Passive Bistatic Radar using Weather Radars and Electromagnetic Vector Sensors

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Abstract

We propose a method to detect a target in a passive bistatic polarimetric radar network, with weather surveillance radar as our illuminator of opportunity (IO). We build our signal model using electromagnetic vector sensors (EMVS) as the receiver, which captures the reflections from a point-like target present in the scene of interest, surrounded with strong clutter. We develop a generalized likelihood ratio test (GLRT) detector which is constant false alarm rate (CFAR) under asymptotic conditions. The proposed detector is robust against the inhomogeneous clutter.

Problem Description and Contributions

Goal:

- Target detection in a bistatic passive polarimetric radar network.

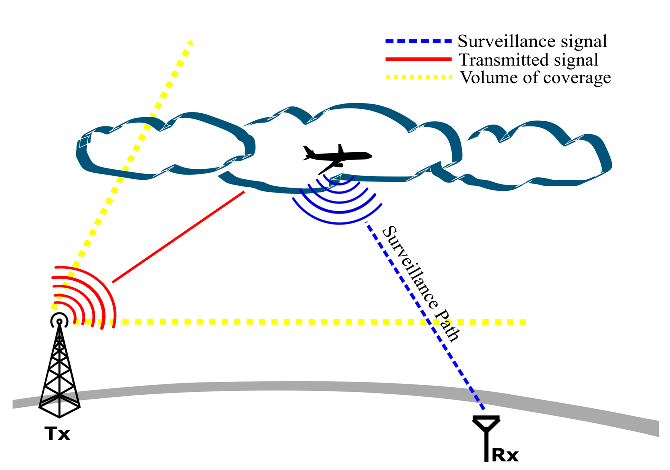

Figure 1: An illustration of passive bistatic weather radar.

Contributions:

- No previous work on bistatic radar addressed employing a weather radar for target detection with unknown signal-subspace.
- First to consider polarization information for mitigating signal-dependent clutter.

Motivation

Coverage area:

- There are 150 nearly identical dual-polarized S-band Doppler weather surveillance radars in the USA, with an observation range of $230 - 460\,\text{km}$ and a range resolution of $0.25 - 1\,\text{km}$, depending on the mode of operation.

Modeling:

- Lack of statistical signal model that considers signal-dependent clutter model for target detection with weather surveillance radar as IO.

Polarized receivers:

- Exploiting the polarimetric information about the target with the help of diversely polarized antennas such as EMVS.

Signal Model

- The surveillance path received signal for a weather radar transmitter and EMVS receiver can be represented as

$$\boldsymbol{y} = \underbrace{\boldsymbol{B}\boldsymbol{S}\boldsymbol{x}_{\text{p}}}_{\text{target signal}} + \underbrace{\boldsymbol{A}\boldsymbol{S}\boldsymbol{x}_{\text{c}}}_{\text{clutter signal}} + \underbrace{\boldsymbol{e}}_{\text{noise signal}}, \quad (1)$$

where

- $\boldsymbol{s} = [s(0), \ldots, s(N-1)]^T$: transmitted signal vector,
- $\bar{\boldsymbol{\epsilon}}_{\alpha,\beta}$: transmitted signal polarization information,
- $\boldsymbol{S} = \boldsymbol{s} \otimes \bar{\boldsymbol{\epsilon}}_{\alpha,\beta} \in \mathbb{C}^{M\times P}$: signal information matrix,
- $\mathcal{D}_{n,\omega} = \boldsymbol{L}_N(\omega)\boldsymbol{F}_N^H\boldsymbol{L}_N(-2\pi n/N)\boldsymbol{F}_N$: delay-Doppler matrix,
- $\boldsymbol{F}_N \in \mathbb{C}^{N\times N}$: unitary discrete Fourier transform (DFT) matrix,
- $\boldsymbol{L}_N(x) = \text{diag}\{e^{j(0)x}, e^{j(1)x}, \ldots, e^{j(N-1)x}\}$: diagonal matrix,
- $\boldsymbol{D}_{\theta,\phi} \in \mathbb{C}^{6\times 2}$: EMVS steering matrix,
- $\boldsymbol{A} = \mathcal{D}_{n_c,0} \otimes \boldsymbol{D}_{\theta,\phi} \in \mathbb{C}^{L\times M}$ and $\boldsymbol{A}^H\boldsymbol{A} = k\boldsymbol{I}_M$,
- $\boldsymbol{B} = \mathcal{D}_{n_p,\omega_D} \otimes \boldsymbol{D}_{\theta,\phi} \in \mathbb{C}^{L\times M}$ and $\boldsymbol{B}^H\boldsymbol{B} = k\boldsymbol{I}_M$.
- EMVS receiver: $L = 6N$, $M = 2N$, $P = 4$, and $k = 2$.

Statistics

- The target detection problem formulated as a hypothesis testing problem is given as

$$\begin{aligned} \mathcal{H}_0 &: \boldsymbol{y}_d \sim \mathcal{CN}\left(\boldsymbol{0}, \boldsymbol{A}\boldsymbol{S}\boldsymbol{\Sigma}\boldsymbol{S}^H\boldsymbol{A}^H + \sigma\boldsymbol{I}_L\right) \\ \mathcal{H}_1 &: \boldsymbol{y}_d \sim \mathcal{CN}\left(\boldsymbol{B}\boldsymbol{S}\boldsymbol{\mu}, \boldsymbol{A}\boldsymbol{S}\boldsymbol{\Sigma}\boldsymbol{S}^H\boldsymbol{A}^H + \sigma\boldsymbol{I}_L\right), \end{aligned} \quad (2)$$

where

- $d \in \{1, \ldots, D\}$ represents the snapshot index,
- $\boldsymbol{S}$ is *deterministic* and *unknown* signal information matrix,
- scattering coefficients of the clutter, $\boldsymbol{x}_c$, are assumed to be distributed as zero mean complex Gaussian random vectors with *unknown* covariance matrices denoted as $\boldsymbol{\Sigma}$,
- polarimetric scattering matrix of the target is rearranged in a coefficient vector, which is assumed deterministic and *unknown*, i.e., $\mathbb{E}[\boldsymbol{x}_{\text{p}}] = \boldsymbol{\mu}$ is unknown, and
- receiver noise vector, $\boldsymbol{e}$, is a zero mean complex Gaussian random vector with covariance $\sigma\boldsymbol{I}_L$, where we assume σ is *known*.

Important Results

Lemma 1: The Hermitian matrix $\boldsymbol{\Sigma}$ that maximizes $-D[L\ln\pi + \ln|\boldsymbol{\Gamma}| + \text{Tr}\{\boldsymbol{\Gamma}^{-1}\boldsymbol{R}\}]$ where $\boldsymbol{\Gamma} = \boldsymbol{A}\boldsymbol{S}\boldsymbol{\Sigma}\boldsymbol{S}^H\boldsymbol{A}^H + \sigma\boldsymbol{I}_L$ is the true covariance matrix, $\boldsymbol{R}$ is the sample covariance matrix, L is the number of samples, and D is the number of snapshots, is given as $\hat{\boldsymbol{\Sigma}} = (\boldsymbol{A}\boldsymbol{S})^\dagger\boldsymbol{R}(\boldsymbol{A}\boldsymbol{S})^{\dagger H} - \sigma(\boldsymbol{S}^H\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{S})$.

Lemma 2: For sufficiently large number of snapshots D, $\sigma^{-1}\,\text{Tr}\{\boldsymbol{P}_{\text{AS}}^{\perp}\boldsymbol{R}\} \approx L - P$ where $\boldsymbol{P}_{\text{AS}}^{\perp}$ is the orthogonal projection matrix and $\text{rank}(\boldsymbol{S}) = \text{rank}(\boldsymbol{P}_{\text{AS}}) = P$.

Lemma 3: The Unitary matrix $\boldsymbol{S}$ that maximizes $-D[L + L\ln\pi + (L-P)\ln\sigma + \ln|\boldsymbol{S}^H\boldsymbol{A}^H\boldsymbol{R}\boldsymbol{A}\boldsymbol{S}| - \ln|\boldsymbol{S}^H\boldsymbol{A}^H\boldsymbol{A}\boldsymbol{S}|]$ is given by $\boldsymbol{W}_1$, where $\boldsymbol{W}\boldsymbol{\Xi}\boldsymbol{W}^H$ is the orthogonal factorization of $\boldsymbol{A}^H\boldsymbol{R}\boldsymbol{A}$, $\boldsymbol{W}$ is an orthogonal matrix partitioned as $[\boldsymbol{W}_1, \boldsymbol{W}_2]$, such that $\boldsymbol{W}_1 \in \mathbb{C}^{M\times P}$ and $\boldsymbol{W}_2 \in \mathbb{C}^{M\times(L-P)}$, and $\boldsymbol{W}_1$ represents the eigenvectors corresponding to P largest eigenvalues.

Lemma 4: The maximum likelihood estimate of $\boldsymbol{\mu}$ in $\ln\left|\boldsymbol{S}^H\boldsymbol{A}^H\boldsymbol{R}_1\boldsymbol{A}\boldsymbol{S}\right|$, where $\boldsymbol{R}_1 = \frac{1}{D}\sum_{d=1}^{D}(\boldsymbol{y}_d - \boldsymbol{B}\boldsymbol{S}\boldsymbol{\mu})(\boldsymbol{y}_d - \boldsymbol{B}\boldsymbol{S}\boldsymbol{\mu})^H$ is given as $\hat{\boldsymbol{\mu}} = (\boldsymbol{B}\boldsymbol{S})^\dagger\bar{\boldsymbol{y}}$, where $\bar{\boldsymbol{y}} = \frac{1}{D}\sum_{d=1}^{D}\boldsymbol{y}_d$.

Generalized Likelihood Ratio Test

- We use generalized likelihood ratio test to solve the hypothesis testing problem in (2). Using Lemma 1–4, the test statistic of the hypothesis testing problem in (2) can be simplified to

$$\xi = \bar{\boldsymbol{z}}^H\boldsymbol{R}_{\boldsymbol{z}}^{-1}\bar{\boldsymbol{z}}, \quad (3)$$

where $\boldsymbol{z}_d = \boldsymbol{U}_1^H\boldsymbol{A}^H\boldsymbol{y}_d$ represents the eigen-transformed observation vector, and $\bar{\boldsymbol{z}}$ and $\boldsymbol{R}_{\boldsymbol{z}}$ are the new sample mean and covariance matrix, respectively.

- The distribution of the test statistic in (3) is given by

$$2(D-P)\xi \sim \begin{cases} \chi^2_{2P}, & \text{under } \mathcal{H}_0 \\ \chi^2_{2P}(\lambda), & \text{under } \mathcal{H}_1 \end{cases}, \quad (4)$$

where the non-centrality parameter is given as

$$\lambda = 2D\boldsymbol{\mu}^H\boldsymbol{S}^H\boldsymbol{B}^H\boldsymbol{A}\boldsymbol{U}_1[\boldsymbol{U}_1^H\boldsymbol{A}^H\boldsymbol{\Gamma}\boldsymbol{A}\boldsymbol{U}_1]^{-1}\boldsymbol{U}_1^H\boldsymbol{A}^H\boldsymbol{B}\boldsymbol{S}\boldsymbol{\mu}.$$

Distribution of the Test Statistic

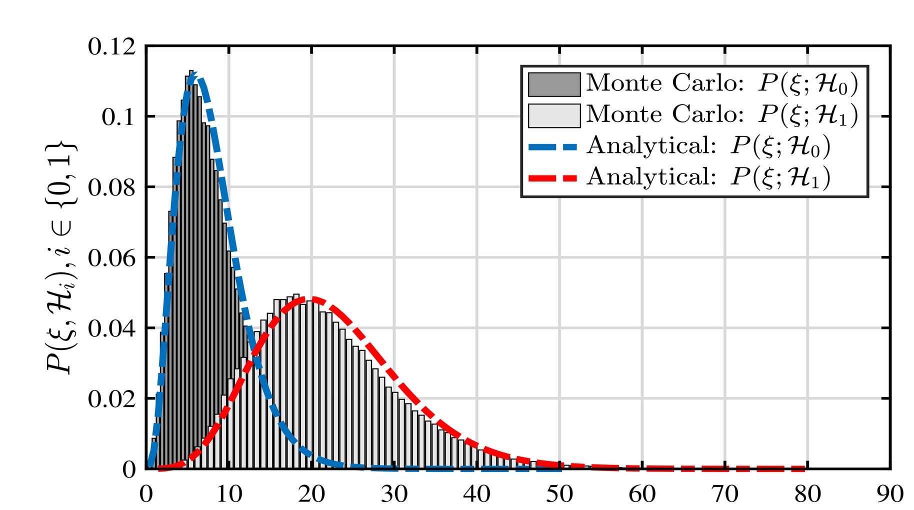

Figure 2: Normalized histogram (empirical PDF) and the analytic PDF under $\mathcal{H}_0$ and $\mathcal{H}_1$, with SNR $= -10\,\text{dB}$, CNR $= 10\,\text{dB}$, number of samples per snapshot $N = 8$, and number of snapshots $D = 200$.

$$\text{SNR} = 10\log_{10}\frac{\boldsymbol{\mu}^H\boldsymbol{S}^H\boldsymbol{S}\boldsymbol{\mu}}{\sigma} \qquad \text{CNR} = 10\log_{10}\frac{\text{Tr}\{\boldsymbol{\Sigma}\}}{\sigma}.$$

Numerical Results

In our simulation results, we fix number of samples $N = 8$, probability of false alarm $P_{\text{FA}} = 10^{-3}$, and CNR $= 10\,\text{dB}$.

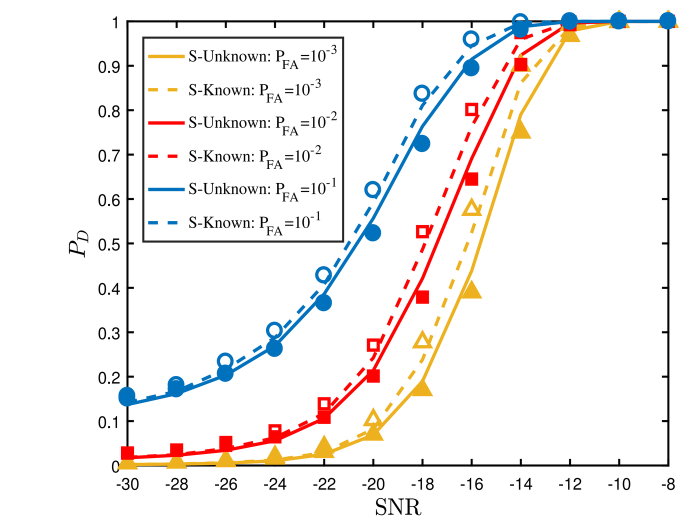

Figure 3: Probability of detection curves for unknown and known signal information matrix.

Observation:
The proposed detector closely matches the performance of the oracle detector, however, it is important to note that the oracle detector does not require large number of snapshots.

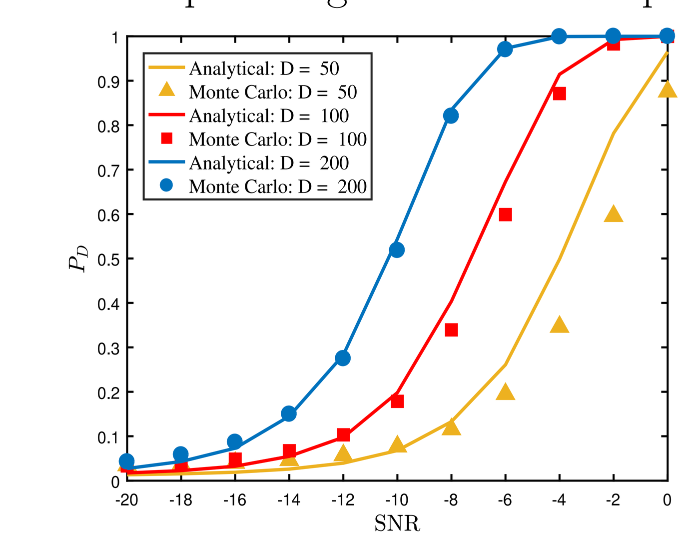

Figure 4: Probability of detection curves for varying number of snapshots.

Observation:
The performance of the detector improves as the number of samples increases, at the expense of longer integration time.

References

[1] G. V. Prateek, M. Hurtado, and A. Nehorai, “Target detection using weather radars and electromagnetic vector sensors,” *Signal Processing*, Vol. 137, pp. 387–397, Aug. 2017.

[2] M. Hurtado and A. Nehorai, “Polarimetric detection of targets in heavy inhomogeneous clutter,” *IEEE Transactions on Signal Processing*, Vol. 56, pp. 1349–1361, Apr. 2008.

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