2B.3 PPP, Fourier Transform and Spectrum Width Measurements of Scanning Radar

Ming Fang, Bruce A. Albrecht
Rosenstiel School of Marine and Atmospheric Science, University of Miami

Nitin Bharadwaj
Pacific Northwest National Laboratory

Pavlos Kollias
Department of Atmospheric and Oceanic Sciences, McGill University

1. Introduction

Radar beam pattern describes the spatial distribution of the power emitted by a radar. The power level of side lobes is usually much lower compared with that of the main lobe and is neglected. The radar beam usually means the beam of the main lobe; the beam width indicates the region that contains about 80% of the power radiated by the main lobe. In a plane passing through the main lobe axis, beam width is an angular distance between two points where the power is half of or 3-dB lower than the peak value in the axis direction. The beam pattern of a still circularly symmetric beam is often represented by a Gaussian function and the beam width is a constant. That is the beam width is independent from the direction in the cross section perpendicular to the beam axis. For a scanning beam, such as the beam of WSR-88Ds and the Atmospheric Radiation Measurement (ARM) program scanning cloud radars, the antenna motion combined with digital signal processing broadens the beam width. If the beam is circularly symmetric in stationary, the effective beam will no longer be circularly symmetric but elliptic symmetric with the major axis in the scanning direction. Zrnic and Doviak (1976) reveal that the effective beam is mathematically the convolution of stationary beam pattern and the impulse response function of the integrator. They give the graphic solution for the one way normalized effective beam width (1993) and show the impact of the broadened beam width on reflectivity measurements. Blahak (2008) proposes that the arithmetic average of the still beam pattern for each transmitted pulse during the dwell time can be used to approximate the effective beam and he also provided the associated equation to calculate the measured reflectivity. Fang and Doviak (2008) find that the effective beam width should been used not only for the reflectivity measurement but also for the radial velocity and spectrum width measurements. This study will show: 1) the effective beam width for higher order velocity moment measurement is different from that for reflectivity measurement; 2) the effective beam width is not only the function of antenna rotation rate but also the function of elevation angle; 3) for scanning beam, different signal processing method, such as pulse-pair processing (PPP) and Fast Fourier Transform (FFT), coincides with different spectrum width equation.

2. Effective Beam Pattern for the Measurement of Z and Other Higher Order Moments

This section will provide an explicit analytical expression of the effective beam pattern for the azimuthally scanning beam at any elevation angle.

2.1 General solution

Our starting point is the correlation coefficient at the output of an integrator with a lagged time mT_s. It can be obtained by replacing T_s in Eq. (B6a) of Fang and Doviak (2008) with $\tau = mT_s$. That is

$$\hat{\rho}^{(o)}\left(\tau,\vec{r}_0,t_n\right)^{(e)} = \overline{\hat{\rho}^{(o)}\left(\tau,\vec{r},t_n\right)}^{(e)}$$

$$= \frac{\rho\left(m\delta\phi_0\right)\int_0^{2\pi} I_{w1}\left(\phi\right)\eta\left(\vec{r}\right)\iint \hat{\rho}_{ost}\left(T_s,\vec{r},t_n\right)G(\vec{r},r_0,\theta_0)dV}{\int_0^{2\pi} I_{w2}\left(\phi\right)\eta\left(\vec{r}\right)\iint G(\vec{r},r_0,\theta_0)dV} \tag{1a}$$

$$G(\vec{r},r_0,\theta_0) = \frac{\left|W_s\left(r,r_0\right)\right|^2}{\ell^2\left(\vec{r}\right)r^4} f^4\left(\theta-\theta_0\right), \tag{1b}$$

$$I_{w1}\left(\phi,mT_s\right) = I_{w1}\left(\phi\right) = \int_{-\infty}^{t=\phi_0/\alpha} h\left(t-\tau\right) f^4\left(\phi-\alpha\tau\right)\exp\left[-\beta_m\left(\phi-\alpha\tau\right)\right]d\tau\,, \tag{1c}$$

$$I_{w2}\left(\phi\right) = \int_{-\infty}^{t=\phi_0/\alpha} h\left(t-\tau\right) f^4\left(\phi-\alpha\tau\right)d\tau\,, \tag{1d}$$

$$\rho\left(m\delta\phi_0\right) = \exp\left[-\frac{1}{4}\left(\frac{m\delta\phi_0}{\sigma_\phi}\right)^2\right], \tag{1e}$$

$$\delta\phi_0 = \alpha T_s\,, \tag{1f}$$

and

$$h\left(t\right) = \begin{cases} \dfrac{1}{MT_s} & \text{if } 0 \le t \le MT_s \\ 0 & \text{otherwise} \end{cases}\,, \tag{1g}$$

$$f^4\left(\phi-\alpha\tau\right) = \frac{\exp\left[-\left(\phi-\alpha\tau\right)^2 \big/ \left(2\sigma_\phi^2\right)\right]}{\sqrt{2\pi}\sigma_\phi}\,, \tag{1h}$$

$$\beta_m = \frac{\alpha\tau\sin^2\theta_0}{2\sigma_\theta^2} = \frac{\delta\phi_0}{2\sigma_\phi^2}\,, \qquad \sigma_\phi = \frac{\sigma_\theta}{\sin\theta_0}\,, \qquad \sigma_\theta = \frac{\theta_1}{4\sqrt{\ln 2}}\,, \tag{1i}$$

where T_s is the pulse repetition time, $\vec{r}$ the position vector, $\vec{r}_0 \equiv \left(r_0,\theta_0,\phi_0\right)$ the center of V_6 at lagged time mT_s in spherical coordinate system, θ_0 the zenith angle of radar beam axis, t_n the ending time of the n^{th} dwell time MT_s, θ and ϕ the angular distance from beam axis in elevation and azimuth direction respectively, W_s the range weighting function, $\ell\left(\vec{r}\right)$ the range propagation path loss, f^2 the one-way radiated power pattern or an angular weighting function in azimuth direction, α the antenna rotation rate, σ_θ^2 the second central moment of the two-way power pattern in elevation direction, σ_ϕ^2 the second central moment of two-way power pattern in the azimuth direction, $\alpha\tau$ the azimuth displacement in T_s and θ_1 the one-way half power beam width for the stationary beam and $\eta\left(\vec{r}\right)$ is the reflectivity. The diacritical circumflex indicates that it is an estimated value not an expected value. The over bar signifies a volumetric mean weighted by beam pattern and reflectivity. The superscript *e* emphasize that the effective beam pattern is used. $\hat{\rho}_{ost}\left(T_s,\vec{r},t_n\right)$ is correlation coefficient due to hydrometer's oscillation or/and wobbling, turbulence and shear of mean wind across V_6. It is an estimated value not an expected value. It changes from one dwell time to the next because of the fluctuation relating to the turbulence.

In Eq. (1a), the denominator is the total power and the reflectivity at a point, i.e. $\eta\left(\vec{r}\right)$ is weighted by I_{w2}, wherefore I_{w2} is the azimuth effective beam pattern for reflectivity measurement. It differs from I_{w1} that is on the nominator in Eq. (1a) and will weigh and influence high order velocity moments, such as radial velocity and spectrum width. Plugging Eqs. (1g) (1h) into Eq. (1c) and (1d), completing integrations and noting Eq. (1i), we have

$$I_{w1}\left(\phi\right)=\sqrt{2\pi}\sigma_\phi \exp\left[\frac{\left(m\delta\phi\right)^2}{8\sigma_\phi^2}\right] f_{en}^4\left(\phi-\phi_0'\right) \tag{2a}$$

$$I_{w2}\left(\phi\right)=\sqrt{2\pi}\sigma_\phi f_{en}^4\left(\phi-\phi_0\right) \tag{2b}$$

$$\phi_0'=\phi_0-\frac{m\delta\phi}{2} \tag{2c}$$

$$f_{en}^4\left(\phi-\phi_0\right)\equiv\frac{1}{2\alpha MT_s}\left\{erf\left[\frac{1}{\sqrt{2}\sigma_\phi}\left(\phi-\phi_0+\alpha MT_s\right)\right]-erf\left[\frac{1}{\sqrt{2}\sigma_\phi}\left(\phi-\phi_0\right)\right]\right\} \tag{2d}$$

where f_{en}^4 is the two way effective radar power beam pattern that is due to the combination impact of antenna rotation and signal processing. Replacing ϕ_0 with ϕ_0' in Eq. (2d), one obtains

$$f_{en}^4\left(\phi-\phi_0'\right)\equiv\frac{1}{2\alpha MT_s}\left\{erf\left[\frac{1}{\sqrt{2}\sigma_\phi}\left(\phi-\phi_0+\frac{\alpha mT_s}{2}+\alpha MT_s\right)\right]-erf\left[\frac{1}{\sqrt{2}\sigma_\phi}\left(\phi-\phi_0+\frac{\alpha mT_s}{2}\right)\right]\right\}. \tag{2e}$$

This is for high order moment measurement and, in fact, is also the generalized form of the effective two way beam pattern in azimuth direction. It reduces to $f_{en}^4\left(\phi-\phi_0\right)$ when m = 0, which corresponds to the zero lagged time of the correlation coefficient of the radar signal.

2.2 Effective beam width for reflectivity measurement

Using Eq. (2d) and following the procedure given by Doviak and Zrnic (1993), one can find the analytical solution that defines the effective beam width for reflectivity measurement. It is read as

$$erf\left[\frac{1}{\sqrt{2}\sigma_\phi}\left(\phi-\phi_0+\alpha MT_s\right)\right]-erf\left[\frac{1}{\sqrt{2}\sigma_\phi}\left(\phi-\phi_0\right)\right]=\frac{1}{2}erf\left(\frac{\alpha MT_s}{2\sqrt{2}\sigma_\phi}\right). \tag{3}$$

Eq. (3) is very similar to Eq. (7.34) of Doviak and Zrnic (1993), but Eq. (3) is not only the function of radar antenna rotationrate, but also the function of the elevation angle of the radar beam because of Eq. (1i). It can be seen that Eq. (3) will reduce to Eq. (7.34) of Doviak and Zrnic (1993) only if θ_o = 90^o, wherefore the existing equation can be only applied to the beam scanning in a flat surface, such as in a horizontal plain or a vertical plain. For the beam azimuthally scanning in a conical surface, Eq. (3) should be applied. Fig. 1 shows the

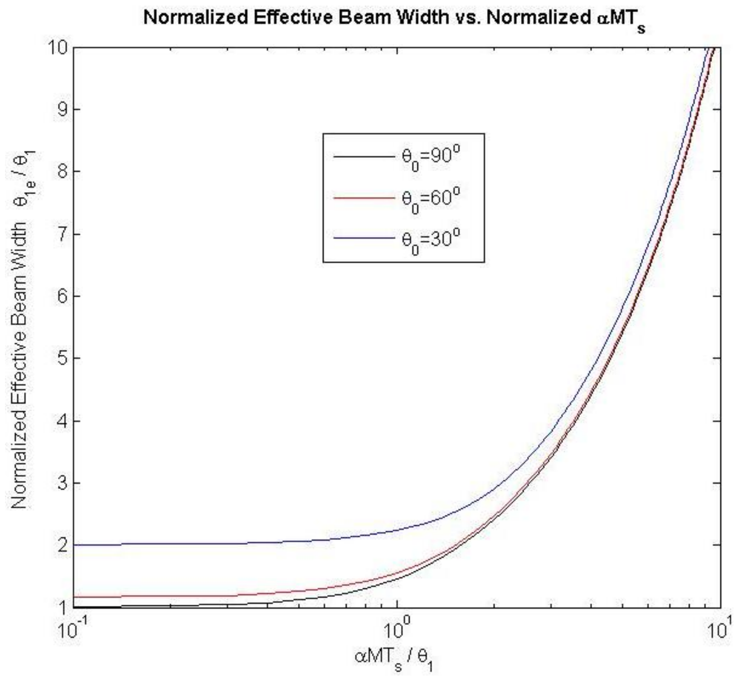

Fig. 1 The dependency of normalized effective beam width on normalized azimuth displacement at different zenith angle.

dependency of the normalized effective beam width on normalized azimuth displacement within a dwell time at three different zenith angles. For the same antenna rotation rate, the effective beam width increases with the increase of the elevation angle.

2.3 Effective beam width for high order moments

Using Eq. (2e) and following the procedure presented in previous section, we obtain the analytical expression that defines effective beam width for high order velocity moment measurement,

$$erf\left[\frac{1}{\sqrt{2}\sigma_\phi}\left(\phi-\phi_0+\frac{\alpha m T_s}{2}+\alpha M T_s\right)\right]-erf\left[\frac{1}{\sqrt{2}\sigma_\phi}\left(\phi-\phi_0+\frac{\alpha m T_s}{2}\right)\right]=\frac{1}{2}erf\left(\frac{\alpha M T_s}{2\sqrt{2}\sigma_\phi}\right) \quad (4)$$

Eq. (4) differs from Eq. (3). It is not only the function of antenna rotation rate and radar beam elevation angle but also the function of lagged time mT_s. However, Eq. (4) will reduce to Eq. (3) if $m \ll 2M$.

3. Spectrum Width Equations Related to FFT and PPP

Substituting Eq. (2a) for Eq. (1a) and Fourier transforming the resultant equation, Fang and Doviak (2008) obtain the expression for normalized Doppler spectrum that can be used to find out the spectrum width equation including various contributors. Among those contributors, the squared spectrum width duo to antenna rotation is written as

$$\sigma_\alpha^2=\int_{-\infty}^{\infty} v^2 S_{n\alpha} dv=\left(\frac{\alpha\lambda\sin\theta_0\sqrt{\ln 2}}{2\pi\theta_1}\right)^2 \quad (5a)$$

$$S_{n\alpha} = \tilde{F}\left[e^{-\frac{1}{8}\left(\frac{\alpha\tau}{\sigma_\phi}\right)^2} \right] = \sqrt{\frac{2\pi\theta_1^2}{\alpha^2\lambda^2 \sin\theta_0^2 \ln 2}} e^{-\frac{2\pi^2\theta_1^2 v^2}{\alpha^2\lambda^2 \sin\theta_0^2 \ln 2}}. \tag{5b}$$

Here, $\tilde{F}$ signifies the Fourier transform; $e^{-\frac{1}{8}\left(\frac{\alpha\tau}{\sigma_\phi}\right)^2} = \rho(\alpha m T_s) \times \exp\left[\frac{(\alpha m T_s)^2}{8\sigma_\phi^2}\right]$ and the exponential function on right side is part of Eq. (2a). However, Eq. (2a) has another term, $f_{en}^4(\phi - \phi_0')$, that is the function of lagged time too as shown in Eq. (2d). In their 2008 work, Fang and Doviak neglect the term $\alpha m T_s/2$. In fact, we do not know if we can neglect this term. The result from $\tilde{F}\left[f_{en}^4(\phi - \phi_0')\right]$ is too complex to obtain an analytical expression and its significance may need to be determined from real observational data, but this discussion implies that in addition to σ_α^2, for the spectrum width obtained using FFT, there should be something else in the spectrum width equation that also relates to antenna rotation and contributes to the spectrum width measurement. That is

$$\overline{\sigma_v^2(\vec{r}, t_n)} = \overline{\sigma_s^2(\vec{r})}^{(e)} + \overline{\sigma_t^2(\vec{r})}^{(e)} + \overline{\sigma_o^2(\vec{r})}^{(e)} + T_c + \sigma_\alpha^2 + something\text{-}else. \tag{6}$$

Here, σ_s, σ_α, σ_o, and σ_t, represent the spectrum widths due to mean wind shear, antenna rotation, hydrometer's oscillation and/or wobbling, and turbulence, respectively; T_c is the coupled term due to the coupling between shear and turbulence. It is noteworthy that the above equation is applicable to the spectrum width obtained from Doppler spectra that is generated using FFT. What is the spectrum width equation for PPP method? For PPP the lagged time or m is fixed in Eq. (2a), and $f_{en}^4(\phi - \phi_0')$ and $\rho(\alpha m T_s) \times \exp\left[\frac{(\alpha m T_s)^2}{8\sigma_\phi^2}\right]$ are no longer the function of the lagged time, but PPP method uses all the lagged-one pairs in a dwell time, wherefore the spectrum width equation should same as Eq. (6) for PPP method. However, PPP needs to assume a symmetric Doppler spectrum, if this assumption is not satisfied, the spectrum width obtained using PPP might be different from that obtained from FFT.

4. Summary and conclusions

Starting from the correlation coefficient at the output of the integrator, this study shows that for a scanning beam the radar effective beam width is not only a function of the radar antenna rotation rate, but also a function of the elevation angle for the reflectivity measurement. The effective beam width is even the function of the lagged time for the measurement of high order velocity moments, such as radial velocity and squared spectrum width. However, if $m \ll 2M$, this equation will reduce to that for the reflectivity measurement. The existing expression that defines the effective beam width is only applicable to the radar beam scanning in a flat surface, such as in a horizontal or vertical plain. For the beam scanning in a conical surface, the effective beam should be calculated using the equation given in this study. This study also shows that for different signal processing method, such as FFT and PPP, the spectrum width equations are different from each other. For the FFT method, in addition to σ_α^2, the spectrum width equation should include something else that also related to the antenna rotation and contribute to the spectrum width measurement. Whether or not this extra contribution is important needs to be determined by analyzing radar measurements. For PPP method, because the lagged time is fixed, there will be no σ_α^2 and extra contribution due to radar antenna rotation. These differences found in this study is particularly important when radar measured spectrum width is used to retrieve turbulence in clouds.

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