IS ADAPTIVE PSEUDOWHITENING COMPATIBLE WITH NEW RADAR-VARIABLE ESTIMATORS?

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1. INTRODUCTION

Adaptive pseudowhitening is a range oversampling processing technique that can reduce observation times without increasing the variance of estimates or that can decrease the variance of estimates using the same observation times. Range oversampling techniques consist of sampling the received signals at a rate faster than the inverse of the transmitted pulse, which produces complex voltages that are correlated in range. The range samples are then transformed with a linear transformation to decorrelate the signals leading to more precise estimates of the radar variables after averaging. Adaptive pseudowhitening applies a different transformation at each range gate that adjusts to the characteristics of the signals and attempts to minimize the variance of estimates.

The current adaptive pseudowhitening implementation relies on explicit expressions for the variance of all the radar-variable estimators. Some recently introduced radar-variable estimators exhibit improved statistical properties compared to conventional estimators. However, they may not have an explicit expression for their variance, rendering them incompatible with the current implementation of adaptive pseudowhitening.

To address this, we introduce a framework that utilizes optimization to produce lookup tables based on a one-parameter version (denoted by p) of adaptive pseudowhitening that can replace the explicit variance expressions for new radar-variable estimators.

2. BACKGROUND

The main idea behind range oversampling processing is using a linear transformation to decorrelate the time series data so that averaging the autocorrelations results in a reduction in estimate variance by the oversampling factor, L. The linear transformation can be written as

$$\mathbf{X} = \mathbf{W}\mathbf{V} \tag{1}$$

where $\mathbf{V}$ is an L-by-M matrix of time series data, $\mathbf{W}$ is an L-by-L linear transformation matrix, $\mathbf{X}$ is the L-by-M matrix of transformed time series data, and M is the number of samples in the dwell. The time series matrix $\mathbf{V}$ corresponds to data from a particular range gate. If the linear transformation fully decorrelates the data, it is called whitening. At high signal-to-noise ratios (SNRs), whitening performs well and reduces the variance by about a factor of L. At low SNRs, the whitening transformation increases the noise, which leads to degraded performance. Fig. 1 shows this noise enhancement effect.

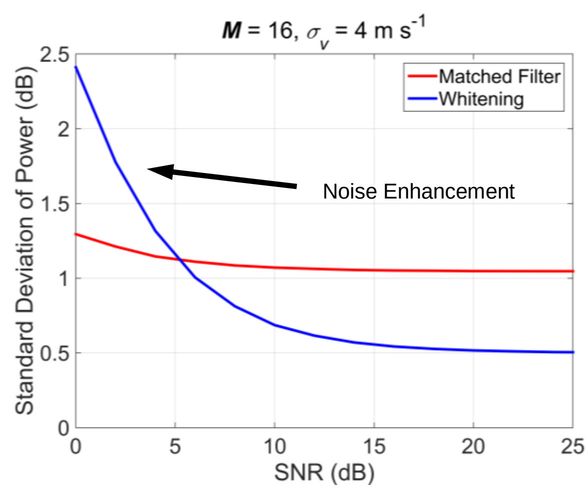

Figure 1. Standard deviation of power for the matched filter and whitening estimators.

To deal with the noise enhancement at low SNR, we developed a new range oversampling processing technique, adaptive pseudowhitening, that acts like the matched filter at low SNR, whitening at high SNR, and better than either estimator in between (Curtis and Torres 2011). The original version of adaptive pseudowhitening uses variance expressions that depend on the

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linear transformation, **W**. The variance expressions have the following form:

$$\mathrm{Var}\left(\hat{\theta}\right)=D\Big[A\ \mathrm{tr}\left(\left(\mathbf{W}^{*}\mathbf{C}_{V_H}\mathbf{W}^{T}\right)^{2}\right)+ B\ \mathrm{tr}\left(\left(\mathbf{W}^{*}\mathbf{C}_{V_H}\mathbf{W}^{T}\right)\left(\mathbf{W}^{*}\mathbf{W}^{T}\right)\right)+C\ \mathrm{tr}\left(\left(\mathbf{W}^{*}\mathbf{W}^{T}\right)^{2}\right)\Big], \quad (2)$$

where θ is the meteorological variable under consideration and $\mathbf{C}_V$ is the normalized range-correlation matrix of the time-series data before the linear transformation (Curtis and Torres 2014). The A, B, C, and D constants are variable-specific. For example, the constants used for the signal power estimator are: $A=1/2\sigma_{vn}\pi^{1/2}$, $B=2/\mathrm{SNR}_0$, $C=1/\mathrm{SNR}_0^2$, and $D=S^2/ML^2$, where σ_{vn} is the normalized spectrum width, SNR_0 is the signal-to-noise ratio at the output of the digital receiver (linear units), and S is the signal power (linear units). The D constant is useful to accurately estimate the variance but is not needed for the minimization, so the A, B, and C constants are used to find the linear transformation that minimizes (2). For the signal power, velocity, and spectrum width estimators, these constants depend on two values: the normalized spectrum width σ_{vn} and the SNR at the output of the digital receiver SNR_0.The dual polarization variance expressions also depend on Z_{DR} and ρ_{HV} in addition to σ_{vn} and SNR_0.

For conventional pseudowhitening, σ_{vn} and SNR_0 are estimated using matched filtered data. The estimates are then used to find a nearly optimal linear transformation. This transformation can be found in terms of **W**, but we can also use the efficient implementation of adaptive psedowhitening described in Curtis and Torres 2011. In this implementation, the transformation is split into two parts: a unitary matrix and a weight vector. The unitary matrix is applied to the time series data (like **W**). The clutter filter can then be applied to the partially transformed data. This is significant because the clutter filter does not have to be applied to data corresponding to each of the radar variables (since the optimal linear transform is different for each estimator). This can reduce the computational complexity, especially when there are several radar variables being estimated. After clutter filtering, the autocorrelations are then calculated. Finally, the radar-variable-specific part of the transformation is applied using a weight vector. This weight vector is the result of minimizing (2) using Lagrange multipliers. For conventional adaptive pseudowhitening, the elements of the weight vector, **d**, are computed using the following formula (Curtis and Torres 2011):

$$d_l=g\frac{\lambda_l}{A\lambda_l^2+B\lambda_l+C} \quad (3)$$

where $0 \le l \le L$, g is a power-preserving constant, and the λ_l are the eigenvalues of the normalized range correlation matrix. A, B, and C are the radar-variable-specific constants from the variance expression. Fig. 2 shows conventional adaptive pseudowhitening compared to both whitening and matched filtering.

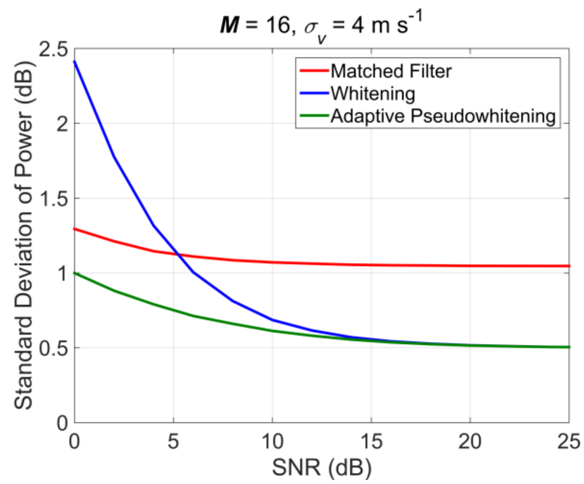

Figure 2. Standard deviation of power for the matched filter, whitening, and adaptive pseudowhitening estimators.

Adaptive pseudowhitening performs like whitening at high SNR, better than the matched filter at low SNR, and better than both in between. As shown in equation (3), the weight vector part of the transformation depends on the variance expression. If there is no explicit variance expression available, we need to find another formula for the weight vector. We propose using a lookup table (LUT) version of adaptive pseudowhitening called LUT adaptive pseudowhitening.

3. LUT Adaptive Pseudowhitening

Based on equation (3), the most natural way to form the lookup tables would be to have three lookup tables for A, B, and C. After studying several possibilities, the simplest method is to use a different one-parameter formula to find the weight vector. This simplifies the optimization when running the Monte Carlo simulations. The one-parameter formula is based on the sharpening filter (Torres et al. 2004):

$$d_l = g\frac{\lambda_l}{(p\lambda_l + (1-p))^2} \tag{4}$$

where everything is the same as equation (3) except that there is only one parameter, p. The p parameter can vary from 0 to 1 where 1 corresponds to whitening, and 0 is close to the digital matched filter.

Just as the A, B, and C values depend on σ_{vn} and SNR_0 (and on Z_{DR} and ρ_{HV} for the dual polarization variables), the p parameter will also. We chose to include M as an independent variable since the variance expressions in general can have an M dependence, but we normally use the approximations that do not include M for the variance expressions. There are three steps for generating the lookup table:

- Simulate 50,000 realizations for different sets of conditions while varying the number of samples (M), the SNR_0, and the normalized spectrum width σ_{vn} (and Z_{DR} and ρ_{HV} for dual polarization variables)
- For each set of conditions, find the optimal p parameter that minimizes the variance of the estimates
- Store the values of p in a look-up table for later use

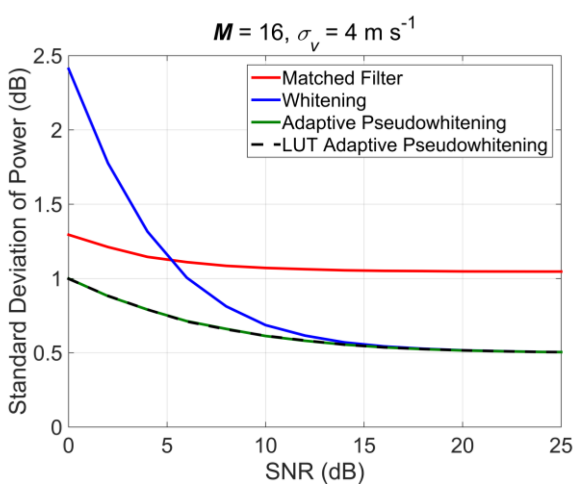

Figure 3. Standard deviation of power for the matched filter, whitening, and both conventional and LUT adaptive pseudowhitening estimators.

To validate LUT adaptive pseudowhitening, we can use an estimator that has an explicit variance expression. In this case, we decided to use the signal power estimator that was used in Fig. 2. The results with LUT adaptive pseudowhitening are shown in Fig. 3. LUT adaptive pseudowhitening performs nearly identically to conventional pseudowhitening. At least in this case, LUT adaptive pseudowhitening seems to work very well and does not need an explicit variance expression.

4. HYBRID SPECTRUM WIDTH ESTIMATOR

As mentioned in the introduction, some estimators have variance expressions that are difficult to derive. One example is the hybrid spectrum width estimator that is currently implemented on the NEXRAD network. For this paper, we will test LUT adaptive pseudowhitening on the hybrid spectrum width estimator. Since there is no explicit variance expression for this estimator, we are unable to validate it directly as we did for the signal power estimator. Instead, we can compare the performance to the conventional R0/R1 estimator with adaptive pseudowhitening and the hybrid spectrum width estimator without adaptive pseudowhitening. The results are shown in Fig. 4.

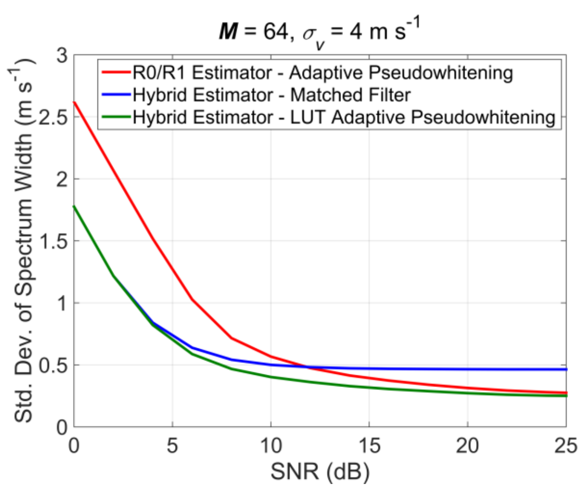

Figure 4. Standard deviation of power for the R0/R1 estimator with adaptive pseudowhitening and the hybrid spectrum width estimator with matched filter and LUT adaptive pseudowhitening.

Even though we do not know if the LUT adaptive pseudowhitening version of the hybrid spectrum width estimator is optimal, it performs better than both the R0/R1 estimator with adaptive pseudowhitening and the matched filter version of the hybrid estimator. One interesting thing to note is the nearly identical performance of the two hybrid spectrum width estimators at low SNR. This occurs because the matched filter version of the hybrid estimator uses the sharpening version of the matched filter with p = 0. This version of the matched filter tends to perform better than the digital matched filter using the eigenvector of the normalized range correlation matrix that corresponds to the maximum eigenvalue.

5. CONCLUSIONS

This research shows that we can still use adaptive pseudowhitening even if there is no explicit variance expression. A lookup table version called LUT adaptive pseudowhitening can be utilized instead. The lookup table is produced using Monte Carlo simulations and a one-parameter version of the weight vector from the efficient implementation of adaptive pseudowhitening. This extends adaptive pseudowhitening to just about any conceivable radar-variable estimator.

6. REFERENCES

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