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Retrieving 2D Wind Field from Aliased Doppler Data by Means of Sliding Windows

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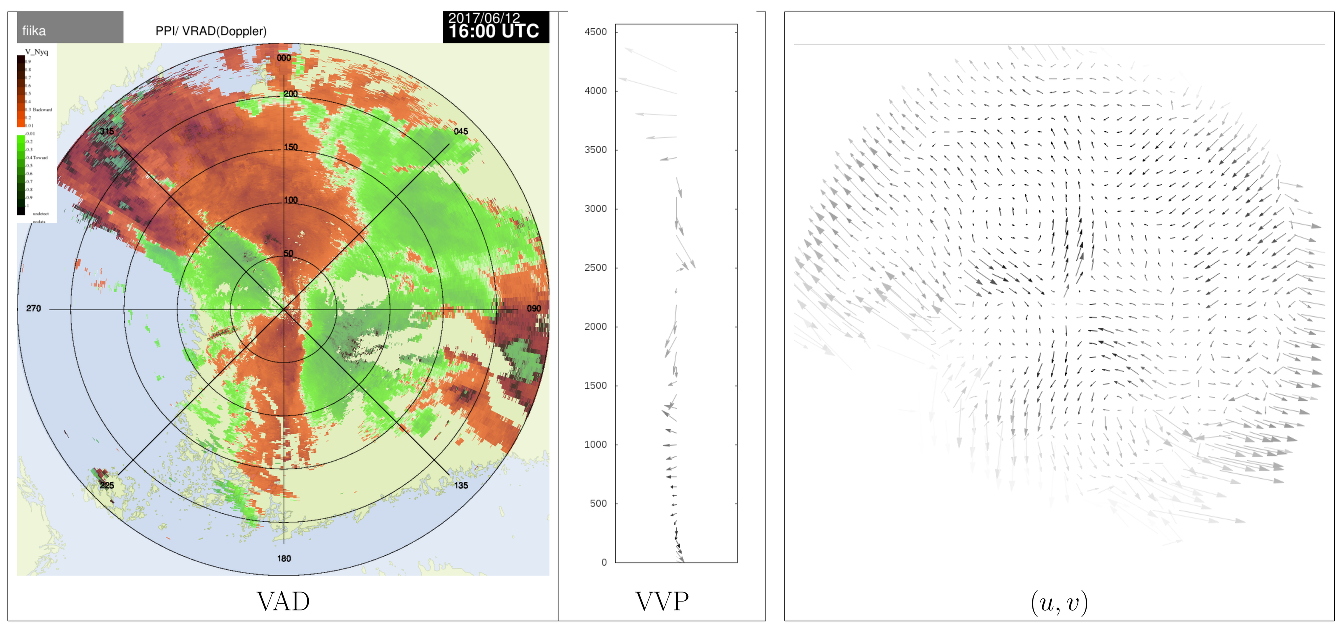
Introduction

In addition to estimating precipitation, Doppler weather radars are routinely used for measuring winds. The most popular wind products are Velocity Azimuth Display (VAD) providing a nearly horizontal intersection of wind field and Velocity Volume Processing (VVP) which averages horizontal wind at altitude ranges above the radar.

However, perceiving actual wind field by visual inspection of VAD requires some experience. This is due to well-known limitations of Doppler:

- Winds are projected on the radar beam, hence reduced to one dimension
- Values exceeding unambiguous speed range $[-v_{\max}, +v_{\max}]$ become wrapped around ie. aliased

These limitations affect also computation of further products. In this paper we present a solution for approximating horizontal wind field directly from aliased Doppler data by means of inverse computation applying an efficient sliding window approach.



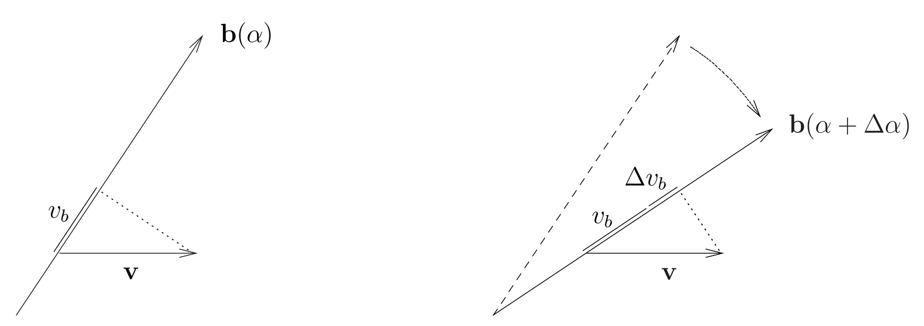
Conventional Doppler products VAD and VVP, as well as the proposed new product: approximated wind field.

Observing *changes* instead of absolute values of Doppler wind

Consider wind $\mathbf{v} = [u\ v]^T$ observed with radar using a low elevation angle. Let us model a radar beam at azimuthal direction α as as unit vector $\mathbf{b} = [\cos\alpha\ \sin\alpha]^T$. Observed wind v_b is the component projected on the beam: $v_b = \mathbf{v}\cdot\mathbf{b} = u\cos\alpha + v\sin\alpha$. Next, consider the rate of the observed beam-to-beam change in projected wind, denoted here as

$$d = \frac{\Delta v_b}{\Delta\alpha} \tag{1}$$

where $\Delta\alpha$ is the azimuthal scanning resolution. Notice that since azimuthal resolutions are typical around one degree, the *changes* of v_b are small compared to absolute values; crossing of the Nyquist velocity range $[-v_{\max}, +v_{\max}]$ in computing Δv_b can be handled safely.



The basic idea is to match d against the theoretical derivative of beam projected wind v_b:

$$d \approx \frac{\partial v_b}{\partial\alpha} = -u\sin\alpha + v\cos\alpha \tag{2}$$

A critical assumption is that the wind field is locally smooth. Then, we can find wind $\mathbf{v} = [u\ v]^T$ that best explains the changes of beam-projected speed in a local window. Assume n Doppler measurements located inside a window, with polar coordinates (α_i, r_i), $i \in \{1, 2, ..., n\}$. Practically, the window should be azimuthally wide, say some tens of degrees, whereas a couple of kilometers in range are sufficient. This leads to solving a set of n equations, in matrix form minimizing $|\mathbf{e}|$ in

$$\begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix} - \begin{bmatrix} -\sin\alpha_1 & \cos\alpha_1 \\ -\sin\alpha_2 & \cos\alpha_2 \\ & \vdots \\ -\sin\alpha_n & \cos\alpha_n \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \mathbf{d} - \mathbf{T}\mathbf{v} = \mathbf{e} \tag{3}$$

where $\mathbf{v} = [u\ v]^T$ is unknown. Its minimum squared error approximation is

$$\hat{\mathbf{v}} = (\mathbf{T}^T\mathbf{T})^{-1}\mathbf{T}^T\mathbf{d} \tag{4}$$

Rewriting $\mathbf{T} = [\mathbf{s}\ \mathbf{c}]$ where $\mathbf{s} = [-\sin\alpha_1 - \sin\alpha_2 \cdots - \sin\alpha_n]^T$ and $\mathbf{c} = [\cos\alpha_1 \cos\alpha_2 \cdots \cos\alpha_n]^T$, we get

$$\hat{\mathbf{v}} = \begin{bmatrix} \mathbf{s}^T\mathbf{s} & \mathbf{s}^T\mathbf{c} \\ \mathbf{c}^T\mathbf{s} & \mathbf{c}^T\mathbf{c} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{s}^T \\ \mathbf{c}^T \end{bmatrix} \mathbf{d} \tag{5}$$

which involves an inversion of a 2×2 matrix easily written out

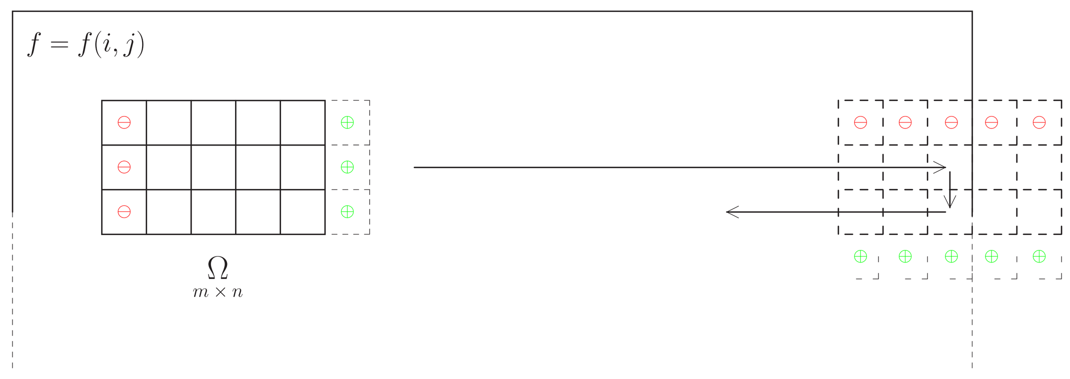
$$\hat{\mathbf{v}} = \frac{1}{(\mathbf{s}^T\mathbf{s})(\mathbf{c}^T\mathbf{c}) - (\mathbf{c}^T\mathbf{s})^2} \begin{bmatrix} \mathbf{c}^T\mathbf{c} & -\mathbf{c}^T\mathbf{s} \\ -\mathbf{c}^T\mathbf{s} & \mathbf{s}^T\mathbf{s} \end{bmatrix} \begin{bmatrix} \mathbf{s}^T\mathbf{d} \\ \mathbf{c}^T\mathbf{d} \end{bmatrix}. \tag{6}$$

Hence, this approximates original wind at a given radar bin. However, if a radar sweep consists of M rays, each having N range bins, and the computational window covers m rays and n range bins, the overall computational effort involves order of $M{\cdot}N{\cdot}m{\cdot}n$ steps – for example $500\cdot360\cdot90\cdot5 \approx 8.000.000$ operations – which may be practically unfeasible. Luckily, the computation can be accelerated, as explained next.

Sliding windows with continuous updating

First, consider a more general problem: smoothing an image $f = f(i, j)$ of size $M \times N$ with an averaging window Ω of size $m \times n$. In the smoothed image, each pixel (i, j) is assigned the average intensity $\frac{1}{mn}\sum_l\sum_k f(i + i_k, j + j_l)$ in its neighborhood, suggesting $M{\cdot}N{\cdot}m{\cdot}n$ operations like for the Doppler inversion above. However, computing an average can be implemented by continuously accumulating a sum: when the window is moved one step, one row of elements is subtracted and one row is added in the sum, resulting in $M{\cdot}N{\cdot}m$ operations. One can design the computation such that the statistics are initialized only once, in the starting corner of image traversal. This leads to "pipelined" forth-and-back row traversal illustrated below.

All the statistics which can derived from accumulated values are suited to pipeline architecture. For example, also variance can be computed using accumulation of sum and squared sum.

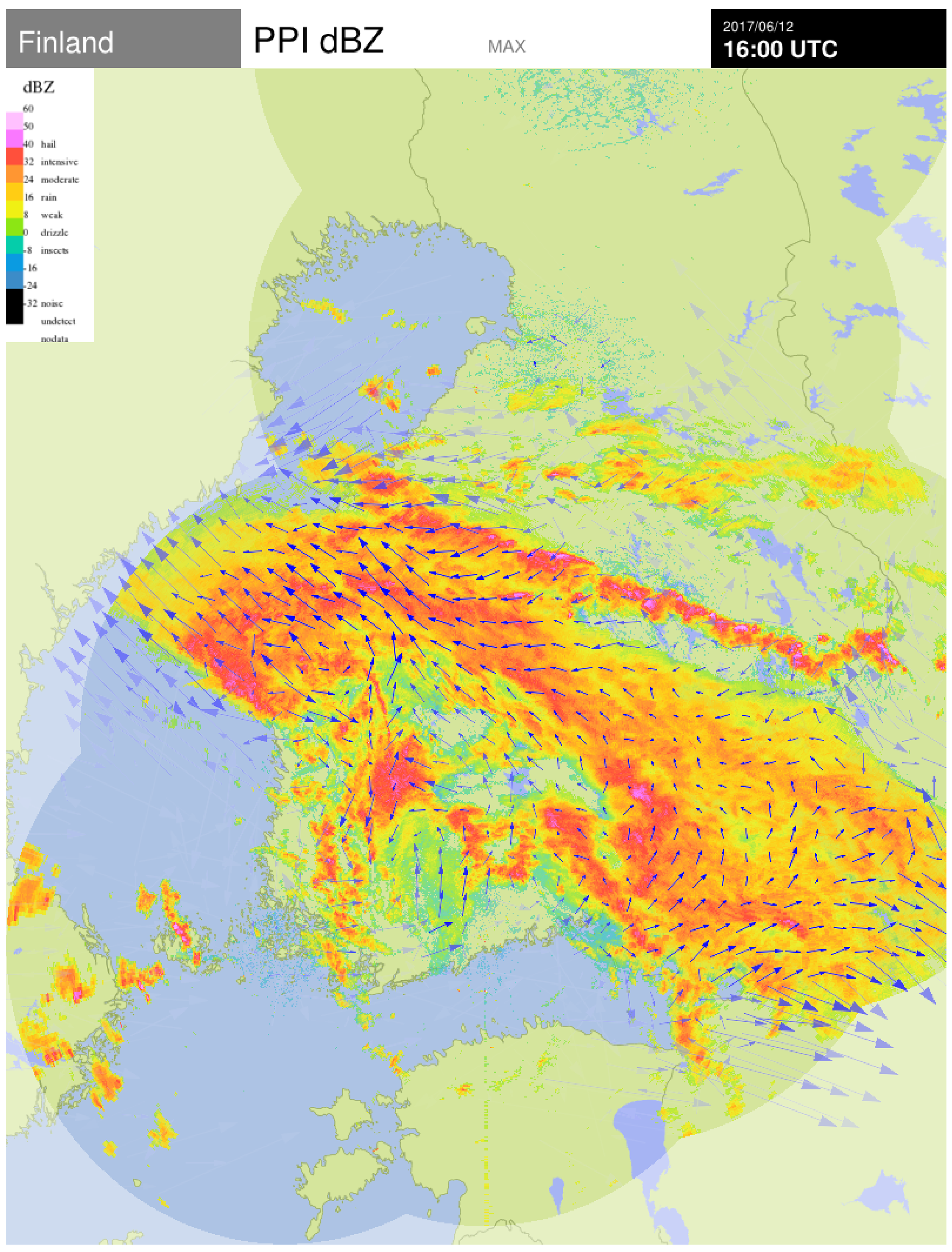


A sliding window Ω traverses each row or column in the image once, reversing the direction at the image edges.

Result

The solution for Doppler wind retrieval (6) involves five summations that can be accumulated continuously: $\mathbf{c}^T\mathbf{c}$, $\mathbf{c}^T\mathbf{s}$, $\mathbf{s}^T\mathbf{s}$, $\mathbf{s}^T\mathbf{d}$, and $\mathbf{c}^T\mathbf{d}$. This means that sliding window technique can be applied for solving $\mathbf{v} = [u\ v]^T$ quickly for each bin of a radar sweep.

This kind of quickly computable wind fields can be utilized directly in meteorological image products as well as in computation of extrapolation based forecasts.



Wind field $\mathbf{v} = [u\ v]^T$ obtained as quality-weighted composite of wind fields solved separately for each radar. The quality (certainty) of the retrieved field, illustrated as opacity of arrows, is readily obtained from determinant $|\mathbf{T}^T\mathbf{T}| = (\mathbf{s}^T\mathbf{s})(\mathbf{c}^T\mathbf{c}) - (\mathbf{c}^T\mathbf{s})^2$. The product has been computed using open source radar software **Rack**.

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https://github.com/fmidev/rack