Compute Bulk Density of winter events based on 2DVD Measurements

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1. INTRODUCTION

Quantitative Precipitation Estimation (QPE) of winter event is very difficult for radar meteorology due to the complexity of variability in particle habits and density. The winter precipitation may include rain, different types of ice crystal and different type of snowflake. Each of these precipitation particles could have very different micro-physical properties. The size of winter precipitation particle can be as small as tens micron (i.e. ice crystal) or as large as hundreds mini-meter (i.e. aggregated snowflake), the shape can be geometric shape as spheroid or conical (i.e. rain drop and conical graupel) or very irregular (i.e. aggregated snow) and the density can be as low as less than 0.1 g cm^{-3} (fluffy snow) or as high as 1 g cm^{-3} (rain drop). Amount these micro-physical properties, the most fundamental property is the density of precipitation. From the hydrology perspective, for given a set of particle size distribution (PSD), the liquid-equivalent snow rate (*SR*) is proportional to the particle density.

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From the meteorological radar perspective, all of radar measurements relate to dielectric constant and dielectric constant can be derived from density (M. E. Tiuri et al. 1984; Huang et al. 2010). Therefore, the density of precipitation plays an important role for QPE of winter event.

It is straightforward to globally estimate precipitation density by combining disdrometer with other instrument. Brandes et al. (2007) used two dimensional video disdrometer (2DVD; Schönhuber et al. 2000) to measure the PSD and co-located Geonor gauge to estimate the power law relationship of density and median volume diameter (D_0) for Colorado fluffy dry snow. Huang et al. (2010) used 2DVD measured PSD and adjusted the coefficient and exponent of density-size power law relation to match the disdrometer's equivalent reflectivity (Z_e) with King City radar horizon reflectivity factor (Z_h). They were able to find the density-apparent diameter (D_{app}) relation for 7 winter events from Canadian CloudSat/CALIPSO Validation Project (C3VP). Recently, Huang et al. (2014) applied hydrodynamic theory (Böhm 1989) to 2DVD single camera data to compute the precipitation particle density on flake-by-flake basis. They analyzed 4 winter events from Light Precipitation Validation Experiment (LPVEx) and obtained the Z_e-SR

relationship for these events. After applied these relationships to operational ground radar, the 2DVD based accumulated *SR* reasonably agreed with Finnish Meteorological Institute (FMI) gauge network.

Although Huang et al. (2014) are able to compute precipitation density on particle-by-particle basis using single camera data; the single camera data set only offers limited information of particle (Huang et al. 2014). This single camera data set does not include the detail of shape. Therefore, some important parameters, such as the area ratio (A_r) and the area of minimum circumscribed ellipse, are only estimated. In this paper, we propose a method to compute particle density based on the actual contour measured by 2DVD. We compare 5 minutes averaged density of this method and single camera method with the results from gauge.

2. METHODOLOGY

a. Mass and fall speed

The fall speed (V_f) of a freely falling particle relates to its mass. When the particle reaches to its terminal fall speed, the drag force is equal to the gravity. There is a common set of equations (Böhm 1989; Heymsfield and Westbrook 2010) to describe the relation between V_f and mass of winter precipitation particle as:

$$\begin{cases} V_f = \dfrac{\eta Re}{\rho_{air} D} \\ Re = \dfrac{\delta_0^2}{4}\left[\left(1 + \dfrac{4\sqrt{X}}{\delta_0^2\sqrt{C_0}}\right)^{1/2} - 1\right]^2 \\ X = \dfrac{8mg\rho_{air}}{\pi\eta^2}\left(\dfrac{A}{A_e}\right)^{\gamma} \end{cases} \dots\dots (1)$$

where η is viscosity, ρ_{air} is air density, *Re* is Reynolds number, *X* is Davies number (also called Best number), *m* is the mass of particle and *g* is gravity constant. The *D* in the first equation of Eq. 1 is the characteristic dimension to represent the size of particle. The A_e in third equation of Eq. 1 is the area of particle projected to anti air flow plane, and *A* is the area of minimum circumscribed ellipse (or circle) which can contain A_e completely. The A_e/A is the area ratio (A_r) which represents the shape factor. The drag coefficient (C_D) can be computed (Abraham 1970) as:

$$\begin{aligned} C_D &= C_0\left(1 + \frac{\delta}{D}\right)^2 \\ &= C_0\left(1 + \frac{\delta_0}{Re^{1/2}}\right)^2 \end{aligned} \dots\dots\dots (2)$$

where C_0 is inviscid drag coefficient, δ is the boundary layer thickness and δ_0 is a dimensionless coefficient.

Theoretically C_0 and δ_0 (or C_D) are the function of *Re*. Böhm (1989) used equi-area circle diameter (D_e) of *A* as characteristic dimension and found that C_0=0.6, δ_0=5.83 and γ=0.25 (the order of A_r; see in Eq. 1) can estimate the V_f for different types of solid precipitation particle with general agreement (Fig.4 of his paper). Heymsfield and Westbrook (2010) used maximum diameter (D_{max}) which is the maximum point-to-point length on particle's contour as characteristic dimension and γ=0.5 and found that C_0=0.35 and δ_0=8.0 given the best fit for previous studies of V_f. Note when the minimum circumscribed ellipse is a circle (the major axis is equal to minor axis), D_e will be equal to D_{max}.

To compare Böhm (1989) equations with Heymsfield-Westbrook (2010) equations, we first assumed that Reynolds number is from 0.1 to 10000. In the first equation of Eq. 1, the *Re* is proportional to characteristic dimension. We used the data from 3 BAECC cases (see section 3a) to show the relation between D_{max} and D_e. Upper panel of Figure 1 shows the colored contour of D_{max} versus D_e along with mean and $\pm 1\sigma$ (black line). The linear fit of mean value is $D_{max} = 1.665D_e+0.2913$. To simplify the computation, we let $D_{max}=1.67D_e$. Insert this relation into the first and second equations of Eq. 1. For given V_f (*Re*), the Davies number of Heymsfield-Westbrook is 1.82 (*Re*=0.1) to 1.64 (*Re*=10000) times of Böhm's (bottom panel of Figure 1). Note that the distribution of D_{max} in each D_e size bin is not symmetric along with mean value but skewed to higher D_{max}. The actual Davies number from Heymsfield-Westbrook could be larger than these numbers. According to the third equation of Eq. 1, the mass is proportional to *X* and A_r^{γ}. The A_r is dependent on types of precipitation. For higher A_r, *X* will dominate the mass estimation. On the other end, A_r^{γ} will play more important role.

b. MATCHING PROCEDURE

Eq. 1 estimates the V_f from the known mass of particle. In our application we revise the equations to estimate the mass from 2DVD measured V_f. Therefore, the accuracy of measured fall speed is extremely important. 2DVD measures V_f by matching the particle image from 2 views. Huang et al. (2010) show that the manufacture's matching algorithms for snow event has serious miss-matching problem. Subsequently, it will cause, in average, overestimation of V_f. In order to avoid this overestimation, they developed a matching algorithms based on Hanesh's (1999) snow shape study. We adapt this method to our matching procedures. Figure 2 shows the concept of our matching procedures as following:

- Obtain the detail information (D_{app}, V_f, horizontal and vertical resolutions, contour of particle from two views and so on) by running manufacture's programs (the circle with horizontal lines in Figure. 2).
- Apply Hanesch's (1999) scan line criteria to take out possible miss-matched particles (overlap of two line filled circles in Figure. 2).
- Take out those particles which have only one scan line or have only one horizontal pixel width. Those particles have severe digitalized error on contour and are not able to fit the minimum circumscribed ellipse.
- Hanesch's scan line criteria allow the difference of height between 2 views to count for digitalized error and particle rotation. We discard those particles whose difference of height is more than 2 scan lines (digitalized error) after rotating contour from -15^{o} to 15^{o}. The remaining particles are the confirmed matched particles (the green round in Figure 2).

c. DISTORTION DUE to HORIZONTAL MOVEMENT of PARTICLE

2DVD records falling particles with a line scan camera. When a particle falls through the optical plane of camera and has a constant horizontal speed, all of the masked pixels will be shifted by a constant distance which is depend on the fall speed and horizontal moving speed. The shape of particle will be distorted because of this shifting. This distortion will cause D_{max} (green line in Figure. 3) and minimum circumscribed ellipse changed (red line in Figure. 3). However, the total amount of masked pixels will maintain the same. So D_{app} will not change. Since the horizontal movement changes the shape of 2DVD images and characteristic dimension, it may induce the error of mass estimation. Unfortunately, the tendency of error depends on the velocity (both direction and intensity) of horizontal movement. As the best knowledge we had, there is no way to correct this distortion for asymmetric particle. So it is important to put 2DVD inside of double-fence international reference (DFIR) to reduce the effect of wind.

3. BAECC TEST SITE and CASE STUDIES

a. Test Site

Biogenic Aerosols Effects on Clouds and Climate (BAECC) field campaign was co-operation of University of Helsinki, the Finnish Meteorological Institute, Colorado State University, and the United States Department of Energy Atmospheric Radiation Measurement (ARM) program from 1 February to 12 September 2014. The test bed is at University of Helsinki Hyytiälä Forestry Field Station, Finland (61°50'37.114"N, 24°17'15.709"E) and equipped with 2DVD, Particle Image Package (PIP), Pluvio^2 200/400 and second ARM Mobile Facility (AMF2) which include X-, Ka- and W-band cloud radar. Figure 4 shows the instruments installed on the test site. The PIP is a new version of Snow Video Imager (SVI). It can capture particle image and compute its fall speed. Based on the image, PIP can also estimate the PSD. There are two snow gages at test site, Pluvio^2 200 and Pluvio^2 400. The Pluvio^2 200 was co-located with 2DVD inside DFIR. We used this snow gage as the reference. We select 3 cases from BAECC field campaign to examine our method. The cases we selected is 12 February 2014 from 05:00 to 09:00 (UTC), 15-16 February 2014 from 22:30 (15) to 01:00 (16), and 21-22 February 2014 19:00 (21) to 07:00 (22).

b. CASES STUDIES

By the definition, the bulk density (ρ) is the total mass divided by total volume in certain time period. The total volume can be obtained by integrating D^3 over PSD (i.e. from PIP) and total mass can be measured by snow gage (i.e. Pluvio). Since PIP can also measure fall speed, we can further compute the volume flux weighted (also called fall speed weighted) bulk density (ρ_v) as:

$$\rho_v = \frac{\int m(D)V_f(D)N(D)dD}{\int D^3 V_f(D)N(D)dD} \ldots\ldots\ldots (3)$$

In Eq. 3, $m(D)$ is the mass-size power-law relationship which can be estimated by combining PIP estimated PSD ($N(D)$) and Pluvio measured mass and using the method similar to Brandes et al. (2007). It is clear that the numerator of Eq. 3 is proportional to the liquid equivalent snow rate. The ρ_v is more correlated to SR than ρ. Moreover, it is well known that $V_f(D)$ is increasing as size increasing (Atlas and Ulbrich 1977; Atlas et al. 1973; Huang et al. 2015), and $\rho(D)$ is decreasing as size increasing (Huang et al 2010; Huang et al. 2015). If we use bulk density instead of density-size relationship, we will underestimate *SR* and *Z* for small ice precipitation particles and overestimate *SR* and *Z* for large particle. It also well known that *SR* and *Z*, especially *Z*, are dominated by large size precipitation particle. Therefore, overall, both *SR* and *Z* will be overestimated by using bulk density. Since ρ_V is weighted by fall speed, it will tend to agree with large particle. The volume flux weighted bulk density is more suitable to compute *SR* and *Z*. The PIP+Pluvio derived ρ_V are the light blue (Pluvio^2 200) and dark blue (Pluvio^2 400) solid lines in Figure 5.

The 2DVD derived ρv is computed in particle-by-particle basis. Assumed *N* matched particles were observed from *T* to $T+\Delta T$ (ΔT is 5 minutes in this study). ρ_V was computed as:

$$\rho_v = \frac{\sum_{i=1}^{N} m_i * v_i}{\sum_{i=1}^{N} V_i * v_i} \dots\dots\dots (4)$$

where m_i is the mass, V_i is volume and v_i is fall speed of ith particle. The mass of each particle was estimated by three different methods as:

- Use single camera method (Huang et al. 2015) to compute mass of each matched particles (Figure 2 circle with vertical lines; the single camera data).
- Get contouring data of matched particles (Figure 2 green round) and find the minimum circumscribed ellipse and circle. Compute mass of each matched particle by the equations of Böhm (1989) and Heymsfield-Westbrook (2010).
- In every 5 minutes interval, compute bulk density by total fall-speed weighted mass divided by total fall-speed weighted volume.

Figure 5 shows the results. The green dash line is the ρ_V derived from single camera. If we use Pluvio^2 200 as reference, this density match the bottom value of reference. ρ_V derived from Heymsfield-Westbrook (2010) equations is magenta dash line and catch the higher value of reference. The red dash line is the ρ_v using Böhm equations which is in between single camera method and Heymsfield-Westbrook method. All three methods generally agree with PIP+Pluvio measurements.

4. CONCLUSIONS and FUTURE WORKS

In this paper, we show that we can compute the density of winter precipitation particle on particle-by-particle basis by using 2DVD measured fall speed and images. Three 5-minute 2DVD volume flux weighted bulk densities compare with PIP+Pluvio volume flux weighted bulk density and they are in general agreement. Heymsfield-Westbrook equations give the highest bulk density and agree with higher PIP-Pluvio bulk density. The single camera method has the lowest density. The best equations should be used may be depend on the type of precipitation. In fact, the boundary layer thickness (δ or δ_0) and inviscid drag coefficient (C_0) all depend on the roughness and shape of precipitation particle. With actual shape observed by 2DVD, we now are able to study these dependencies and the error structure of hydrodynamic equations.

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FIGURES

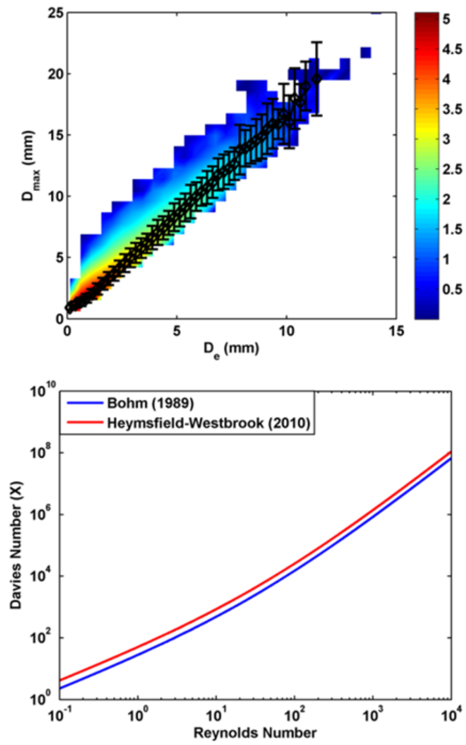

Figure 1: (Upper) Colored contour of D_{max} vs. D_e along with mean and standard deviation (Black line) of D_{max} in each D_e size bin. (Bottom) The Böhm (blue) and Heymsfield-Westbrook (red) Davies number vs. Reynolds number assumed that $D_{max} = 1.67*D_e$.

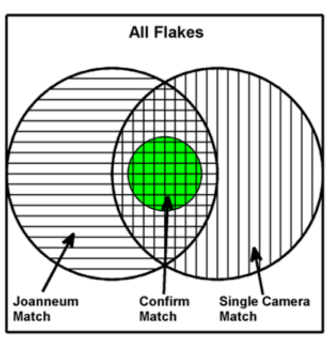

Figure 2: Concept of matching procedure.

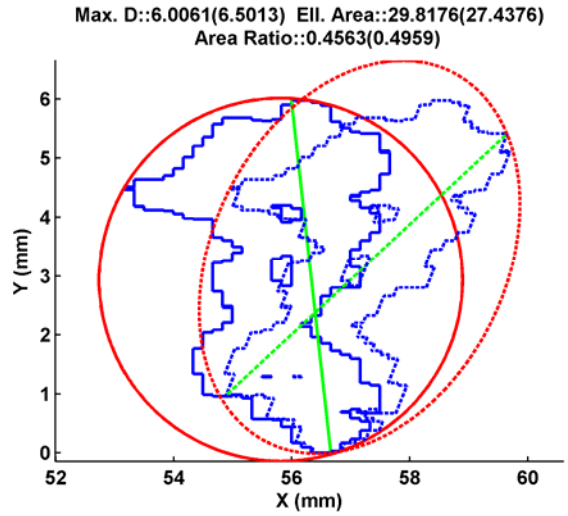

Figure 3: A example of snowflake falling in 0.7 m/s and observed by 2DVD. The blue line is the contour of flake, green line is maximum diameter and red line is the minimum circumscribed ellipse. Solid line represents the true shape of flake. Dash line is the distorted shape by 0.3 m/s horizontal movement toward to positive X direction. On the figure title, the number inside bracket is measured from distorted shape.

Figure 4: Instruments install in BAECC test site. Note that Pluvio^2 200 was inside the DFIR. We used it as reference.

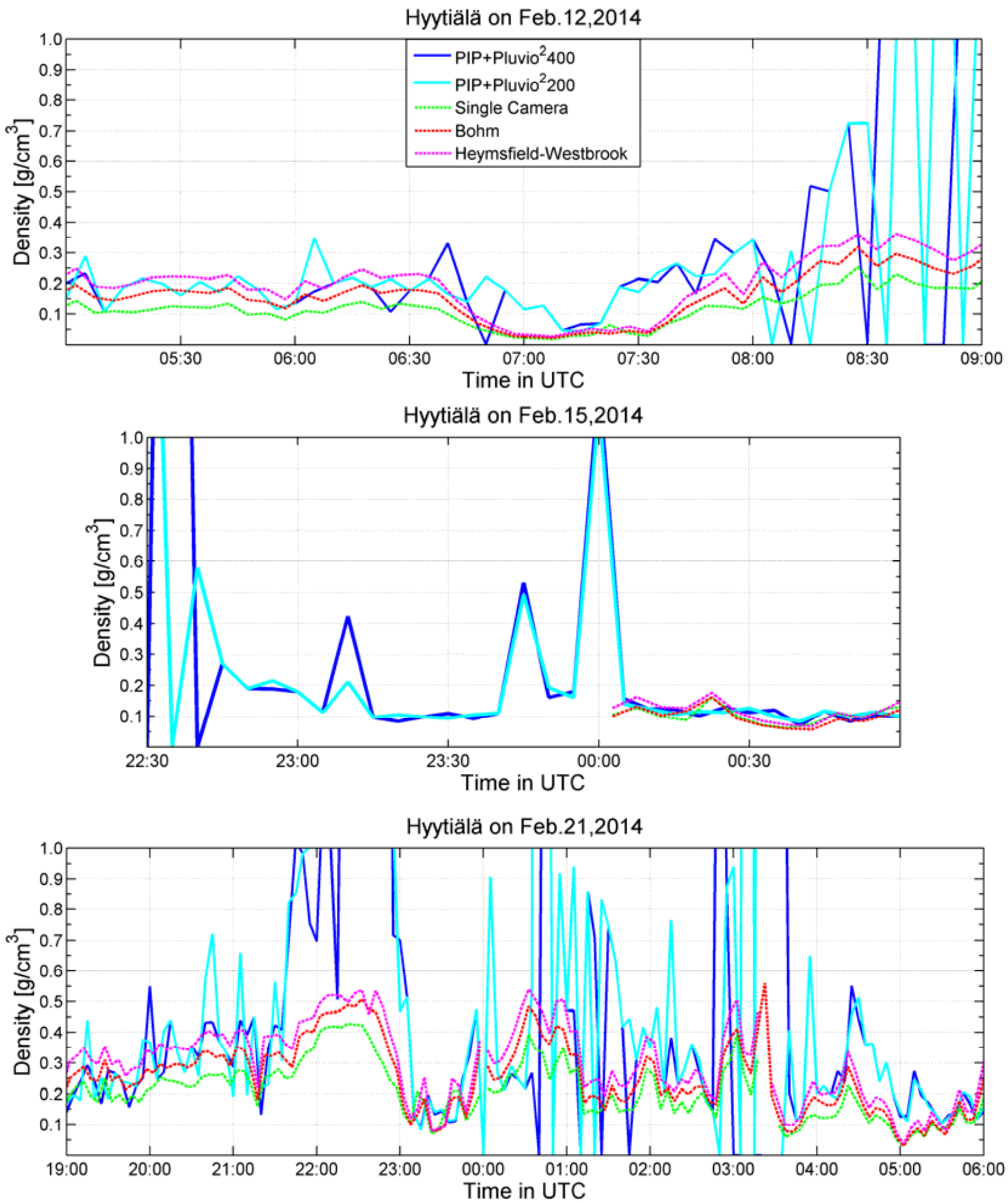

Figure 1: Results of 3 cases from BAECC field campaign. The Solid lines are the volume flax weighted bulk density based on PIP+Pluvio method. The dash lines the volume flux bulk density derived from 2DVD using single camera (green), Böhm equations (red) and Heymsfield-Westbrook equations (magenta).

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