1. INTRODUCTION

Accurate characterization of raindrop size distribution (DSD) and the estimation of DSD parameters over large spatial and temporal scales is a long-standing goal for the studies based on polarimetric radar measurements (e.g., Gorgucci et al. 2008). Indeed, many studies have proposed methods to estimate DSD parameters as a part of rainfall rate estimation algorithms. However, most of those methods require external reference data such as 2DVD measurements to derive relations that are essential for the DSD parameter retrieval algorithms (e.g., Gorgucci et al. 2002; Brandes et al. 2003; Kalogiros et al. 2013). Making use of the physical-based ad hoc or empirical relations derived from the reference data may cause error because of the raindrop temperature, shape, and size distribution dependency when used in different conditions, including seasons, locations, and precipitation types. Moreover, the fact that the sampling volume of the reference data may cause error because of the method ad hoc or empirical relations derived from the DSD retrieval algorithm that does not require external reference DSD data is needed to estimate DSD parameters and rainfall rate from polarimetric measurements with high reliability.

In the present study, we propose a new algorithm to estimate the three DSD parameters and rainfall rate for polarimetric radar at the attenuation frequency using consistency among the polarimetric measurements. The proposed algorithm requires no external reference data such as 2DVD measurements for attenuation corrections because it retrieves co-polar and differential specific attenuation from the interrelation among the polarimetric measurements.

2. METHODOLOGY

The attenuation correction algorithm used in the present study assumes that the raindrop temperature and operating frequency are given and that all polarimetric variables including attenuation are determined by the DSD, which is represented by the modified gamma distribution. Also, no ice hydrometeors (such as hail and/or graupel) are assumed to be included in the range profile. The attenuation-corrected \( Z_a \) and \( Z_{on} \) obtained with the proposed method are used to derive the three DSD parameters, i.e., the shape parameter \( \mu \), the median volume diameter \( D_0 \), and the raindrop concentration \( N_0 \) as shown below.

### 2.1 Attenuation Correction

Horizontal reflectivity \( Z_h \) and vertical reflectivity \( Z_v \) can be expressed in terms of the raindrop concentration \( N_0 \) as

\[
Z_{hv} = N_0 \times F_{hv}(\mu, D_0), \tag{1}
\]

where

\[
F_{hv}(\mu, D_0) = 10^{i\kappa} \times \frac{\lambda^4}{\pi^3} \int_{D_{min}}^{D_{max}} 4\pi K(D) D^4 \exp\left(-\frac{3.67 + \mu}{D_0} D\right) dD. \tag{2}
\]

Note in the derivation of Eq. (1), \( N_0 \) was moved from the inside of integral in Eq. (2) to the first term of Eq. (1) because \( N_0 \) is a constant and is independent of \( D \).

The differential reflectivity, \( Z_{dr} \) is given from Eq. (1) by

\[
Z_{dr} = 10\log_{10} \left( \frac{N_0 \times F_{hv}(\mu, D_0)}{N_0 \times F_{hv}(\mu, D_0)} \right) = G_{hv}(\mu, D_0). \tag{3}
\]

Equation (3) clearly shows that \( Z_{dr} \) is independent of \( N_0 \). Similarly, the specific attenuation of horizontal polarization \( A_h \) can be expressed in terms of \( N_0 \) as

\[
A_h = N_0 \times B_h(\mu, D_0), \tag{4}
\]

where

\[
B_h(\mu, D_0) = 8.686 \times 10^{-3} \lambda \times \text{Im} \int_{D_{min}}^{D_{max}} f_{hv}(D) D^4 \exp\left(-\frac{3.67 + \mu}{D_0} D\right) dD. \tag{5}
\]

Additionally, the differential specific attenuation \( A_{dr} \) can also be expressed in terms of \( N_0 \) as

\[
A_{dr} = N_0 \times B_{dr}(\mu, D_0), \tag{6}
\]

where

\[
B_{dr}(\mu, D_0) = 8.686 \times 10^{-1} \lambda \times \text{Im} \int_{D_{min}}^{D_{max}} \{ f_{hv}(D) - f_{hv}(D) \} D^4 \exp\left(-\frac{3.67 + \mu}{D_0} D\right) dD. \tag{7}
\]
The terms of $A_H/Z_r$ and $A_D/Z_r$ are given from Eqs. (1), (4), and (6) by

$$\frac{A_H}{Z_H} = \frac{N_r \times B_{rH}(\mu, D_0)}{N_r \times F_H(\mu, D_0)} = L_H(\mu, D_0),$$

and

$$\frac{A_D}{Z_H} = \frac{N_r \times B_{rD}(\mu, D_0)}{N_r \times F_H(\mu, D_0)} = L_D(\mu, D_0).$$

Eqs. (3), (8), and (9) show that both $A_H/Z_r$ and $A_D/Z_r$ can be expressed as a function of $Z_{\text{refr}}$ by use of $D_0$ as an intermediate variable for a given value of $\mu$, as shown in Fig. 1.

Figure 1 indicates that the consistency curve of $A_H/Z_r$ has low temperature and shape-parameter dependencies, especially for C-band. However, this figure also shows that the consistency curve of $A_D/Z_r$ has slightly larger dependencies not only on temperature but also on shape parameter, especially for X-band. These small dependencies on both temperature and shape parameter could make large differences in the retrieval of rainfall rate, particularly in heavy rainfall, because the attenuation effects are defined as path integrals of the co-polar and differential specific attenuation given by

$$Z_{\text{H}}^{\text{obs}}(r) = Z_{\text{H}}^{\text{true}}(r) - 2 \int_{s}^{1} A_H(s) ds - C_H$$

and

$$Z_{\text{D}}^{\text{obs}}(r) = Z_{\text{D}}^{\text{true}}(r) - 2 \int_{s}^{1} A_D(s) ds - C_D,$$

where $Z_{\text{H}}^{\text{true}}(r)$ (dBZ) and $Z_{\text{D}}^{\text{true}}(r)$ (dB) represent true reflectivity and differential reflectivity after attenuation correction at a range of $r$, respectively, $r_i$ is the distance of the first range resolution volume, and $C_H$ and $C_D$ are the correction terms for the reflectivity and differential reflectivity profiles, respectively.

We have opted for a simple gate-to-gate attenuation correction scheme based on (Aydin et al. 1989). The true reflectivity and differential reflectivity at range $r_i$ can be obtained by recurrence formulas derived from Eqs. (10) and (11) as

$$Z_{\text{H}}^{\text{true}}(r_i) = Z_{\text{H}}^{\text{obs}}(r_i) + 2 \sum_{n=1}^{N_{\text{r}}-1} A_H(r_i, r_{n+1}) \delta s_n + C_H,$$

and

$$Z_{\text{D}}^{\text{true}}(r_i) = Z_{\text{D}}^{\text{obs}}(r_i) + 2 \sum_{n=1}^{N_{\text{r}}-1} A_D(r_i, r_{n+1}) \delta s_n + C_D,$$

where $r_n$ represents the distance of the $n$th range gate, $\delta s$ (km) is the range resolution of the radar measurements and

$$A_H(Z_{\text{H}}^{\text{true}}(r_i), Z_{\text{D}}^{\text{true}}(r_i)) = A_D(Z_{\text{H}}^{\text{true}}(r_i), Z_{\text{D}}^{\text{true}}(r_i)) = 0.$$

Note that the co-polar and differential specific attenuation are inferred from true $Z_r$ and $Z_{\text{H}}$ with the consistency curves in Figs. 1a and 1b, respectively. The term $C_{\text{D}}$ is the sum of relative bias error in $Z_{\text{H}}$. and excess differential attenuation. In the method used here, it is also assumed that the systematic bias in $Z_{\text{H}}$ measurements is negligible (by calibrating with vertical measurements in rain) and that excess differential attenuation can be neglected assuming that both the $H$ and $V$ signal powers are affected almost equally by rain on the radome. Therefore, the value of $C_{\text{D}}$ is set to zero, as assumed in Bringi et al. (2006). Similarly, the bias $(C_H)$ in the observed $Z_r$ corrected with the proposed method is the sum of the radar constant calibration error and any excess attenuation from the radar to the first range resolution volume, including excess attenuation due to rain on the radome, as shown in Eqs. (12) and (14). Note that the value of $C_H$ could vary with the beam direction and time because it contains excess attenuation due to the wet radome of the antenna. We estimated the value of $C_H$ by use of the autocalibration of $Z_r$, introduced by Goddard et al. (1994).

![Fig. 1. (a) Relationships of horizontal specific attenuation per unit linear horizontal reflectivity and (b) specific differential attenuation per unit linear horizontal reflectivity as a function of differential reflectivity at raindrop temperatures of 10 °C and 20 °C at C-band (5.370 GHz) and X-band (9.375 GHz) with shape parameters of 0 and 5 for a modified gamma distribution with the axis ratio of Brandes et al. (2005).](image-url)
2.2 Retrieval of the DSD Parameters

The DSD parameters are derived from the attenuation-corrected Z_D and Z_H observed with the proposed method. The shape parameter is estimated by comparing the theoretical \( \Phi_D \) with the smoothed-observed \( \Psi_D \) through a rain path in the radial direction, as shown later in Section 3. Once the shape parameter is determined, the median volume diameter \( D_0 \) can be derived from the attenuation-corrected \( Z_{2H} \) at each range gate because \( Z_{2H} \) is independent of \( N_0 \) and is a function of \( D_0 \) and \( \mu \), as given by Eq. (3). Then, \( N_0 \) can be derived from the true \( Z_{2H} \) with the retrieved \( D_0 \) and \( \mu \) from Eq. (1). Other rain parameters including rainfall rate can be derived theoretically from the DSD parameters.

3. MRI C-BAND POLARIMETRIC RADAR AND RETRIEVAL OF DSD PARAMETERS

3.1. MRI C-BAND POLARIMETRIC RADAR

The Meteorological Research Institute (MRI) advanced C-band solid-state polarimetric radar (MACS-POL radar) is mounted on top of the MRI building in Tsukuba, Japan (Adachi et al. 2013). The radar routinely collects a full suite of dual-polarization measurements, including the reflectivity factor (Z_H), differential reflectivity (Z_{2H}), differential propagation phase (\( \Psi_{DP} \)), and correlation coefficient at zero lag (\( \rho_{HV}(0) \)). This system employs two solid-state amplifier units to transmit horizontally and vertically polarized waves. The radar is operating in the simultaneous transmission and reception (STAR) mode for polarized waves. The radar is operating in the simultaneous transmission and reception (STAR) mode for polarized waves. The radar is operating in the simultaneous transmission and reception (STAR) mode for polarized waves.

### Table 1. Operating characteristics of the MRI advanced C-band solid-state polarimetric radar.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>5370 MHz</td>
</tr>
<tr>
<td>Occupied band width</td>
<td>&lt; 4.5 MHz</td>
</tr>
<tr>
<td>Peak power</td>
<td>3.5 kW (for each channel, simultaneous transmission)</td>
</tr>
<tr>
<td>Duty</td>
<td>20 % (Max)</td>
</tr>
<tr>
<td>Pulse length</td>
<td>1 μs (range &lt; 20 km) and 129 μs (≥ 20 km) for Elv. &lt; 8° and 1 μs (range &lt; 7.5 km) and 47 μs (≥ 7.5 km) for Elv. ≥ 8°</td>
</tr>
<tr>
<td>Pulse compression</td>
<td>Linear FM chirp for long-pulse observations</td>
</tr>
<tr>
<td>Antenna diameter</td>
<td>Parabolic dish, ( \Phi = 4 ) m</td>
</tr>
<tr>
<td>Antenna speed</td>
<td>4 rpm for Elv. &lt;8° and 6 rpm for Elv. ≥ 8°</td>
</tr>
<tr>
<td>Signal minimum</td>
<td>&lt; –110 dBm</td>
</tr>
<tr>
<td>Antenna gain (H and V)</td>
<td>&gt; 42 dB</td>
</tr>
<tr>
<td>Beam width</td>
<td>1.01°</td>
</tr>
<tr>
<td>Azimuth spacing</td>
<td>0.7°</td>
</tr>
<tr>
<td>Transmitter</td>
<td>GaAs Power FET</td>
</tr>
<tr>
<td>Number of linear sampling</td>
<td>30</td>
</tr>
<tr>
<td>Range Gate Spacing</td>
<td>150 m</td>
</tr>
<tr>
<td>PRF</td>
<td>624/780 Hz (Elv. &lt; 8°) and 936/1170 Hz (Elv. ≥ 8°)</td>
</tr>
<tr>
<td>Observation parameters</td>
<td>( Z_{DR}, Z_{V}, ) radial velocity, ( \rho_{HV}(0) ) and ( \Psi_D )</td>
</tr>
<tr>
<td>Vendor</td>
<td>TOSHIBA</td>
</tr>
</tbody>
</table>

3.2. Autocalibration and Retrievals of Rain Microphysical Parameters

The radar reflectivity field observed by the MACS-POL radar at 0754 JST on 3 December 2010 indicates that a very heavy convective rain line was approaching the MRI site from the southwest with a speed of about 18 m s⁻¹ (Fig. 2). This figure shows that the rain line passed at this time over Sekiyado (SYD), where a Parsivel optical disdrometer (Löfler-Mang and Joss 2000) was installed. Thus, we explore radial profiles of the radar data at an azimuth of 279° in the experiments so that the radial profile extends toward the Sekiyado site, as shown in the figure by the thick line.

Range profiles of observed \( \Psi_{DP} \), running mean of observed \( \Psi_{DP} \), and theoretical \( \Phi_{DP} \) estimated from attenuation corrected \( Z \) and \( Z_{2H} \) with the proposed method are shown in Fig. 3. We applied a running mean to the observed \( \Psi_{DP} \) to mitigate the high frequency fluctuations including the backscatter differential phase \( \delta \) and retain the mean trend for ease of viewing. Note that this running mean applied to the observed \( \Psi_{DP} \) does not have any influence on the theoretical estimations of \( \Phi_{DP} \). The values of measured \( Z \) were scaled so that the theoretical \( \Phi_{DP} \) fits the smoothed-observed \( \Psi_{DP} \) with range using the autocalibration technique.

Figure 3 shows a good agreement with range between the smoothed-measured \( \Psi_{DP} \) and theoretical \( \Phi_{DP} \) with attenuation correction procedures profiles.
A correction factor of 0.98 dB was applied to the values of measured $Z$ for this profile. This correction factor ($C_H$) could reflect a bias due to the wet radome of the antenna. Indeed, the correction factor for $Z$ needed to match the two profiles varies with time from 0.00–0.98 dB in the comparisons. This range of bias variation in $Z$ due to a wet radome agrees well with the result of Thompson et al. (2011). In the retrievals of the theoretical $\Phi_{DP}$ profiles, we assumed a raindrop temperature of 10 °C and a shape parameter of 5. The temperature was estimated from surface observations at Sekiyado with the dry adiabatic lapse rate, and the value of the shape parameter was estimated by comparing the profiles of smoothed-measured $\Psi_{DP}$ and theoretical $\Phi_{DP}$ as shown below.

In the retrieval of the DSD parameters, we assume that the shape parameter is constant in a range profile, although this assumption may not be satisfied if the radar is sampling mixed convective/stratiform echoes that simultaneously exist in a single profile. Because the shape parameter is one of the parameters that determine the DSD, it influences attenuation, which may affect the profiles of theoretical $\Phi_{DP}$. We found that dependency of the theoretical $\Phi_{DP}$ profile on the shape parameter is evident when multiple rainfall peaks existed in a rain path in the radial direction. An example of a $\Phi_{DP}$ range profile with attenuation correction procedures is shown in Fig. 4. The gradient of the smoothed-measured $\Psi_{DP}$ profile becomes flat locally with range at around 40 km, suggesting that this range was located between heavy rainfall regions. Note that in that range, the smoothed-measured $\Psi_{DP}$ profile locates between the theoretical $\Phi_{DP}$ profiles with shape parameters of 0 and 8, despite the fact that the values of the theoretical $\Phi_{DP}$ at the first and last range gates coincide with those of the smoothed-measured $\Psi_{DP}$. The shape parameter of 5 made all the theoretical $\Phi_{DP}$ profiles analyzed in the comparisons fit best with the smoothed-measured $\Psi_{DP}$ profiles associated with the line-shaped convective system. This value may reflect the mean of the shape parameter of this storm. Indeed, the mean value of $\mu$ derived from the disdrometer data measured at Sekiyado by the method proposed by Zhang et al. (2003) for the rain associated with the passage of the storm indicates almost the same value, as shown in Fig. 4.

### 3.3. Comparison of the Retrieved Microphysical Parameters of Raindrops with Disdrometer Measurements

To evaluate the reliability of the rain microphysical parameters retrieved with the DSD auto-retrieval technique proposed, we compared these with those derived from the Parsivel disdrometer measurements at Sekiyado, which is located about 31.8 km west-northwest of the MRI site (Fig. 2). Because this type of disdrometer has been reported to have an overestimation tendency, especially in heavy (e.g., $R>30$ mm h$^{-1}$) rainfall (e.g., Thurai et al. 2011; Tokay et al. 2013), we reprocessed and applied a quality control to the disdrometer data (for the details, see Appendix A of Adachi et al. (2013)) before the comparisons. The radar-estimated microphysical parameters available for the single point nearest the Sekiyado station were used for the comparisons.
Time series of (a) rainfall rate, (b) reflectivity, (c) differential reflectivity, and (d) median volume diameter derived from disdrometer measurements (thick line) and estimated from polarimetric radar data (circles) at the Sekiyado station from 0645 to 0815 JST on 3 December 2010. $D_0$ ($Z_{wp}$) in (d) was derived by use of Bringi et al. (2006).

The comparisons show that the parameters derived with the proposed method generally have good agreement with measurements on the ground, suggesting that the DSD estimated with this method is also reliable. A sample comparison of DSD estimated with the proposed method appears in Fig. 6. This comparison was made at the Kumagaya (67.9 km from the radar) site (Fig. 2) during a passage of typhoon, where attenuation effect was much severer than that of the rain-line event. Black dots represent the DSD data measured on the ground with a Parsivel disdrometer, while thick blue line (dashed red line) represents the DSD estimated with our method (Marchall-Palmer DSD without attenuation correction). Note the DSD derived with the proposed method is represented by a straight line because the value of the shape parameter was estimated to 0. The DSD estimate with our method agrees well with the disdrometers measurements especially for the raindrops with the diameter more than 2 mm. As a result, the estimated rainfall rate is very close to the disdrometers measurements with an error of less than 3% despite large attenuation associated with heavy rainfall. In contrast, the DSD estimated from the measured reflectivity with the radar differs substantially...
from the ground observation. Consequently, the rainfall rate is seriously underestimated (−82%), suggesting unreliability of the estimated rainfall rate based solely on the Z–R relation with conventional radar.

4. CONCLUSION

We developed an algorithm for rain attenuation correction of the reflectivity factor and differential reflectivity measured by polarimetric radar at attenuating frequency to retrieve DSD parameters and rainfall rate. It does not require any assumptions of relationship among DSD parameters and/or simplifications of relationship between the axis ratio and diameter of raindrops, which were used in previous studies. Moreover, the proposed algorithm needs no external reference data such as 2DVD measurements for attenuation corrections because it retrieves the co-polar and differential specific attenuation from interrelation among the polarimetric measurements. Additionally, the algorithm retrieves three parameters of the modified gamma distribution, from which rain parameters including rainfall rate can be theoretically estimated.

The performance of this algorithm was evaluated by comparison with optical disdrometers. The evaluation of the algorithm showed fairly good agreement between the retrieved three DSD parameters of raindrops and both reflectivity and differential reflectivity with those obtained by surface measurements. Additionally, the algorithm demonstrated significant improvement in performance for rainfall rate estimation compared with rates estimated using the so-called Z–R relationship.

For details of this study, see Adachi et al. (2015).

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