Should interpolation of reflectivity be performed in Z or dBZ?

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Summary

- The relative accuracy of performing linear interpolation of radar reflectivity in units of Z and $\mathrm{dB}Z$ is assessed
- Interpolating in $\mathrm{dB}Z$ produces smaller errors overall; however, at high reflectivities interpolating in Z is better, particularly when the reflectivity difference is large
- A simple blending of interpolation in Z and $\mathrm{dB}Z$ produces small errors across the full range of reflectivities considered

Introduction

- Spatial interpolation of radar reflectivities is commonly required when remapping data from its native spherical coordinates to a Cartesian or latitude–longitude grid (e.g. Zhang et al. 2005) or estimating echo top heights (Lakshmanan et al. 2013)
- It is often argued that this processing should be performed in the measurement units, Z (mm^6 m^{-3}), rather than $\mathrm{dB}Z$
- However, for linear interpolation, the better quantity to operate on is whichever varies more linearly in space
- Using a simple analysis method applied to reflectivity data from a single WSR-88D site, Lakshmanan (2012) showed that interpolation in $\mathrm{dB}Z$ produces consistently smaller errors
- Here, we extend Lakshmanan's analysis to examine in detail the dependence of interpolation errors on reflectivity, reflectivity difference, interpolation direction, and geographic location

Data and Methodology

- We use data for one year (2011) from three operational radars in Australia, covering diverse rainfall regimes (Table 1)
- Days with precipitation in the vicinity of each site were identified using gridded rain gauge data
- To reduce computational expense, only eight radar scans were used per day: those closest to 00, 03, 06, 09, 12, 15, 18, and 21 UTC

Table 1. Details of the three radars used in this study. Climates are defined using the Köppen–Geiger classification system. Symbols have the following meaning: ω = angular beam width, $r_{\max}$ = maximum range, Δr = range gate spacing, $\Delta\phi$ = angular beam spacing.

Location (Climate)	Band	ω	r_{max} (km)	Δr (m)	$\Delta\phi$ (°)	Tilts
Brisbane (Humid Subtropical)	S	1.0	150	250	1.0	14: 0.5–32.0°
Darwin (Tropical Savannah)	C	1.0	150	250	1.0	15: 0.5–43.1°
Melbourne (Temperate Oceanic)	S	1.0	150	250	1.0	14: 0.5–32.0°

- Triads (sets of three neighbouring bins) with monotonically increasing or decreasing reflectivity were identified in the three coordinate directions: range, azimuth, and elevation (Fig. 1)
- The three reflectivity values were stored together with the distances (in metres or degrees) between the central and two outer bins
- The true reflectivity at the central bin, $\mathrm{dB}Z_i$, was then compared with estimates found by linearly interpolating between the two neighbouring bins, $\mathrm{dB}Z_{i\pm1}$, in both Z and $\mathrm{dB}Z$
- The error associated with linearly interpolating in Z is

$$\varepsilon_Z = 10\log_{10}\left[w\,10^{\mathrm{dB}Z_{i-1}/10} + (1-w)\,10^{\mathrm{dB}Z_{i+1}/10}\right] - \mathrm{dB}Z_i$$

- The error associated with linearly interpolating in $\mathrm{dB}Z$ is

$$\varepsilon_{\mathrm{dBZ}} = w\,\mathrm{dB}Z_{i-1} + (1-w)\,\mathrm{dB}Z_{i+1} - \mathrm{dB}Z_i$$

- Here, w is the interpolation weight given by

$$w_\chi = \frac{\chi_{i+1}-\chi_i}{\chi_{i+1}-\chi_{i-1}} = \frac{\Delta\chi_2}{\Delta\chi_1+\Delta\chi_2}$$

where $\chi = r, \phi$, and θ for triads in the range, azimuth, and elevation directions, respectively
- Range gate and azimuthal spacing are fixed so $w_r = w_\phi = 0.5$; however, since tilts are unevenly spaced, w_θ varies

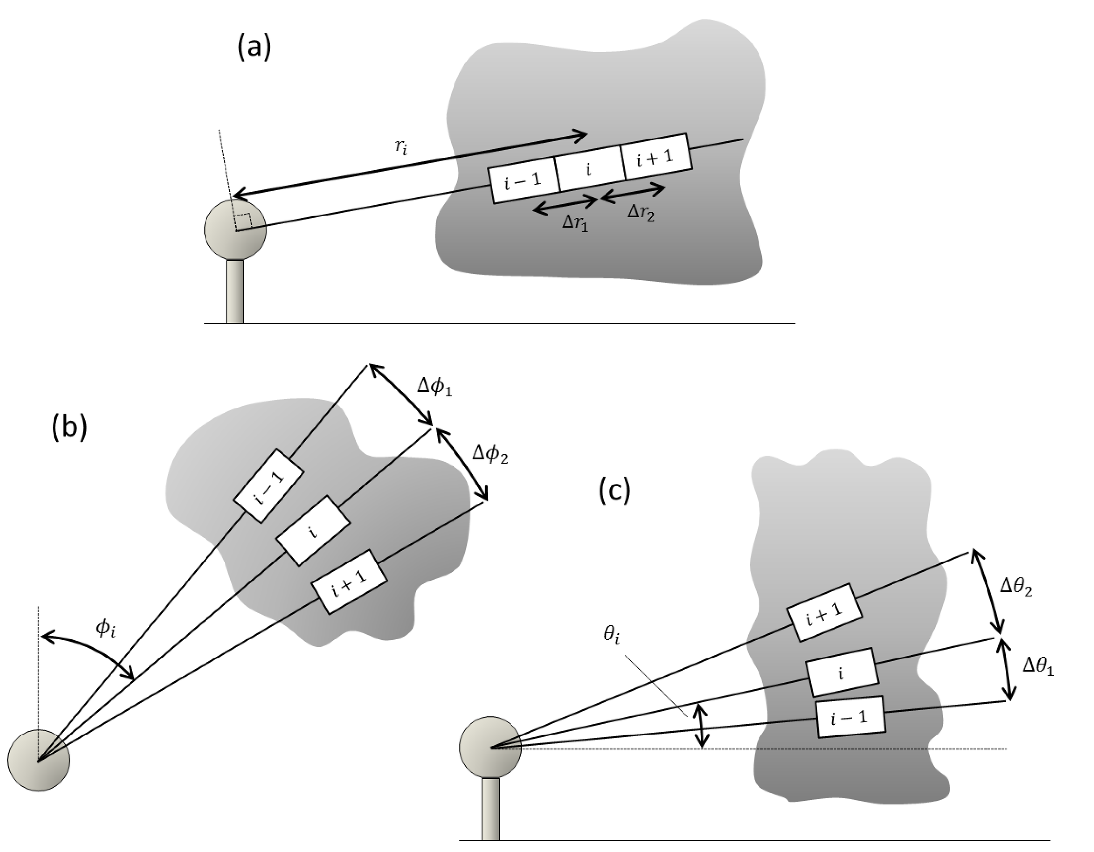

Figure 1. Schematic illustrating the identification of triads in the (a) range, (b) azimuth, and (c) elevation directions. Panels (a) and (c) show a side view while panel (b) shows a plan view.

Results

- Results computed over all triads (Table 2) are consistent with those of Lakshmanan (2012)
- For all radars and coordinate directions, interpolating in $\mathrm{dB}Z$ gives lower mean absolute error (MAE)
- Bias is positive for interpolation in Z and negative for interpolation in $\mathrm{dB}Z$
- Errors for θ are larger than those for ϕ which in turn are larger than those for r; this is due to the large distances involved when interpolating in the azimuth and elevation directions at long range
- On average, interpolating in $\mathrm{dB}Z$ is better for around 55 % of triads

Table 2. Summary of interpolation errors computed across all triads for each radar and coordinate direction. The bias and mean absolute error (MAE) are given in dB. The last column shows the percentage of triads for which interpolation in dBZ produces a smaller absolute error.

Radar	Dir.	No. triads	Interp. Z		Interp. $\mathrm{dB}Z$		% dBZ
			Bias	MAE	Bias	MAE	better
Brisbane	r	1.9 x 10^7	0.8	1.3	-0.3	1.2	54.5
	ϕ	2.3 x 10^7	1.3	1.8	-0.6	1.6	54.0
	θ	2.0 x 10^7	2.2	2.7	-0.8	2.3	55.7
Darwin	r	3.7 x 10^7	0.9	1.3	-0.2	1.2	55.0
	ϕ	4.2 x 10^7	1.4	1.9	-0.4	1.6	55.9
	θ	2.9 x 10^7	2.1	2.6	-0.6	2.1	56.8
Melbourne	r	2.5 x 10^7	0.8	1.3	-0.2	1.2	54.4
	ϕ	3.2 x 10^7	1.2	1.8	-0.5	1.5	54.9
	θ	2.7 x 10^7	2.4	2.9	-0.9	2.4	55.6

- When the results are broken down according to the central bin reflectivity and the absolute reflectivity difference between the two outer bins, $\Delta\mathrm{dB}Z$, a different picture emerges (Fig. 2)
- As we would expect, errors increase with increasing $\Delta\mathrm{dB}Z$, but not uniformly across the range of reflectivities
- Interpolation in Z produces positive biases which decrease with increasing reflectivity
- Interpolation in $\mathrm{dB}Z$ produces negative biases which increase with increasing reflectivity
- Overall, interpolation in $\mathrm{dB}Z$ is better at low reflectivities (< 20 dBZ) and interpolation in Z is better at high reflectivities (> 40 dBZ), consistent with Mohr and Vaughan (1979)
- Virtually identical patterns are seen for the other coordinate directions and radars (not shown)
- Low reflectivities occur much more frequently and therefore dominate the results for all triads

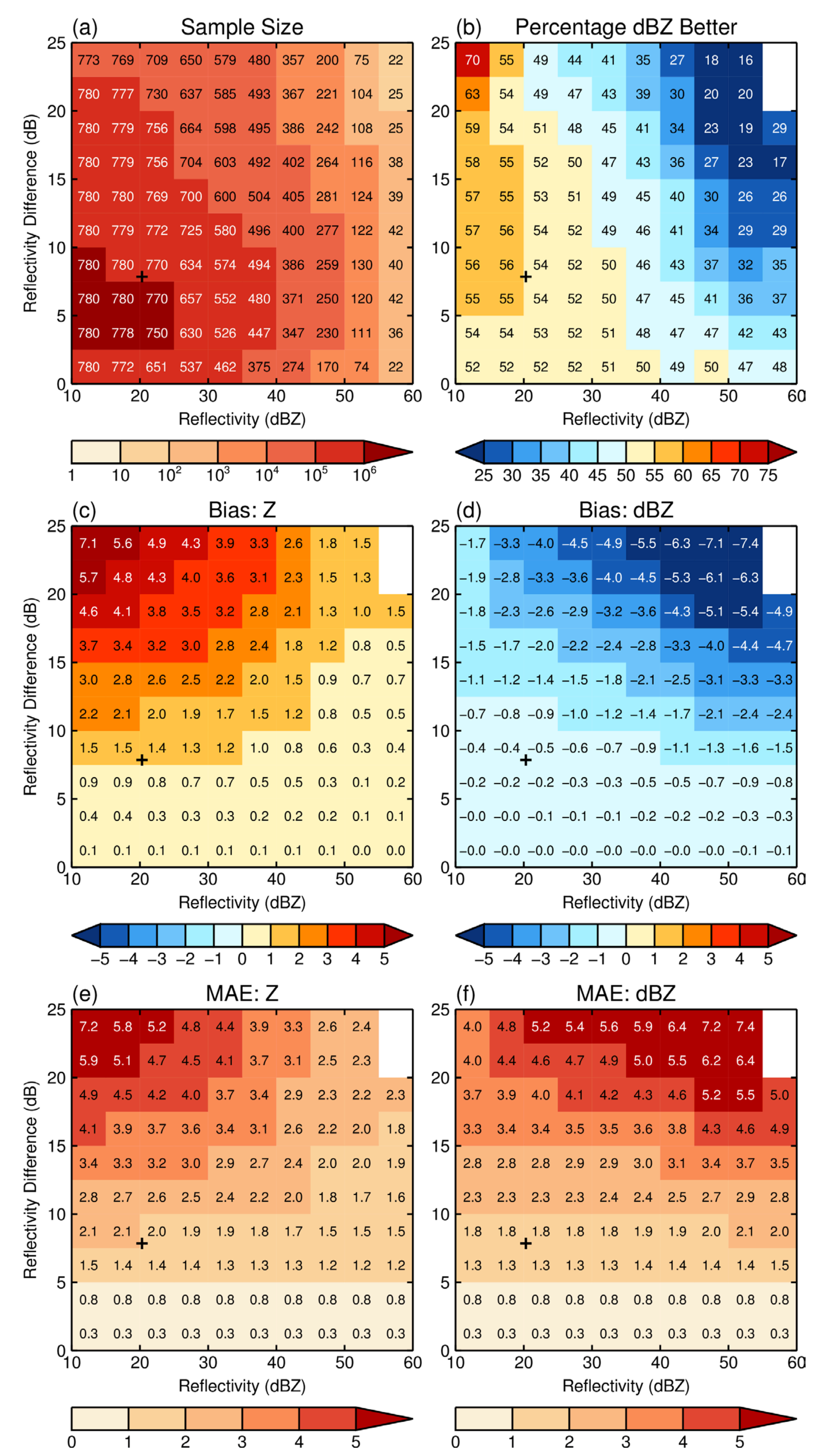

Figure 2. Interpolation results for azimuth triads from the Brisbane radar expressed as a function of the central bin reflectivity and the absolute reflectivity difference between the two outer bins. Variables shown are (a) sample size, (b) percentage of triads where interpolation in $\mathrm{dB}Z$ produces a smaller absolute error, bias (dB) for interpolation in (c) Z and (d) $\mathrm{dB}Z$, and mean absolute error (MAE; dB) for interpolation in (e) Z and (f) $\mathrm{dB}Z$. Crosses indicate the mean reflectivity and reflectivity difference.

Interpretation

- We can understand our results by examining the curvature of the reflectivity field in Z and $\mathrm{dB}Z$
- For the central bin of each triad, we compute the normalised distance, δ, and normalised reflectivity, ζ, in Z and $\mathrm{dB}Z$:

$$\delta = \frac{\chi_i - \chi_{i-1}}{\chi_{i+1} - \chi_{i-1}}$$

$$\zeta_Z = \frac{Z_i - Z_{i-1}}{Z_{i+1} - Z_{i-1}}$$

$$\zeta_{\mathrm{dBZ}} = \frac{\mathrm{dBZ}_i - \mathrm{dBZ}_{i-1}}{\mathrm{dBZ}_{i+1} - \mathrm{dBZ}_{i-1}}$$

- We then bin the triads by reflectivity and reflectivity difference, average the values in each bin, and plot ζ_Z and ζ_{dBZ} versus δ (Fig. 3)

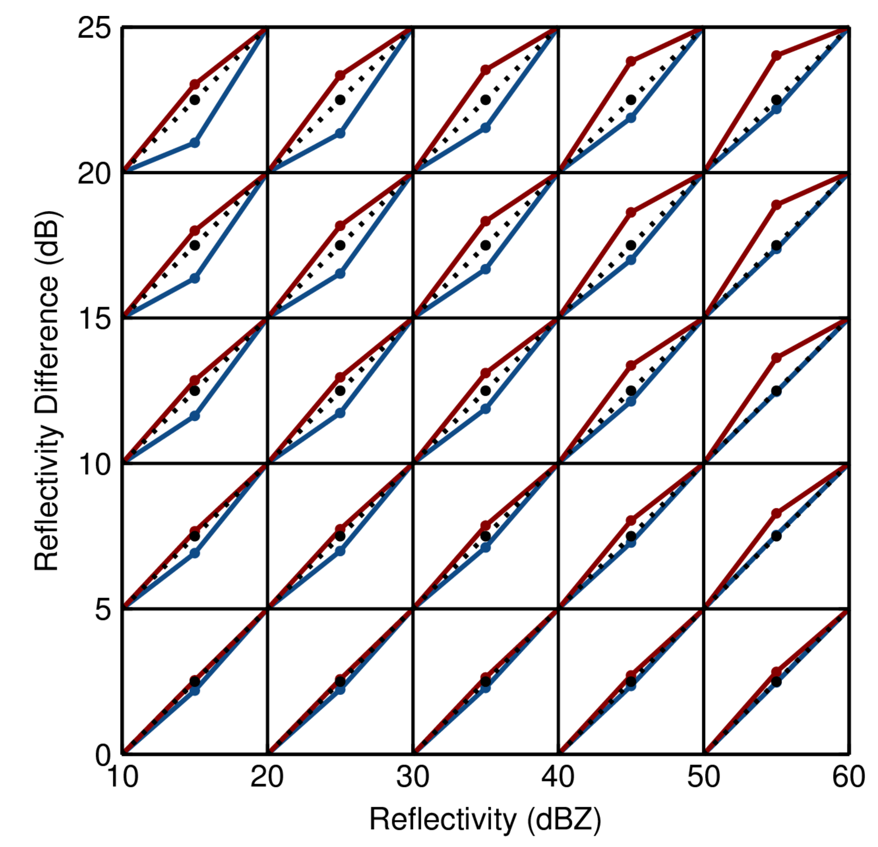

Figure 3. Illustration of the curvature of the reflectivity field in Z and $\mathrm{dB}Z$ as a function of the central bin reflectivity and the absolute reflectivity difference between the two outer bins. In each of the 25 panels, the normalised reflectivity, ζ, is plotted as a function of normalised distance, δ, with both variables on a scale of 0–1. The bottom-left and top-right corners thus correspond to the first and last bin in each triad, respectively. Blue and red dots show the central bin values in Z and $\mathrm{dB}Z$, respectively, with the associated coloured lines indicating the curvature in those units. The black dot which lies on a straight line connecting the bottom-left and top-right corners (dotted) shows the result of linear interpolation. When the black dot lies *above* the blue/red dot it indicates that interpolation in $Z/\mathrm{dB}Z$ will produce an *overestimate*. When the black dot lies *below* the blue/red dot it indicates that interpolation in $Z/\mathrm{dB}Z$ will produce an *underestimate*. The distance between the black and coloured dots measures the normalised interpolation error, $\varepsilon/\Delta\mathrm{dB}Z$. Values shown here are for azimuth triads from the Brisbane radar.

- For low $\mathrm{dB}Z_\mathrm{i}$, reflectivity has positive curvature in Z and near-zero curvature in $\mathrm{dB}Z$; thus, interpolation in Z produces an overestimate while interpolation in $\mathrm{dB}Z$ is quite accurate
- For high $\mathrm{dB}Z_\mathrm{i}$, reflectivity has negative curvature in $\mathrm{dB}Z$ and near-zero curvature in Z; thus, interpolation in $\mathrm{dB}Z$ produces an underestimate while interpolation in Z is quite accurate
- In general, curvature increases with increasing $\Delta\mathrm{dB}Z$, so the normalised interpolation error ($\varepsilon/\Delta\mathrm{dB}Z$) also increases

Blended Interpolation

- Based on Fig. 3, we propose a weighted average of the interpolation estimates in Z and $\mathrm{dB}Z$, with the weight a function of reflectivity
- For this new method, the interpolation error is

$$\varepsilon_{\mathrm{new}} = W\varepsilon_Z + (1-W)\,\varepsilon_{\mathrm{dBZ}}$$

where W increases from 0 at 10 dBZ to 1 at 60 dBZ
- This method was tested using both the Z- and $\mathrm{dB}Z$-based estimates for $\mathrm{dB}Z_i$ to calculate W, but the former gave better results
- The new method produces more accurate results with bias and MAE values generally less than ±1 dB and 4 dB, respectively (Fig. 4)
- Future work could investigate more sophisticated interpolation methods as an alternative to this simple blending

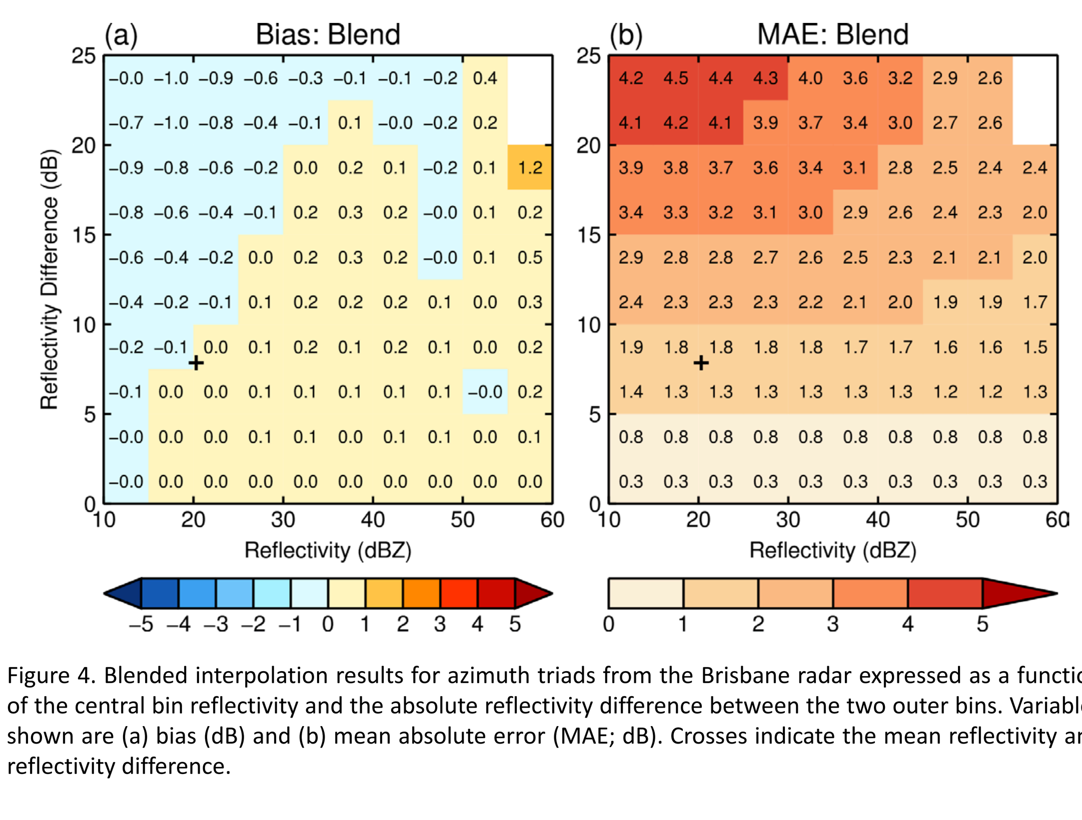

Figure 4. Blended interpolation results for azimuth triads from the Brisbane radar expressed as a function of the central bin reflectivity and the absolute reflectivity difference between the two outer bins. Variables shown are (a) bias (dB) and (b) mean absolute error (MAE; dB). Crosses indicate the mean reflectivity and reflectivity difference.

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