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THEORY AND VERIFICATION OF BIAS CORRECTION FOR POLARIMETRIC PHASED ARRAY RADAR

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Abstract — This paper provides a brief system overview depicting the key features and limitations of the X-band dual polarization phased array radar developed at the University of Massachusetts. Then a discussion on origin of polarization rotation and its impact on the quality of polarimetric products follows. Next, we will define a rotation matrix and show how unbiased measurements can be recovered. Finally, polarimetric data collected in a stratiform rain event will be presented and evaluated.

Index Terms—*phased array, dual polarization, weather radar*

1. INTRODUCTION

Phased array technology in radar meteorology has gained a lot of attention in the past decade due to the multiple benefits it provides, such as a smart scanning strategy and more rapid updates. Similarly, the upgrade to dual polarization capability is a common trend observed in many operational weather radar networks around the world. Implementation of a second polarization channel allows significant improvement in the data quality and provides additional information for accurate weather forecasting. The Microwave Remote Sensing Laboratory (MIRSL) at the University of Massachusetts has developed low-cost, mobile, dual-polarized **P**hase-**T**ilt **W**eather **R**adar (see Figure 1). Development and calibration of PTWR are described in Orzel et al (2013). Main radar system specifications are presented in Table I.

The PTWR is able to provide polarimetric radar products using the alternate transmission and alternate reception (ATAR) mode of operation. However, this is only possible if polarization channels are properly calibrated. There is a major difference between mechanically-scanned reflector antennas and electronically scanning arrays, where the polarimetric properties of an antenna vary with scan angle. Biases in phased array polarimetry are discussed in Zhang et al (2009), Zrnić et al (2011) and Sanchez-Barbetty et al (2012), among others.

Fig 1. MIRSL phase-tilt mobile weather radar.

The common source of errors intrinsic to the array antenna itself is a non-optimal cross-polarization isolation or mismatch of the beam patterns in the two polarizations. These are defined at the system design stage and can vary over time due to aging, temperature changes, or other effects. The other error is related to the misprojection of the co- and cross-polar fields onto the local horizontal and vertical directions. In case of the PTWR architecture, synthesized beams remain in the principle plane of the array and hence the "H" and "V" polarizations remain orthogonal across the scan. However, they rotate as one scans off-boresight at non-zero elevation angle introducing a constant canting angle. Fortunately, this bias can be corrected by an appropriate multiplication of the measured scattering matrix with rotation matrix.

Table I MIRSL phase tilt weather radar parameters.

Parameter	Specifications
Radar system size [m]	1.47 x 0.82 x 0.30
Weight [kg]	82
Frequency [GHz]	9.3-9.4
Transmit peak power [W]	60
Duty cycle	up to 30%
Beam width [°]	2 (az) x 3.5 (el)
Mode of operation	ATAR

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2. SCANNING GEOMETRY

In this section we will describe the dependence of polarization on scan angle in terms of the radiation patterns of a pair of crossed-dipoles located at the coordinate system origin presented in Figure 2. For consistency we assume the same conditions and use the same terminology as in Zhang et al (2009), who described polarization properties for 2D phased array architecture. On the other hand, the PTWR is an example of 1-D array architecture, which enables electronic scanning in the azimuth plane, while the array is mechanically tilted in elevation.

In Figure 2, the y-z plane is perpendicular to the ground. The antenna aperture is in y'-z' plane. Coordinate system X'Y'Z' is obtained by rotation about y-axis by the elevation angle θ_e. The x'-axis represents the boresight of the radar system at elevation angle θ_e.

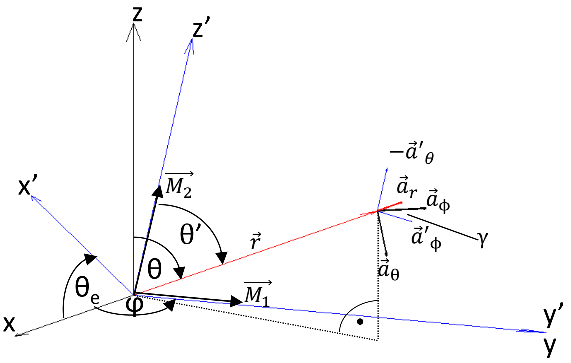

Fig 2. Spherical coordinate system for electric fields radiating from a pair of dipoles having moments $\vec{M}_1$ and $\vec{M}_2$. Unit vectors $\vec{a}_\varphi$, $\vec{a}_\theta$, $\vec{a}'_\varphi$ and $\vec{a}'_\theta$ lie in the same plane perpendicular to $\vec{r}$. In case of PTWR, $\theta' = 90°$.

The following transformation relates level and tilted coordinates in the Cartesian system:

$$\begin{aligned} \vec{a}'_x &= \vec{a}_x \cos\theta_e + \vec{a}_z \sin\theta_e \\ \vec{a}'_y &= \vec{a}_y \\ \vec{a}'_z &= -\vec{a}_x \sin\theta_e + \vec{a}_z \cos\theta_e \end{aligned} \tag{1}$$

In the spherical coordinate system, unit vectors are defined as follows:

$$\begin{aligned} \vec{a}'_\varphi &= -\vec{a}_x \cos\theta_e \sin\varphi' + \vec{a}_y \cos\varphi' - \vec{a}_z \sin\theta_e \sin\varphi' \\ \vec{a}'_\theta &= \vec{a}_x \sin\theta_e - \vec{a}_z \cos\theta_e \end{aligned} \tag{2}$$

And in terms of the tilted coordinate system:

$$\begin{aligned} \vec{a}'_\varphi &= -\vec{a}'_x \sin\varphi' + \vec{a}'_y \cos\varphi' \\ \vec{a}'_\theta &= -\vec{a}'_z \end{aligned} \tag{3}$$

Projections of $\vec{a}'_\varphi$ and $\vec{a}'_\theta$ onto the horizontal ($\vec{a}_\varphi$) and vertical ($-\vec{a}_\theta$) directions yield:

$$\begin{aligned} \vec{a}_\varphi \cdot \vec{a}'_\varphi &= \cos\theta_e \sin\varphi \sin\varphi' + \cos\varphi \cos\varphi' = \cos\gamma \\ -\vec{a}_\theta \cdot \vec{a}'_\varphi &= -\sin\gamma \\ \vec{a}_\varphi \cdot \vec{a}'_\theta &= \sin\gamma \\ -\vec{a}_\theta \cdot \vec{a}'_\theta &= \cos\gamma \end{aligned} \tag{4}$$

$\vec{M}_1$ is the magnetic current density of a horizontally polarized radiating element, and $\vec{M}_2$ is the magnetic current density of a vertically polarized radiating element. Following the method described by Zhang et al (2009), one can define the electric field at $\vec{r}$ from $\vec{M}_1$ and $\vec{M}_2$ as:

$$\begin{aligned} \vec{E}_1 = E_{t1}\big[&-\vec{a}_x \cos\theta_e \cos\varphi' \sin\varphi' + \vec{a}_y cos^2\varphi' \\ &- \vec{a}_z \sin\theta_e \cos\varphi' \sin\varphi'\big] \\ &= E_{t1} \cos\varphi' \, \vec{a}'_\varphi \end{aligned}$$

$$\begin{aligned} \vec{E}_2 = E_{t2}[&-\vec{a}_x \sin\theta_e + \vec{a}_z \cos\theta_e] \\ &= -E_{t2}\vec{a}'_\theta \end{aligned}$$

where

$$E_{tq} = \frac{k^2 e^{-jkr}}{4\pi\varepsilon r} M_q \tag{5}$$

The factor $\cos\varphi'$ in (5) indicates a reduction of intensity of the H and V fields as the beam is directed away from broadside. This effect can be measured during initial array calibration and included in the radar calibration constant on a beam by beam basis. The definition of electric fields $\vec{E}_1$ and $\vec{E}_2$ shown in (5) proves that both "H" and "V" polarizations remain orthogonal across the scan. However, note that $\vec{a}'_\varphi$ expressed in terms of the level coordinate system reveals a vertical component, which is produced by a horizontal dipole $\vec{M}_1$. This is the source of bias in polarimetric products. The rotation of the 'horizontal' unit vector out of the horizontal plane is given by γ, defined in (4).

Finally, it is important to point out that the 1-D phased array radar PPI scan is a single face of pyramid. This is principally different from a case of a mechanically rotated dish antenna system, which scans a section of a cone. For example, in the case of PTWR the effective beam elevation decreases from requested θ_e as the beam is directed away from broadside. Radar-defined beam direction (φ',θ_e) is related with beam direction in the level coordinate system (φ,θ) as:

$$\varphi = \arctan(\tan(\varphi')\sec(\theta_e))$$
$$\theta = \arccos(\cos(\varphi')\sin(\theta_e)) \qquad (6)$$

3. ROTATION AND SCATTERING MATRIX

It is generally assumed that in most meteorological observations of interest, water drops take a form of oblate spheroids with a zero mean canting angle, that is, the angle between the incident electric field and the axis of symmetry of a water drop. In this case the off-diagonal components of a backscattering matrix can be ignored. However, in case of PTWR there is a constant non-zero canting angle due to rotation of H and V fields out of horizontal-vertical polarization basis. This is a rather unusual phenomena observed in radar data (Ryzhkov 2007). Canted hydrometeors cause depolarization of linearly polarized H and V waves, but the effect of depolarization on co-polar variables is negligible if canting angle is of the order of a few degrees (Ryzhkov 2001). Furthermore, Zrnić et al. (2010) showed that bias due to cross-polar and copolar radiation coupling is of higher importance in the case of the simultaneous transmit simultaneous receive mode of operation. Wang et al (2006) also investigated a case of antenna feed alignment error, concluding that in the alternate mode of operation, orientation error up to 5 degrees can be tolerated if an error ΔZ_{dr} of 0.2 dB can be accepted. Although PTWR operates in ATAR mode, the self-induced canting angle can be higher than 5 degrees (see Figure 3). For example, if beam direction is set 45 degrees away from broadside and the array aperture is tilted by 10 degrees, then the canting angle equals 7.1 degrees, which is equivalent to the antenna cross-polarization isolation level (CPL) of around -18 dB.

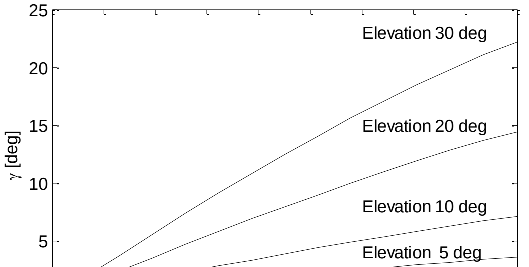

Fig 3 Canting angle, γ, as a function of azimuth scan angle,φ , for various elevation tilts, θ_e.

The retrieval biases increase rapidly if CPL > -30 dB in STSR mode, and CPL > -20 dB in ATAR mode. The correction to the scattering matrix can be applied as derived by Zhang et al (2009), if the projection matrix is modified for a 1-D phased array architecture. The backscattering electric field $\vec{E}_s$ can be expressed as (Doviak 2006):

$$\vec{E}_s = \begin{bmatrix} E_{sh} \\ E_{sv} \end{bmatrix} = S^{(b)} E_i \times \frac{\exp(-jkr)}{r} \qquad (7)$$

where $S^{(b)}$ is a backscattering matrix of hydrometeors defined as (Bringi 2001):

$$S^b = P(\gamma)SP(-\Upsilon) = P(\gamma)\begin{bmatrix} S_{11}(\Psi) & 0 \\ 0 & S_{22}(\Psi) \end{bmatrix} P(-\Upsilon) \qquad (8)$$

$$P(\gamma) = \begin{bmatrix} cos\gamma & sin\gamma \\ -sin\gamma & cos\gamma \end{bmatrix} \; and \; P(-\gamma) = P^{-1}(\gamma) = P^t(\gamma) \qquad (9)$$

$P(\gamma)$ is the rotation matrix and γ is a mean canting angle. If the propagation effect is included and assuming uniformly canted particles, transmission matrix is:

$$T_c = P(\gamma)TP(-\gamma) = P(\gamma)\begin{bmatrix} T_{hh} & 0 \\ 0 & T_{vv} \end{bmatrix} P(-\gamma) \qquad (10)$$

Finally, combining (8)-(10), the biased backscattering matrix for a 1-D polarimetric PAR is defined as:

$$S^{(p)} = P(\gamma)TSTP(-\gamma) = P(\gamma)S'P(-\gamma) \qquad (11)$$

The unbiased backscattering matrix S’ can still be recovered by a multiplication with the inverse of the rotation matrices:

$$S' = P(\gamma)^{-1} S^{(p)} P(-\gamma)^{-1} \qquad (12)$$

Note that in ATSR mode of operation the backscattering matrix has to be corrected for the Doppler effect as described by Zrnić et al (2011).

4. OBSERVATIONS

PTWR is mounted on a mobile platform and was routinely operated in a region of Western Massachusetts, USA during summer 2013. The sample radar products provided by PTWR are demonstrated in Figure 4. The observed widespread rainfall event exhibits moderate weather conditions with mean reflectivity around 20 dBz and mean spectrum width below 2 m/s. Since PTWR is mounted on a pedestal, it is possible to imitate the operation of a “classic weather radar”, by scanning mechanically using only a broadside beam of the phased array antenna. In order to compare PPI scan collected by 1-D phased array, the pedestal was programmed to scan an arc in accordance with equation (6). This type of scan, which is not affected by rotation of polarizations, was performed 30 seconds after collecting data presented in Figure 4. Figure 5 compares the reflectivity data measured in three subsequent PPI scans for beam direction φ' = 45°. Scan 1 and 2 were collected using electronic beam forming, while scan 3 was collected using a broadside beam and a proper pedestal azimuth/elevation setting. Note that due to beam broadening effect scans 1-2 illuminate a slightly larger precipitation volume. Nevertheless, all profiles stand in fair agreement. This supports the theory derived by Wang et al (2006), who showed that in a light rain event and assuming the most stringent conditions, the reflectivity bias $\Delta Z_h < 0.1$ dB if CPL < -18 dB.

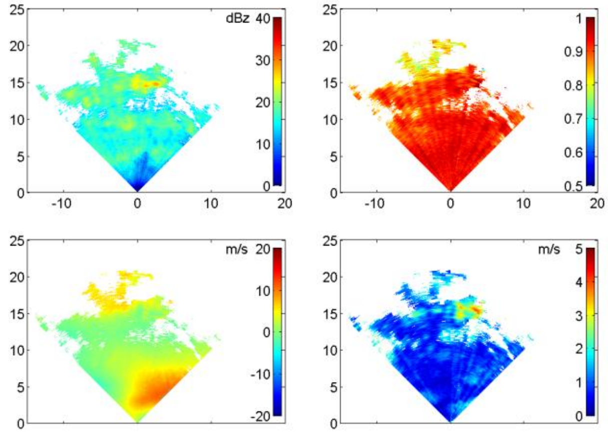

Fig 4. Widespread stratiform rainfall observations collected on 8/26/2013 in Amherst, MA, USA. PTWR was operated in electronic scan mode tilted by 10 degrees. Panels are (clockwise from upper left): reflectivity Zh, correlation coefficient ρ_{HV}, spectrum width, and velocity.

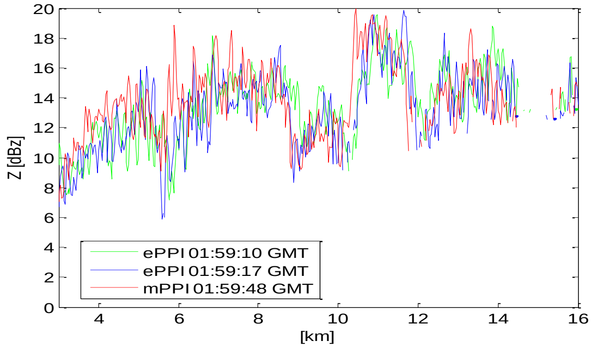

Fig 5 Comparison of reflectivity measurements collected in electronic scan mode (ePPI) and mechanical scan mode (mPPI). The data represents beam direction φ' = 45°.

5. SUMMARY

A dual-polarized, low-cost, solid-state X-band phased array weather radar has been developed and tested at University of Massachusetts. We described the scanning geometry of a 1-D phased array architecture that enables mechanical tilt in elevation. The effect of polarization rotation on data quality has been discussed, and a possible bias correction scheme has been suggested. Finally, we verified the effect of polarization rotation on reflectivity data collected by PTWR.

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