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CIRCULATION AND AREAL CONTRACTION RATE AS DETECTED AND MEASURED BY DOPPLER RADAR

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INTRODUCTION

- Davies-Jones and Stumpf (1977, 28th Conf. on Radar Meteorology, pp. 313-314) advocated using the detection and measurement of significant circulation around and areal contraction rate of a curve as a potential method for giving advanced warnings of tornadoes.
- Circulation (rate of areal contraction) is the line integral around a closed curve of velocity tangential (inward normal) to the curve.
- Circulation and areal contraction rate may be more useful than differential single-Doppler velocities in the characteristic velocity couplet for detecting and measuring the strength of convergent tornadic mesocyclones at low altitudes because of the following reasons:
  a. Doppler rotational velocity (half the velocity difference between the two peaks in the velocity couplet) does not separate vortical and convergent flow,
  b. circulation and areal contraction rate are (1) less scale dependent, (2) are tolerant of noisy Doppler velocity data, and (3) are relatively insensitive to range and azimuth.

3-D VELOCITY COMPONENTS OF THE RADAR TARGETS

- Since a Doppler radar scans the 3-D velocity components of a target with a volume scan, it is expeditious and computationally economical to determine the kinematic properties of the velocity field in the conical surfaces of constant elevation angle.
- Mathematically, the velocity components defined in a right-hand spherical coordinate system are

$$\mathbf{V} = V_R\hat{\mathbf{R}} + V_\alpha\hat{\boldsymbol{\alpha}} + V_\beta\hat{\boldsymbol{\beta}}, \qquad (1)$$

where V_R is the radial component of the target in the radar viewing direction, V_α and V_β are both perpendicular to the radar viewing direction. V_α is along the upward inclined normal in the vertical plane containing the radar beam, and V_β is horizontal and perpendicular to the right of the beam in a constant α_o-surface.
- Here, $\hat{\mathbf{R}}$, $\hat{\boldsymbol{\alpha}}$, $\hat{\boldsymbol{\beta}}$ are the unit basis vectors of the spherical coordinate system.

CIRCULATION

- By Stokes' theorem, a circulation Γ around the boundary C of an area A in the α_o-surface is given by

$$\Gamma = \oint_C (V_R dR + V_\beta R\cos\alpha_o d\beta), \qquad (2)$$

where the first term on the right-hand side of (2) is the observed circulation around C with V_β ignored.
- The so-called "cell circulation" Γ_{ob} around the i,k grid cell is computed from the first term on the right-hand side of (2) at the midpoint of each cell as

$$(d\Gamma_{i+1/2,k+1/2})_{ob} = \frac{\Delta R}{2}[V_R(R_i,\alpha_o,\beta_{k+1}) + V_R(R_{i+1},\alpha_o,\beta_{k+1}) - V_R(R_{i+1},\alpha_o,\beta_k) - V_R(R_i,\alpha_o,\beta_k)], \qquad (3)$$

where i is the index in the range direction, k is the index in the azimuth direction, and $\Delta R = R_{i+1} - R_i$ is the range increment.
- Circulation in (3) is additive because the circulation around the outer perimeter of a union of contiguous grid cells is simply the sum of the circulations around the perimeter of each grid cell.
- Thus, the circulation around C by the line-integral method of Davies-Jones (1993, MWR, vol. 121, pp. 713-725) is computed, given by

$$\Gamma = \frac{\Lambda(V_R,R)}{2} - \cos\alpha_o\frac{\Lambda(RV_\beta,\beta)}{2}, \qquad (4)$$

where $\Lambda(V_R,R)/2$ is the observed circulation, and $\Lambda(f,g) \equiv (f_1g_2 - f_2g_1 + f_2g_3 - f_3g_2 + \cdots + f_{2M+3}g_{2M+4} - f_{2M+4}g_{2M+3} + f_{2M+4}g_1 - f_1g_{2M+4})$.
- The method fits a piecewise-linear velocity field to the observation (linear between adjacent points on the circle).

AREAL CONTRACTION RATE

- The divergence (δ) in a constant α_o-surface is percentage areal expansion per unit time of the fluid element, given by

$$\delta = \frac{1}{A}\frac{dA}{dt}, \qquad (5)$$

where A is the area of the fluid element, and $dA/dt \equiv A_t > 0\ (<0)$ is the rate at which the area of the curve expands (contracts).
- For a horizontal convergent flow, negative areal expansion rate ($A_t < 0$) corresponds to contraction.
- The element of area A in (5) on the α_o-surface (Hildebrand 1962, p. 304) is

$$dA = \cos\alpha_o R\, dR\, d\beta. \qquad (6)$$

- Differentiating (6) with respect to time t and integrating by parts give the rate of areal expansion (the negative of the areal contraction rate) on the α_o-surface,

$$A_t = -\cos\alpha_o\oint_C V_R R d\beta + \oint_C V_\beta dR, \qquad (7)$$

where the first and second terms on the right-hand side of (7) are the observed and unobserved effluxes.
- Using Davies-Jones' line-integral method, the expansion rate of the area A enclosed by C is

$$A_t = -\cos\alpha_o\frac{\Lambda(RV_R,\beta)}{2} + \frac{\Lambda(V_\beta,R)}{2}, \qquad (8)$$

where the last term on the right-hand side of (8) is unobserved.
- Ignoring the last term on the right-hand side of (7), the so-called "cell areal expansion rate" represents the observed expansion rate of the i,k grid cell calculated at the midpoint of the cell and is therefore obtained by

$$\left(\frac{dA_{i+1/2,k+1/2}}{dt}\right)_{ob} = \frac{1}{2}\cos\alpha_o(\beta_{k+1} - \beta_k) \times \{R_{i+1}[V_R(R_{i+1},\alpha_o,\beta_{k+1}) + V_R(R_{i+1},\alpha_o,\beta_k)] - R_i[V_R(R_i,\alpha_o,\beta_{k+1}) + V_R(R_i,\alpha_o,\beta_k)]\}. \qquad (9)$$

SIMULATED DOPPLER RADAR & RANKINE MODEL

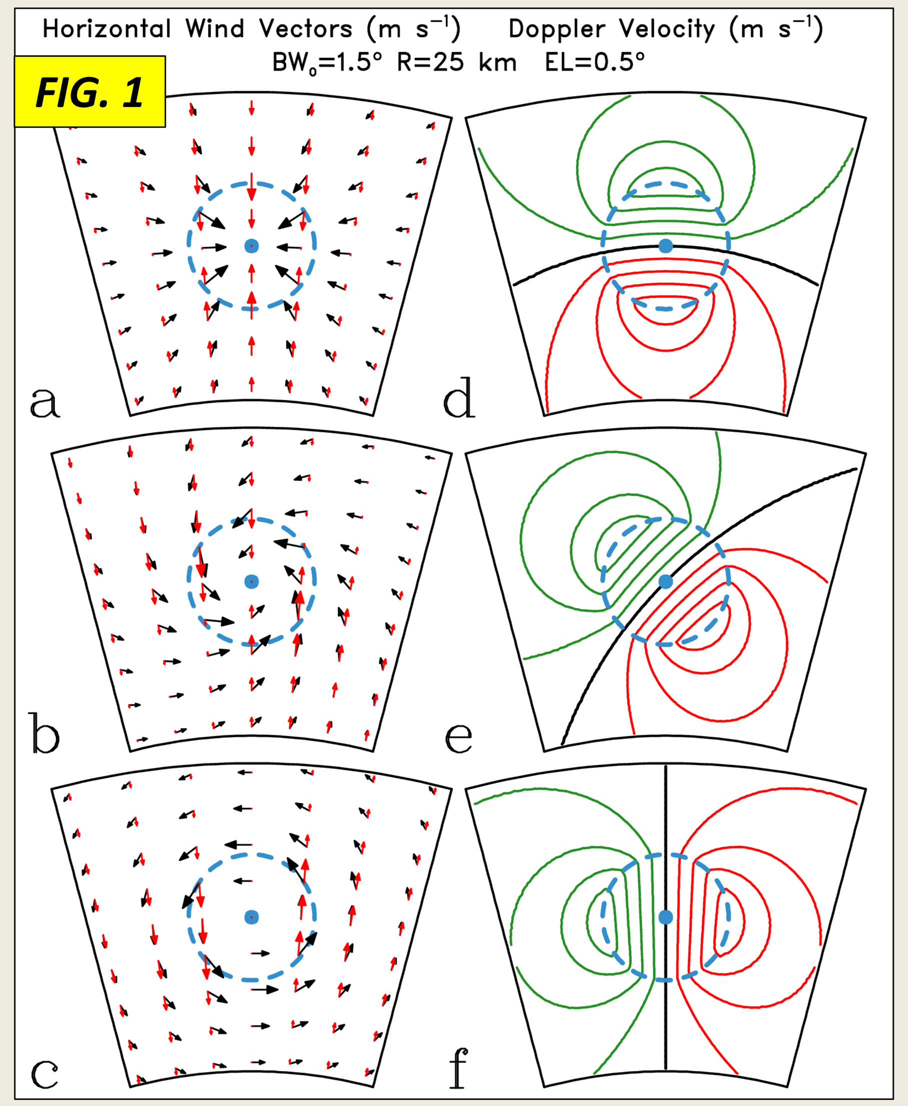
- Simulated Doppler velocities are produced by scanning a radar across Rankine vortical and convergent flows.
- In (1), V_R at lowest elevation angle is computed from

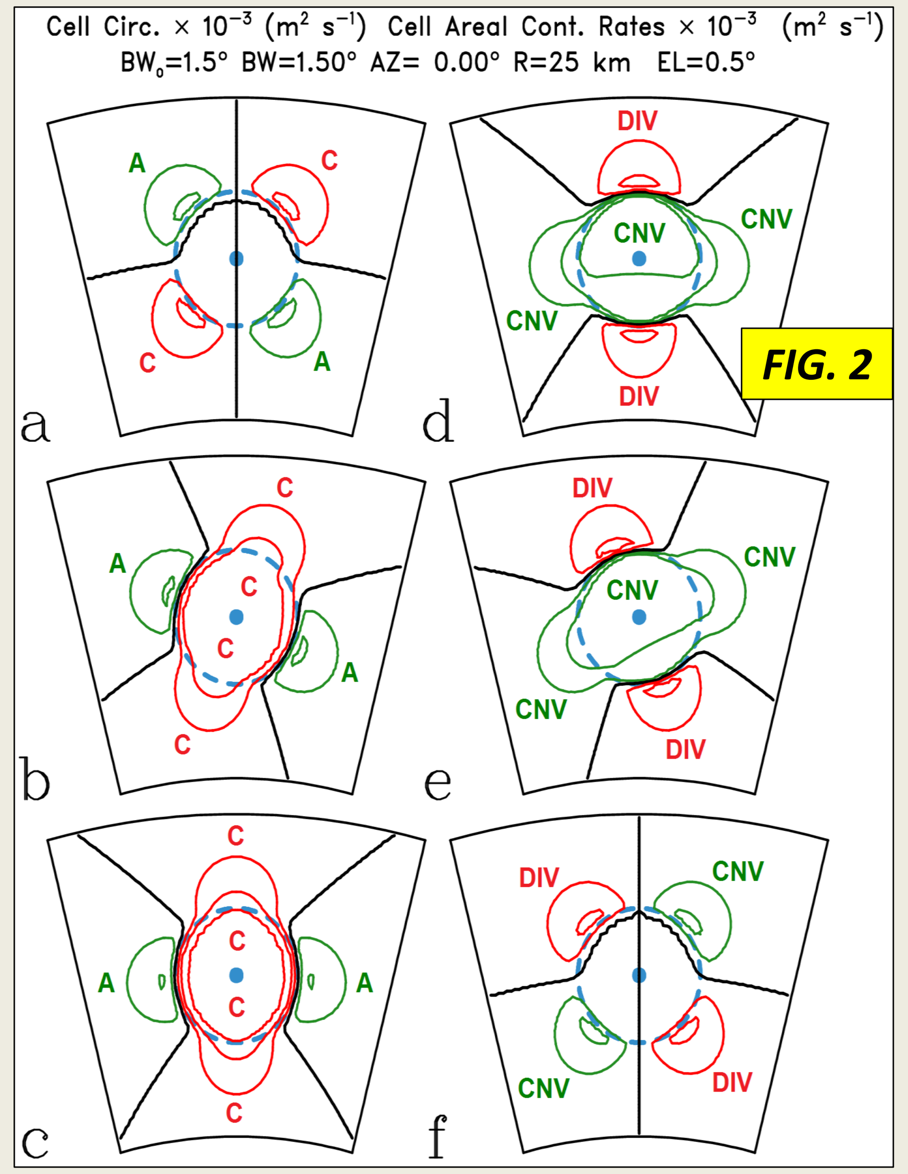
$$V_R(R_i,\alpha_o,\beta_k) = (U\sin\gamma + V\cos\gamma)\cos\alpha_o, \qquad (10)$$

where U is the radial component of an axisymmetric convergent flow, V is the tangential component of an axisymmetric cyclonic vortex, and γ is the angle between the tangential direction component V and the radar viewing direction β_k at a point.
- U and V can be modeled using the Rankine formulas, given by

$$U = U_x\left(\frac{\rho}{R_x}\right)^\varepsilon, \quad V = V_x\left(\frac{\rho}{R_x}\right)^\varepsilon \qquad (11)$$

where U_x, V_x are, respectively, radial and tangential velocity peaks, R_x is core radius, ρ is radius from center, $\varepsilon(=+1,-1)$ is the exponent that governs the inner velocity ($\rho \le R_x$) and outer velocity profiles ($\rho > R_x$), respectively.





CONCLUSIONS AND ON-GOING WORK

- Circulation and areal contraction rate are useful because (a) they help separate vortical and convergent flows, and (b) circulation around a circle is a better measure of vortex strength than Doppler rotational velocity.
- A potential vortex (the outer part of the Rankine vortex) and a sink each produce spurious "quadrupole" patterns in both observed vorticity and divergence. These patterns produce false readings in the potential vortex.
- For a circle centered on the vortex, the quadrupole contributions to circulation and areal contraction rate cancel.
- In our on-going work, we will demonstrate that circulation is much less range and azimuth dependent than Doppler rotational velocity is.

FIG. 1 *Simulated, storm-relative horizontal (black) and Doppler (red) wind vectors with (a) convergent flow (U_x=-25 m s^{-1}, V_x=0), (b) combined convergence and rotation (U_x=-17.8 m s^{-1}, V_x=17.8 m s^{-1}), and (c) rotational flow (U_x=0, V_x=25 m s^{-1}) are plotted at 0.5°-elevation angle. Simulated, positive (negative) Doppler velocities represent flow away from (toward) the radar, shown by red (green) contours with contour interval of 5 m s^{-1}. Simulated, zero Doppler velocity contour (black) represents flow perpendicular to the radar viewing direction. Blue Doppler signature center is located at 25 km north of a simulated Doppler radar. Blue, dashed circle represents an axisymmetric Rankine core diameter ($2R_x$) of 5 km with its radial (U_x) and tangential (V_x) velocity peaks. Range increment is 240 m. The Doppler velocity pattern in (d)-(f) rotates counterclockwise, reflecting a change from (a) convergent flow through (b) combined convergence and rotation to (c) rotational flow.*

FIG. 2 *Cell circulations in (a)-(c) and cell areal contraction rates in (d)-(f) are calculated. Positive (red) and negative (green) contours with zero (black) contour are indicated with contour interval of 0.5 m^2 s^{-1}. **C** stands for Doppler cyclonic shear; **A** for Doppler anticyclonic shear; **CNV** for Doppler convergence; and **DIV** for Doppler divergence, as they correspond to red Doppler wind vectors (Figs. 1a-c).*

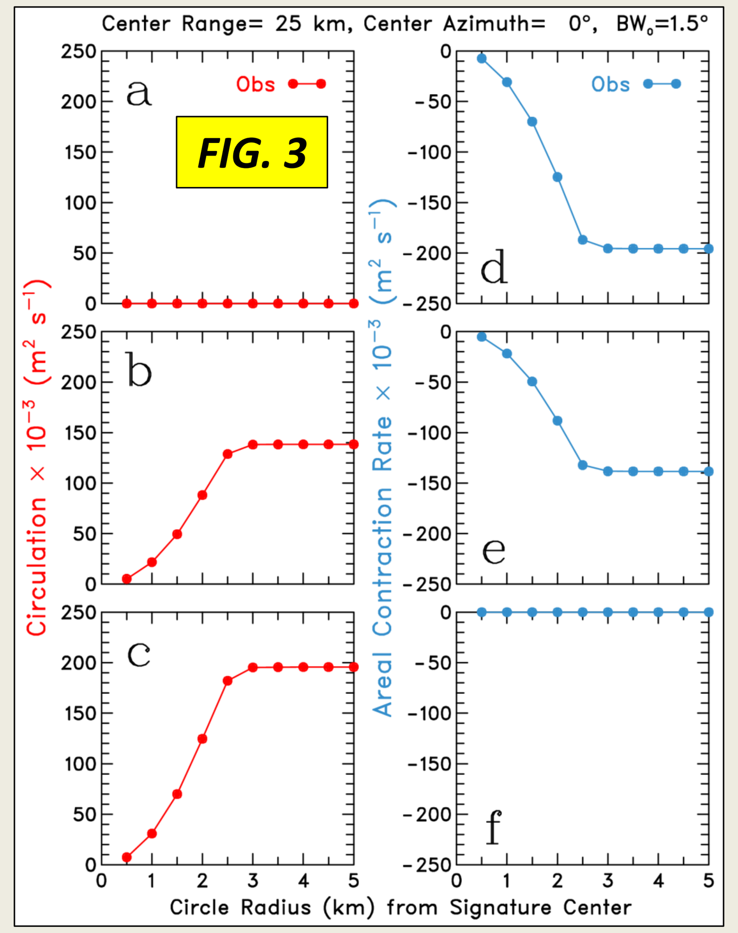


FIG. 3 *Circulations [red dotted curves in (a)-(c)] and areal contraction rates [blue dotted curves in (d)-(f)] are functions of circle radius from the signature center, as are calculated from the first term on the right-hand side of (4) and (8), respectively.*

37th Conference on Radar Meteorology
American Meteorological Society
16 September 2015, Norman, OK

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