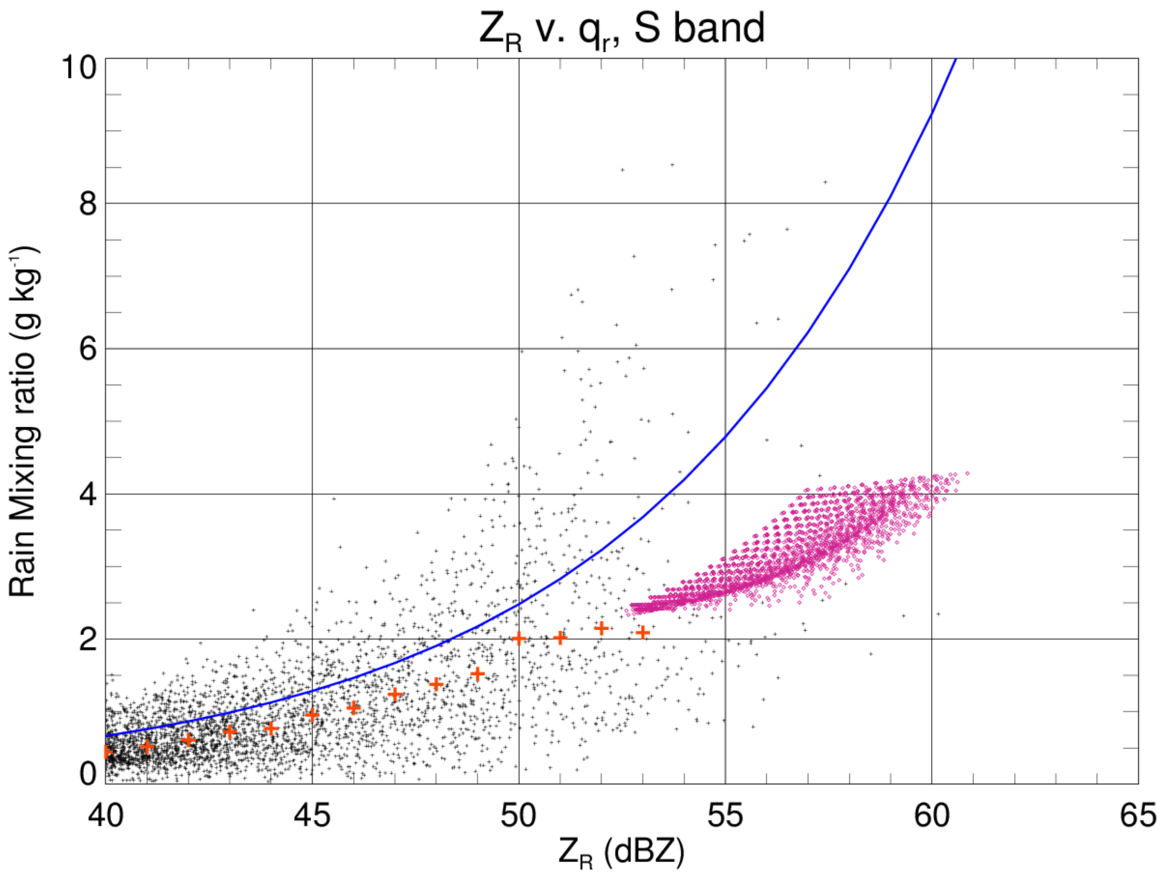


*Fig. 1: Comparison of expected $Z_R$-$q_r$ relation with observations and model results: disdrometer observations (black), disdrometer median values (orange), model runs (pink), and the traditional $Z_R$-$q_r$ relation (blue).*

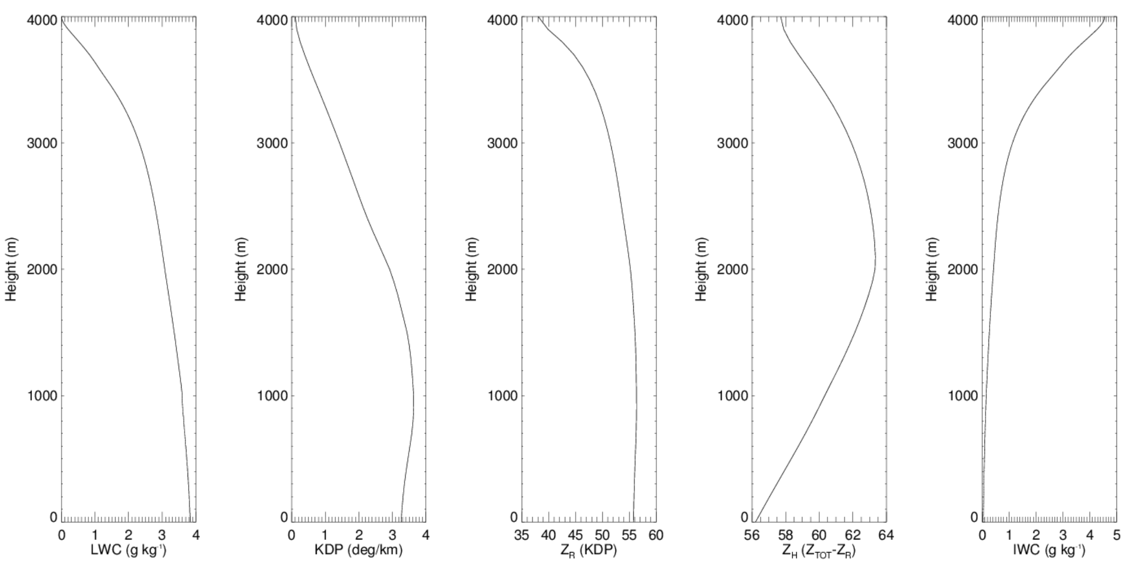


*Fig. 2: Vertical profiles of LWC, $K_{DP}$, $Z_R(K_{DP})$, $Z_H$, and IWC from a hail distribution described by $\Lambda = 0.42$ mm$^{-1}$, $N_0 = 400\Lambda^{4.11}$, and $D_{max} = 24$ mm.*

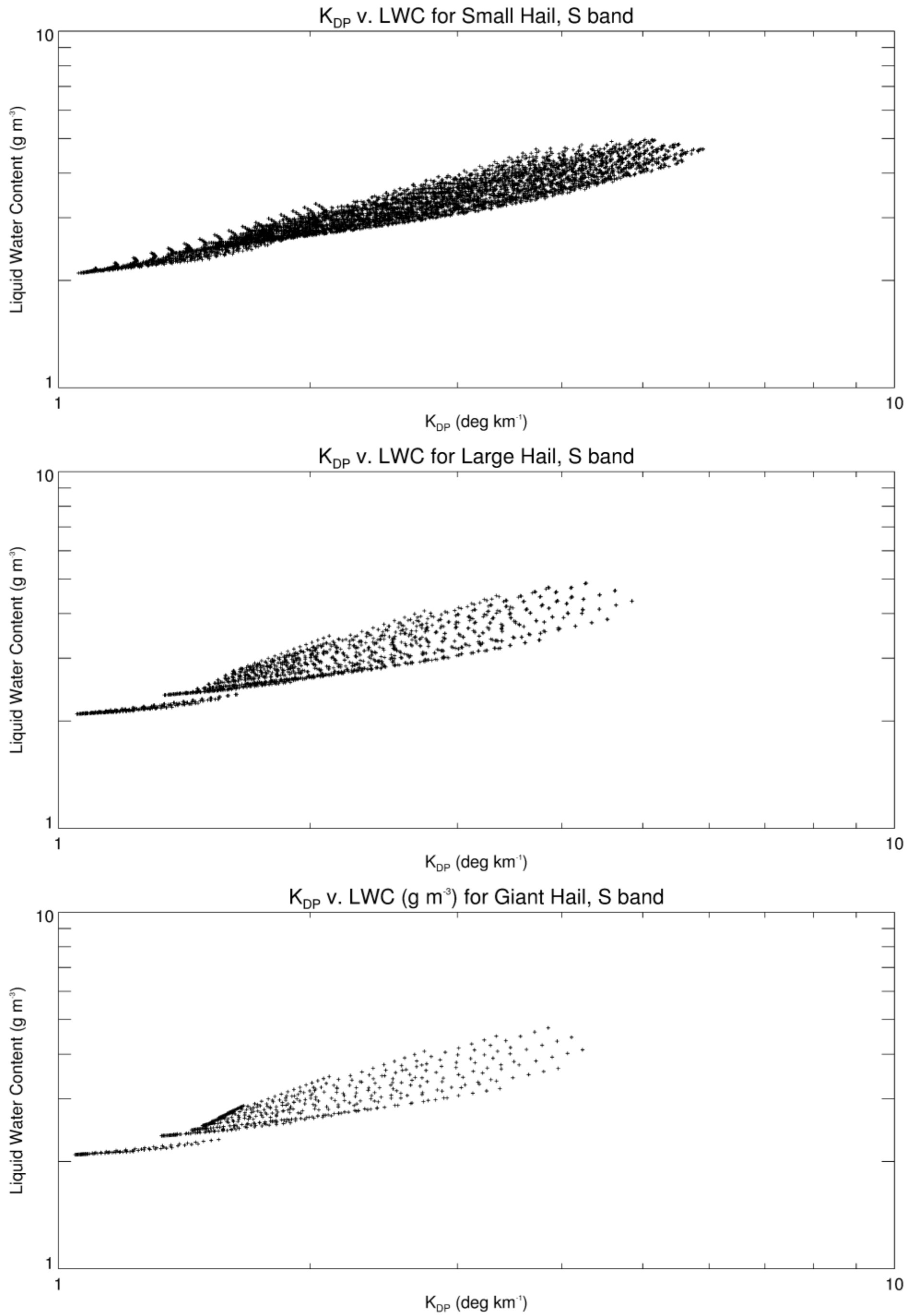


*Fig. 3: Model derived $K_{DP}$-LWC relations for small, large, and giant hail.*

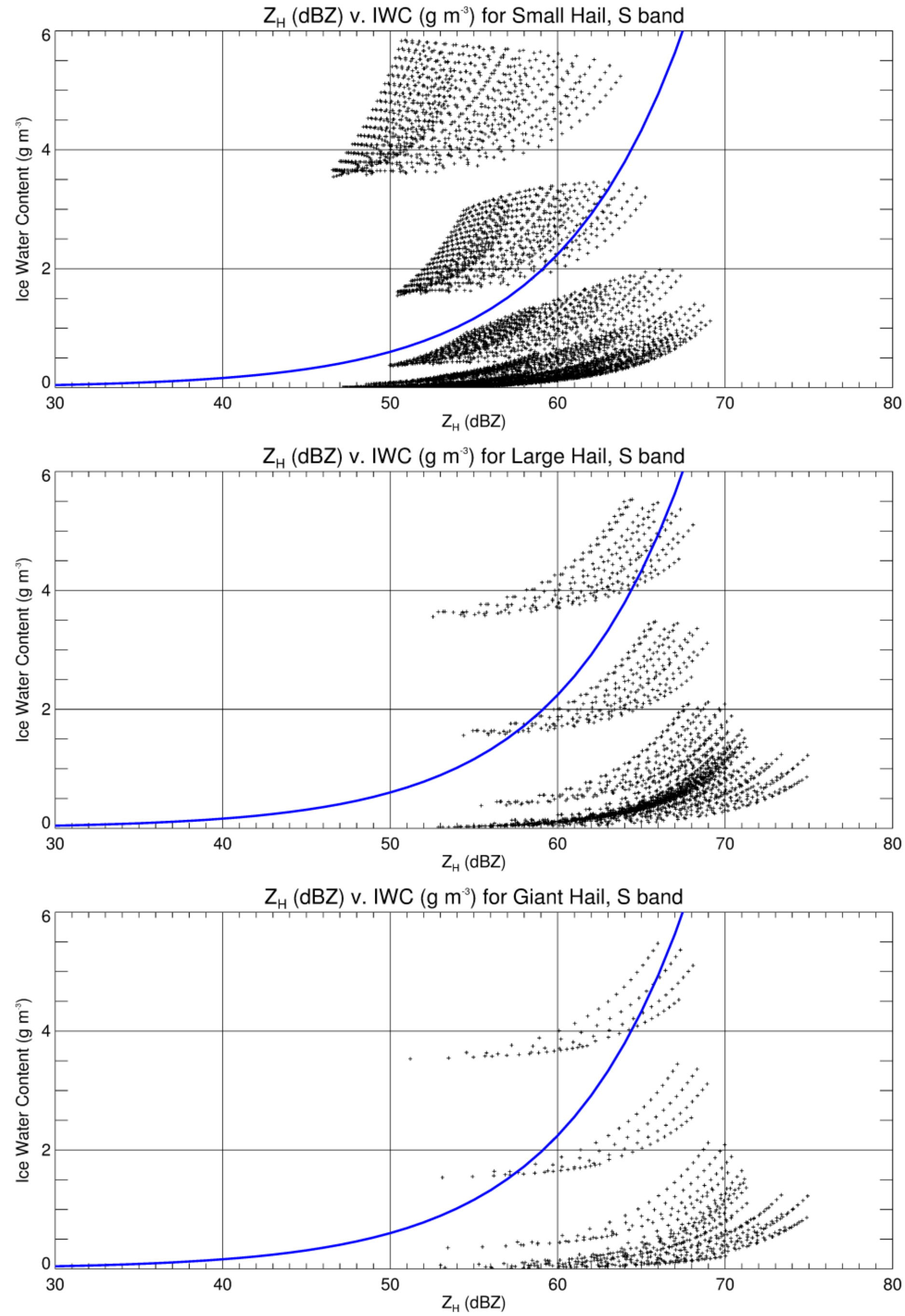


*Fig. 4: Reflectivity from hail (dBZ) vs. ice water content (IWC, g m$^{-3}$) for small, large, and giant hail. The blue line is the currently used $Z_H$-$q_h$ relation.*

well-correlated relationship between $K_{DP}$ and LWC, the nonlinearity in the $q_h(Z_H)$ relation is quite evident. It is obvious that different $q_h(Z_H)$ relations should be utilized for hail at different heights with respect to the freezing level or for hail with different degrees of melting.

This is revealed in Fig. 4 where the results of simulations for small, large, and giant hail are shown. The top cluster of points in each plot is at the freezing level, with the clusters descending toward the abscissa as the hail partially melts and falls toward the ground. This result makes sense intuitively, as the large number of melting graupel particles and small melting hail will quickly decrease the IWC while the reflectivity is heavily weighted toward the largest hailstones which melt most slowly relative to their size. These examples clearly reveal that separate $q_h(Z_H)$ relations should be used for different clusters of the $q_h$-$Z_H$ pairs. The currently used $Z_H$-$q_h$ relation (2) is overlaid in blue, demonstrating the large errors undoubtedly present in using such a parameterization.

The recently developed algorithm for polarimetric hail detection and discrimination of its size can provide the guidance on which of the $q_h(Z_H)$ relations should be chosen in a particular area of the storm depending on the hail size and the height of the radar resolution volume with respect to the freezing level (Ryzhkov et al. 2013b). Since these situations can be fairly well distinguished in terms of the differential reflectivity, $Z_{DR}$, it is quite possible that $q_h(Z_H,Z_{DR})$ relations can be devised.

## 4. SUMMARY AND FUTURE WORK

To investigate the current state of radar reflectivity assimilation, a wide range of hail distributions were modeled using the model and radar operator put forth by Ryzhkov et al. (2011). These modeled distributions were combined with a large set of observations from a disdrometer with 7 years of data from Oklahoma. For rain, it was found that the $Z_R$-$q_r$ relation currently used in assimilation is acceptable for light and moderate rain. However, errors in LWC begin to grow and exceed 1 g m$^{-3}$ past 50 dBZ and subsequent retrievals quickly become off by a factor of two. Future work will include investigating an improved $Z_R$-$q_r$ relation for cases of heavy rain with reflectivities exceeding 50 dBZ. This could be readily implemented via the Heavy Rain classification in the HCA, whose membership probability function for reflectivity ranges from 40 dBZ to 60 dBZ and peaks between 45 and 55 dBZ, consistent with our results.

The possibility of determining $q_r$ and $q_h$ separately in rain-hail mixtures using polarimetric data was also investigated. The rain mixing ratio $q_r$ can be quite reliably estimated from $K_{DP}$ in the rain-hail mixture and the $Z_r(K_{DP})$ relation can be utilized for the computation of $Z_H$ (i.e., the hail contribution to Z). Then multiple $q_h(Z_H)$ relations should be applied depending on the results of polarimetric discrimination between small, large, and giant hail at different heights with respect to the freezing level. Future work will include investigating whether the $Z_H$-$q_h$ relation can be parameterized by height and what role the differential reflectivity ($Z_{DR}$) could play in constraining these relations further.

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