Multilag Estimators for the Alternating Mode of Dual-Polarimetric Weather Radar Operation

David L. Pepyne

pepyne@ecs.umass.edu

Center for Collaborative Adaptive Sensing of the Atmosphere (CASA)

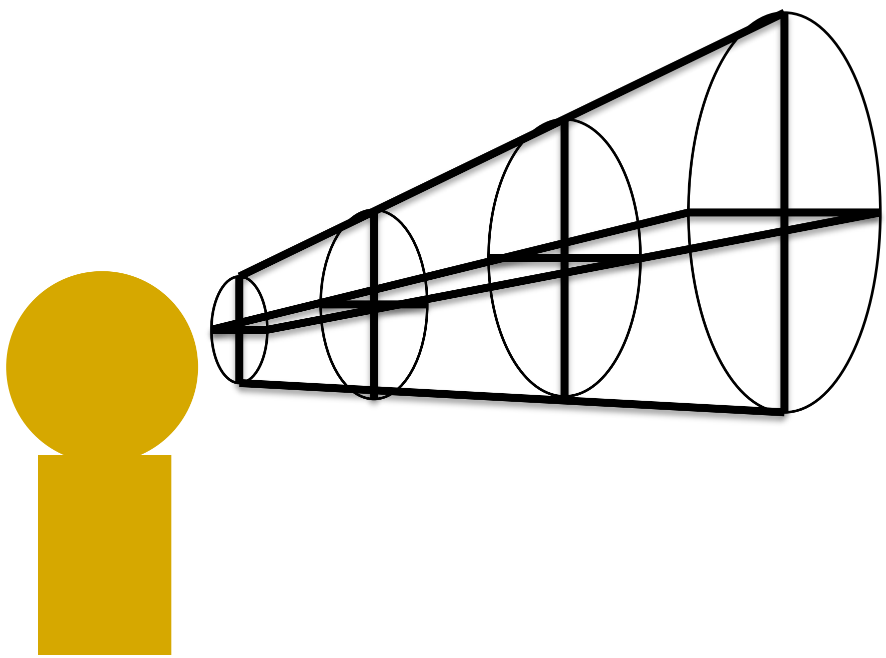
University of Massachusetts, Amherst

Presented at the AMS 38th Conference on Radar Meteorology, Chicago, IL

29 August 2017

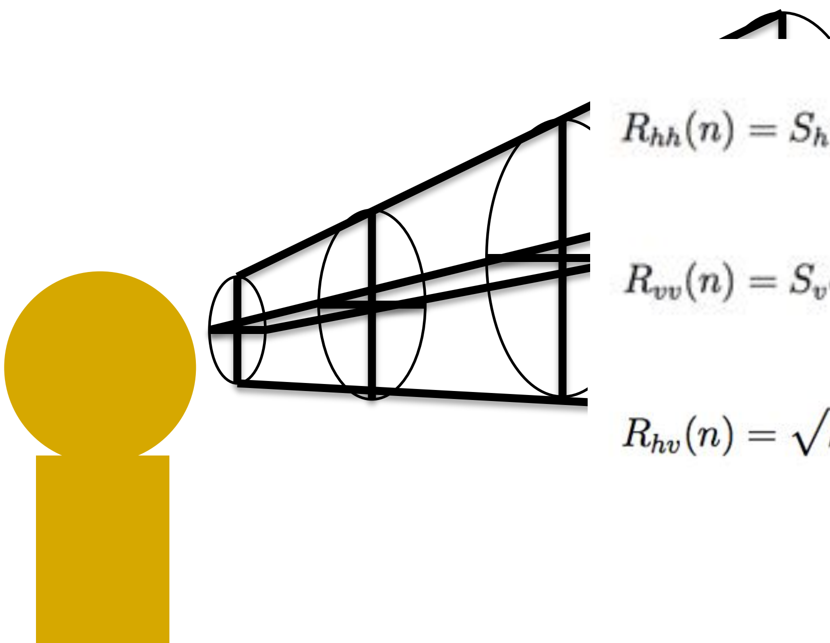
Polarimetric Weather Radar

- Weather radars scan the sky with a narrow "pencil" beam
- Polarimetric weather radars transmit two beams
  - One horizontally polarized, another vertically polarized
  - Enables them to measure the shape features of hydrometeors, e.g., the flattening of raindrops as they fall
  - Facilitates hydrometeor classification and precipitation estimation



Polarimetric Weather Radar

- The beams are partitioned into range bins
  - The volumes are determined by the beamwidths and range bin spacing
  - Range in volume from thousands of cubic meters in size close to the radar to millions of cubic meters in size far from the radar
  - Contain huge numbers of hydrometers
- The co-polar "hh" and "vv" I,Q data streams from a hydrometer filled range bin are random processes whose auto- and cross-correlation functions are well modeled by Gaussians of the forms:



$$R_{hh}(n) = S_h exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V n T_s}{\lambda}\right] + N_h \delta(n)$$

$$R_{vv}(n) = S_v exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V n T_s}{\lambda}\right] + N_v \delta(n)$$

$$R_{hv}(n) = \sqrt{S_h S_v}\rho_{hv} exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V n T_s}{\lambda} + j\phi_{dp}\right]$$

Polarimetric Weather Data Processing

$$R_{hh}(n) = S_h exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V n T_s}{\lambda}\right] + N_h \delta(n)$$

$$R_{vv}(n) = S_v exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V n T_s}{\lambda}\right] + N_v \delta(n)$$

$$R_{hv}(n) = \sqrt{S_h S_v}\rho_{hv} exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V n T_s}{\lambda} + j\phi_{dp}\right]$$

- **In the equations,**
  - **n = lag index in number of pulses**
  - **T_s = pulse repetition interval (here we assume constant PRI)**
  - **λ = radar wavelength**
  - **N_h, N_v = noise powers, e.g., due to thermal sources**
- **The goal of weather data processing is to estimate the parameters of the correlation functions:**
  - **S_h = h-pol signal power (used to estimate reflectivity)**
  - **S_v = v-pol signal power (used together with S_h to estimate differential reflectivity Z_{dr} = $10\log_{10}(S_h/S_v)$)**
  - **V = radial velocity**
  - **W = spectrum width**
  - **ρ_{hv} = correlation coefficient**
  - **ϕ_{dp} = differential phase**

Conventional Estimators

- "Conventional" estimators for signal power S_h, differential reflectivity Z_{dr}, and correlation coefficient ρ_{hv} require we know the noise powers N_h and N_v:

$$|R_{hh}(0)| = S_h + N_h \Rightarrow S_h = |R_{hh}(0)| - N_h$$

$$|R_{vv}(0)| = S_v + N_v \Rightarrow S_v = |R_{vv}(0)| - N_v$$

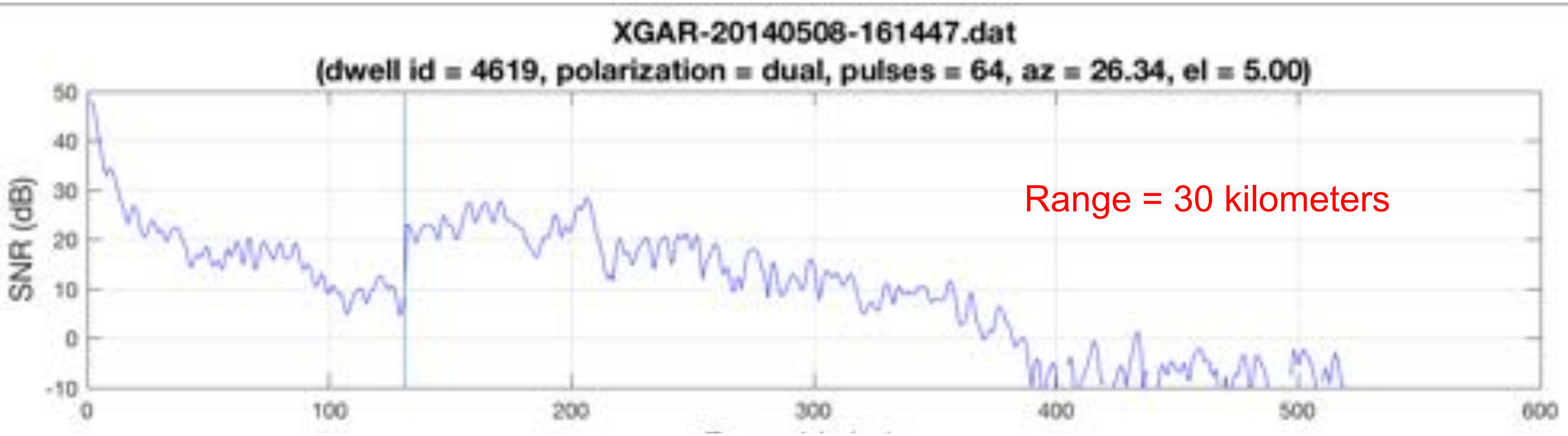
$$Z_{dr} = 10log_{10}\left(\frac{S_h}{S_v}\right)$$

$$|R_{hv}(0)| = \sqrt{S_h S_v}\rho_{hv} \Rightarrow \rho_{hv} = \frac{|R_{hv}(0)|}{\sqrt{S_h S_v}}$$

- The standard way to estimate the noise powers is with "receive only dwells"
  - Estimate the noise from dwells obtained with the transmitter turned off
  - No transmitted pulse → all range bins are signal free → estimate the noise powers from these signal free bins

Noise Power Estimation

- **For radars that don't generate receive only dwells, algorithms are needed for "noise bin detection"**
  - **Use some method to find signal free noise bins and estimate the noise powers from the powers in those bins**
- **Finding noise free bins this way is a problem for short range, pulse-compressed X-band radars that CASA works with – the entire domain of a pulse can be covered by weather!**
  - **If I change the waveform, e.g., pulse duration/receiver bandwidth, during weather I have to guess the noise powers → errors!**



Short pulse region – covered with weather

Long pulse region – too few "signal free" range bins for a good noise estimate

Multi-lag Estimators

- The papers by L. Lei et al 2009, 2012 developed a number of multi-lag estimators – including estimators for signal power, differential reflectivity, and correlation coefficient:

$$S_h = \frac{|R_{hh}(1)|^{4/3}}{|R_{hh}(2)|^{1/3}}$$

$$Z_{dr} = 10log_{10}\left(\frac{|R_{hh}(1)|}{|R_{vv}(1)|}\right)$$

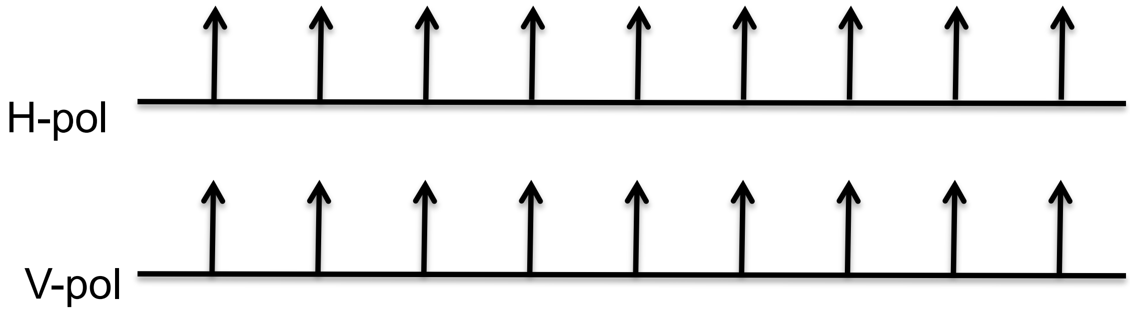
$$\rho_{hv} = \frac{|R_{hv}(1)|}{\sqrt{|R_{hh}(1)||R_{vv}(1)|}}$$

- Because these estimators don't use the lag-0 autocorrelations, they are independent of the noise powers → they do not require noise estimation

- L. Lei, G. Zhang, R. Palmer, B.-L. Cheong, M. Xue, and Q. Cao, "A Multi-Lag Correlation Estimator for Polarimetric Radar Variables in the Presence of Noise," *Proc. of the 34th AMS Conf. on Radar Meteorology*, 2009.
- L. Lei, G. Zhang, R.J. Doviak, R. Palmer, B.-L. Cheong, M. Zue, Q. Cao, and Y. Li, "Multilag Correlation Estimators for Polarimetric Radar Measurements in the Presence of Noise," *Journal of Atmospheric and Oceanic Technology*, Vol. 29, pp. 772-795, June 2012.

Mechanically Scanned Dish Weather Radars

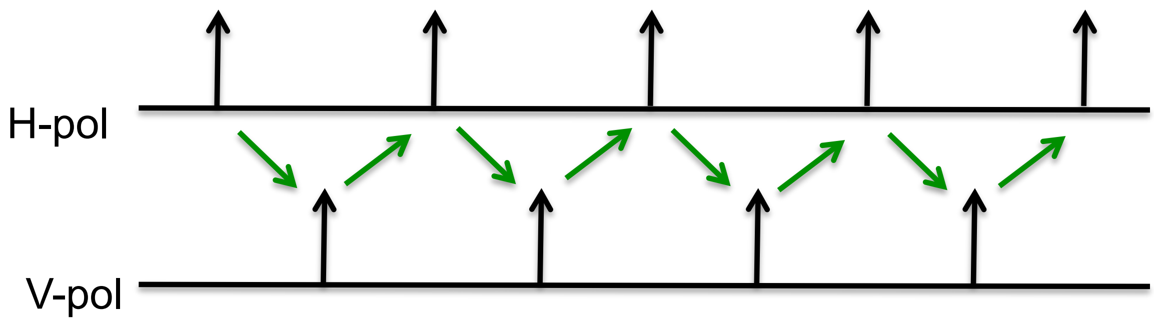
- **The Lei Lei et al results are for the SIMULTANEOUS mode of dual-pol weather radar operation**
  - **In this mode the radar effectively transmits two beams with each pulse – one horizontally polarized, the other vertically polarized:**

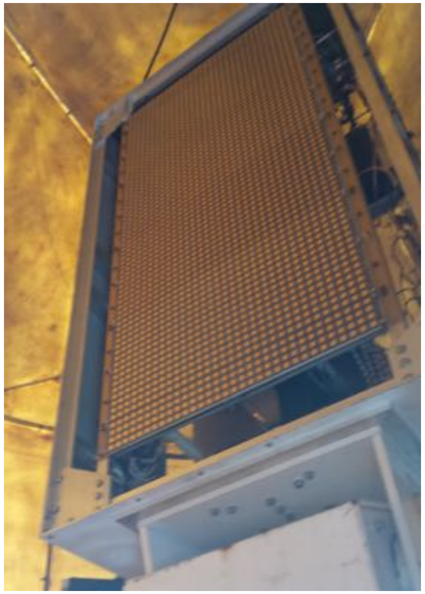


- **The SIMULTANEOUS is used by mechanically scanned dish radars**
  - **“Easy” to build a dish that can achieve the required cross-polar isolation**
  - **“Easy” to generate the two polarizations with a dual-pol waveguide**
  - **Examples include NEXRAD and CASA’s mechanically scanned radars**

Phased-Array Weather Radars

- Future weather radars will use solid-state phased-array antenna where the beam is steered electronically
  - The “inertia-less” beams of such radars will allow them to perform multiple missions, e.g., weather and aircraft surveillance at the same time
- Because it results in a simpler, lower cost array, phased-array radars will likely use the ALTERNATING mode of dual-pol radar operation
  - In this mode, the radar transmits a single beam that alternates between the h and v polarizations on consecutive pulses
  - Examples include CASA’s X-band phased-array radars





Multi-Lag Extensions to the ALTERNATING Mode

- **Denote the ALTERNATING ACFs and CCF as**
  - $R_{xx}(n)$, $R_{yy}(n)$, and $R_{xy}(n)$
- **Relating the ALTERNATING correlations to the SIMULTANEOUS correlations we get,**

$$R_{xx}(n) = R_{hh}(2n)$$

$$R_{yy}(n) = R_{vv}(2n)$$

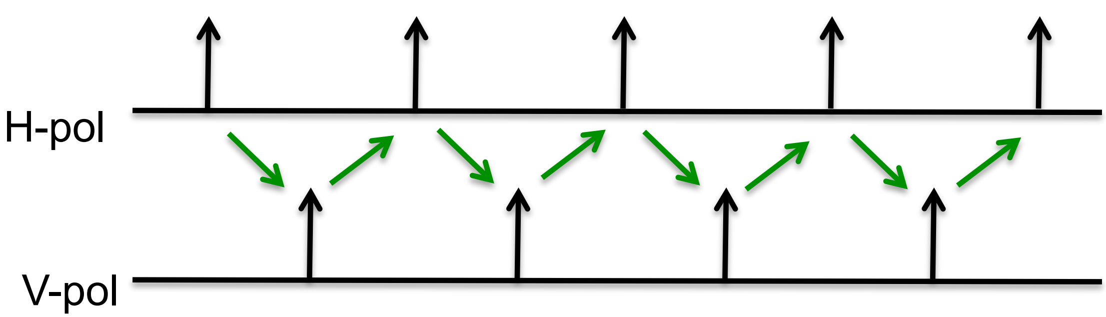
$$R_{xy}(n) = R_{hv}(2n-1)$$

where

$$R_{hh}(n) = S_h exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V nT_s}{\lambda}\right] + N_h\delta(n)$$

$$R_{vv}(n) = S_v exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V nT_s}{\lambda}\right] + N_v\delta(n)$$

$$R_{hv}(n) = \sqrt{S_h S_v}\rho_{hv} exp\left[-\frac{8\pi^2 W^2 (nT_s)^2}{\lambda^2}\right] exp\left[-j\frac{4\pi V nT_s}{\lambda} + j\phi_{dp}\right]$$



Signal Power Estimator

- **Follows directly from the multilag estimator for the SIMULTANEOUS mode,**

$$
\begin{aligned}
\frac{|R_{xx}(1)|^{4/3}}{|R_{xx}(2)|^{1/3}} &= \frac{|R_{hh}(2)|^{4/3}}{|R_{hh}(4)|^{1/3}} \\
&= \frac{|S_h exp\left[-\frac{8\pi^2 W^2 (2T_s)^2}{\lambda^2}\right]|^{4/3}}{|S_h exp\left[-\frac{8\pi^2 W^2 (4T_s)^2}{\lambda^2}\right]|^{1/3}} \\
&= \frac{S_h^{4/3} exp\left[-\frac{8\pi^2 W^2 T_s^2}{\lambda^2}\right]^{4(4/3)}}{S_h^{1/3} exp\left[-\frac{8\pi^2 W^2 T_s^2}{\lambda^2}\right]^{16(1/3)}} \\
&= S_h
\end{aligned}
$$

Differential Reflectivity Estimator

- **Follows directly from the multilag estimator for the SIMULTANEOUS mode,**

$$
\begin{aligned}
10log_{10}\left(\frac{|R_{xx}(1)|}{|R_{yy}(1)|}\right) &= 10log_{10}\left(\frac{|R_{hh}(2)|}{|R_{vv}(2)|}\right) \\
&= 10log_{10}\left(\frac{|S_h exp\left[-\frac{8\pi^2W^2(2T_s)^2}{\lambda^2}\right]|}{|S_v exp\left[-\frac{8\pi^2W^2(2T_s)^2}{\lambda^2}\right]|}\right) \\
&= 10log_{10}\left(\frac{S_h}{S_v}\right) \\
&= Z_{dr}
\end{aligned}
$$

Correlation Coefficient Estimator

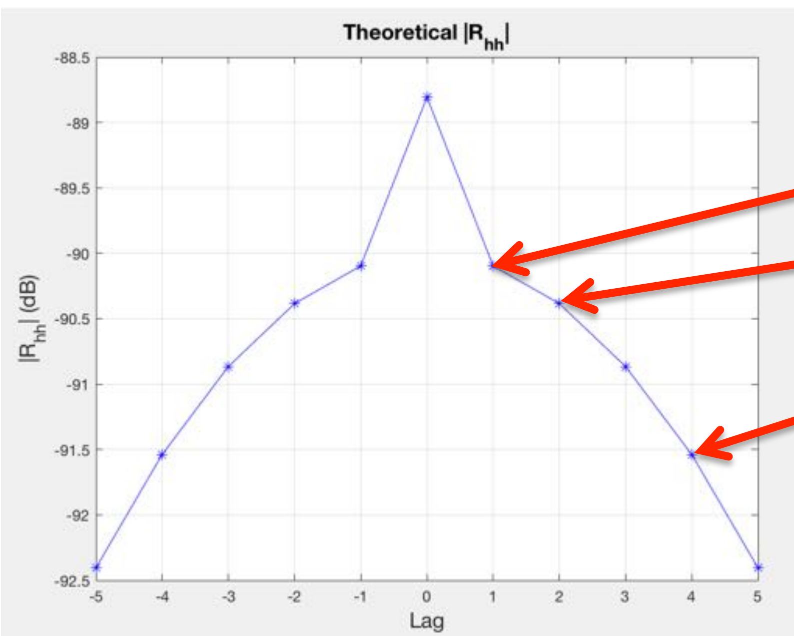
- **Starts with the multilag estimator for the SIMULTANEOUS mode,**

$$\rho_{hv} = \frac{|\hat{R_{hv}}(1)|}{\sqrt{|\hat{R_{hh}}(1)||\hat{R_{vv}}(1)|}}$$

- **Numerator – use the relationship between R_{xy} and R_{hv},**

$$R_{xy}(n) = R_{hv}(2n-1) \longrightarrow |\hat{R_{hv}}(1)| = |R_{xy}(1)|$$

- **Denominator – curve fit the $|R_{hh}(n)|$ autocorrelation curves**



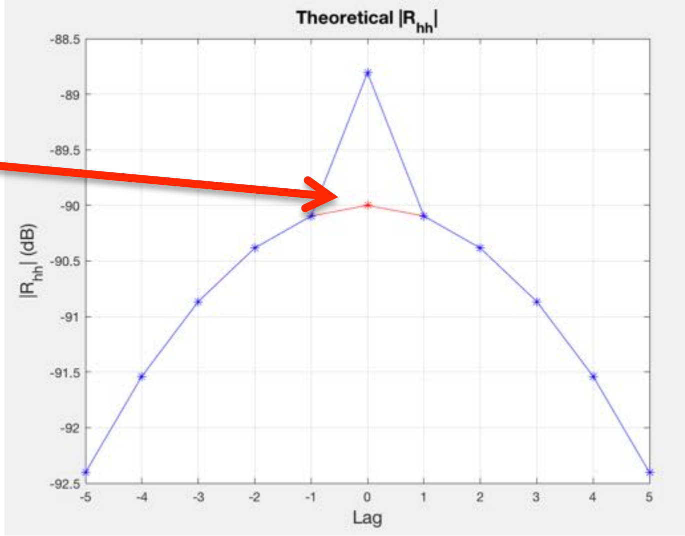
Estimate $|R_{hh}(1)|$ with $|R_{xx}(1)| = |R_{hh}(2)|$ and $|R_{xx}(2)| = |R_{hh}(4)|$

Ditto for $|R_{vv}(1)|$

Correlation Coefficient Estimator

- **Assume no noise magnitude curve has form,**

$$|R_{hh}(n)| = aexp\left[-\frac{n^2}{2c^2}\right]$$



- **Use $|R_{xx}(1)| = |R_{hh}(2)|$ and $|R_{xx}(2)| = |R_{hh}(4)|$ to determine *a* and *c*,**

$$|R_{xx}(2)| = aexp\left[-\frac{8}{c^2}\right]$$

$$= |R_{xx}(1)|exp\left[+\frac{2}{c^2}\right] exp\left[-\frac{8}{c^2}\right]$$

$$= |R_{xx}(1)|exp\left[+\frac{2}{c^2} - \frac{8}{c^2}\right]$$

$$= |R_{xx}(1)|exp\left[+\frac{-6}{c^2}\right]$$

$$\Rightarrow ln\left(\frac{|R_{xx}(2)|}{|R_{xx}(1)|}\right) = \frac{-6}{c^2}$$

$$\Rightarrow c^2 = \frac{-6}{ln(\frac{|R_{xx}(2)|}{|R_{xx}(1)|})}$$

$$\Rightarrow c = \sqrt{\frac{-6}{ln(\frac{|R_{xx}(2)|}{|R_{xx}(1)|})}}$$

$$a = |R_{xx}(1)|exp\left[+\frac{2}{c^2}\right]$$

- **Use the resulting *a* and *c* to estimate $|R_{hh}(1)|$,**

$$|\hat{R_{hh}}(1)| = aexp\left[-\frac{1}{2c^2}\right]$$

$$|\hat{R_{hh}}(1)| = |R_{xx}(1)|^{5/4}|R_{xx}(2)|^{-1/4}$$

Correlation Coefficient Estimator

- **Put it all together to get,**

$$|\hat{R_{hv}}(1)| = |R_{xy}(1)|$$

$$\rho_{hv} = \frac{|\hat{R_{hv}}(1)|}{\sqrt{|\hat{R_{hh}}(1)||\hat{R_{vv}}(1)|}}$$

$$\rho_{hv} = \frac{|R_{xy}(1)||R_{xx}(2)|^{1/8}|R_{yy}(2)|^{1/8}}{|R_{xx}(1)|^{5/8}|R_{yy}(1)|^{5/8}}$$

$$|\hat{R_{hh}}(1)| = |R_{xx}(1)|^{5/4}|R_{xx}(2)|^{-1/4}$$

$$|\hat{R_{vv}}(1)| = |R_{yy}(1)|^{5/4}|R_{yy}(2)|^{-1/4}$$

Simulation Results

- **Here we use simulated I,Q data to compare our "multilag" estimators to the "conventional" estimators**

  - **The correlation functions tell us how to simulate weather target I,Q data (cf Galati & Pavan, 1995)**
    - **Generate two white noise processes**
    - **Correlate them according to ρ_{hv}**
    - **"Color" them to give them Gaussian spectra with parameters *V* and *W***
    - **Scale them to give them the signal powers S_h and S_v**
    - **Add to each one white noises with powers N_h and N_v respectively**
    - **Extract the ALTERNATING I,Q sequence from the resulting SIMULTANEOUS one**

- **G. Galati and G. Pavan, "Computer Simulation of Weather Radar Signals," *Simulation Practice and Theory*, Vol. 3, No. 1, pp. 17-44, July 1995.**

Simulation Results

- The estimators we compare are:

Conventional

$$S_h = |R_{xx}(0)| - N_h$$

$$Z_{dr} = 10log_{10}\left(\frac{|R_{xx}(0)| - N_x}{|R_{yy}(0)| - N_y}\right)$$

$$\rho_{hv} = \frac{|R_{xy}(0)| + |R_{xy}(1)|}{2(S_x S_y)^{3/8}|R_{xx}(1)R_{yy}(1)|^{1/8}}$$

Multilag

$$S_h = \frac{|R_{xx}(1)|^{4/3}}{|R_{xx}(2)|^{1/3}}$$

$$Z_{dr} = 10log_{10}\left(\frac{|R_{xx}(1)|}{|R_{yy}(1)|}\right)$$

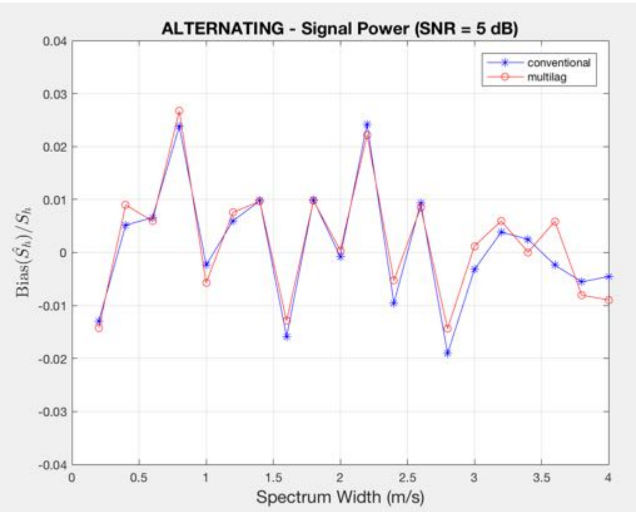
$$\rho_{hv} = \frac{|R_{xy}(1)||R_{xx}(2)|^{1/8}|R_{yy}(2)|^{1/8}}{|R_{xx}(1)|^{5/8}|R_{yy}(1)|^{5/8}}$$

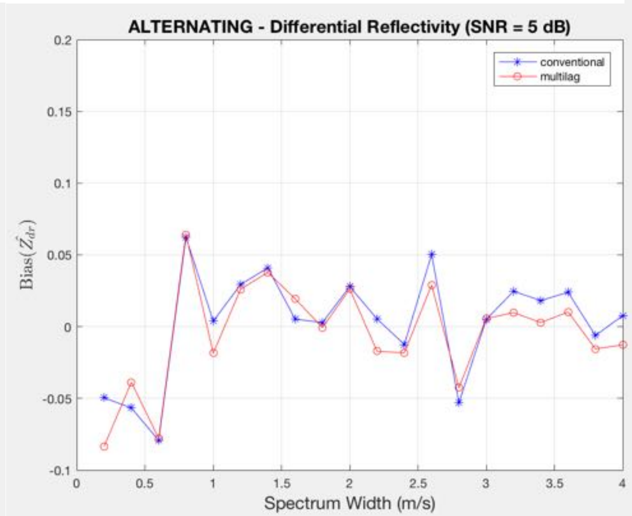
- For the simulation experiments we assume we know the noises N_x and N_y exactly – in simulation we can do this!

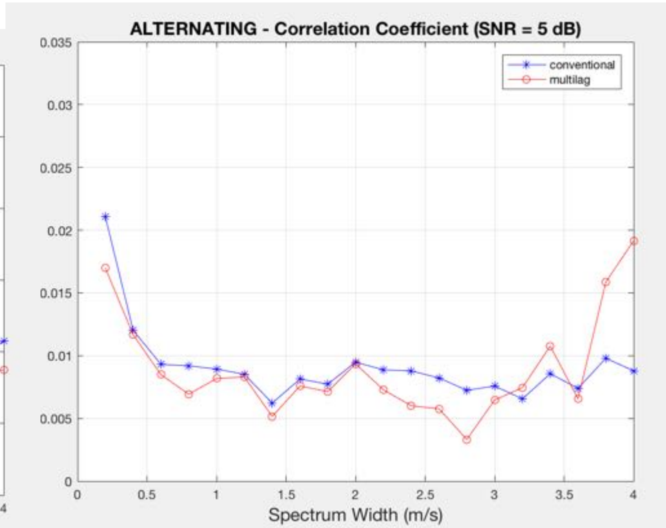
- V.M. Melnikov and D.S. Zrnic, "On the Alternate Transmission Mode for Polarimetric Phased Array Weather Radar," *Journal of Atmospheric and Oceanic Technology*, Vol. 32, February 2015

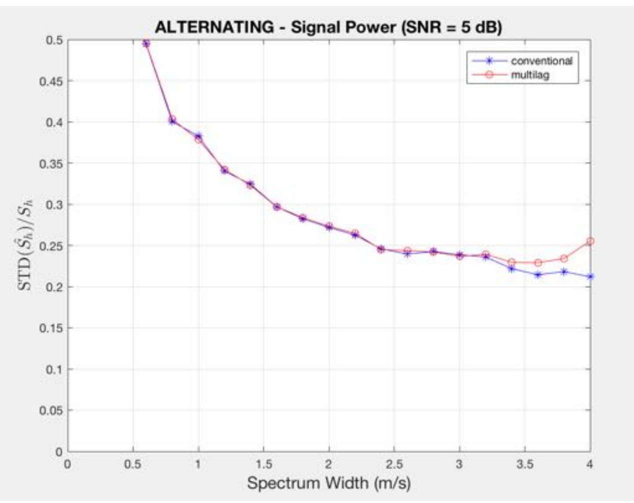
Simulation Results

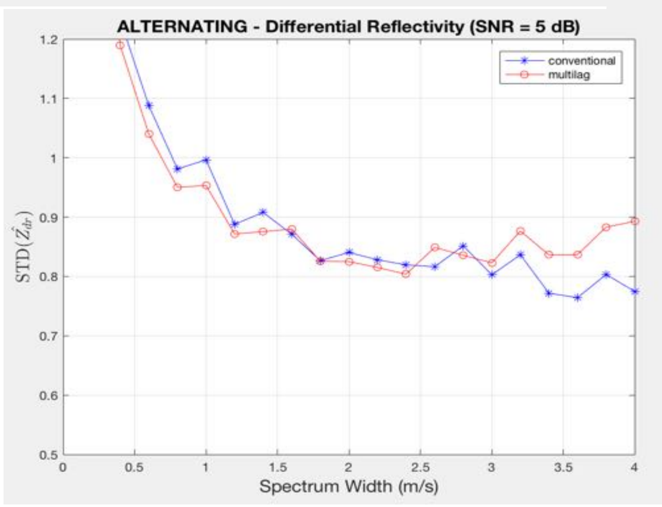
- **Simulation Parameters**: M = 128 pulses/dwell, SNR = 5 dB, Z_{dr} = 1, ρ_{hv} = 0.97; frequency = 9.41 GHz (X-band), PRF = 3750 Hz (40 km range)
  - Except for STD(ρ_{hv}), the estimators give nearly identical performance

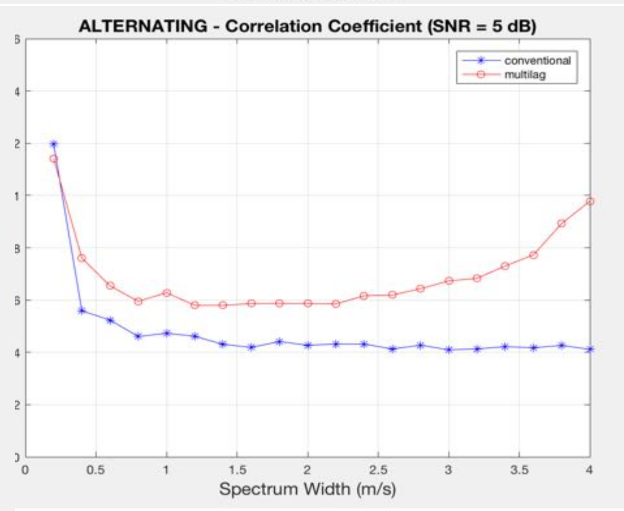






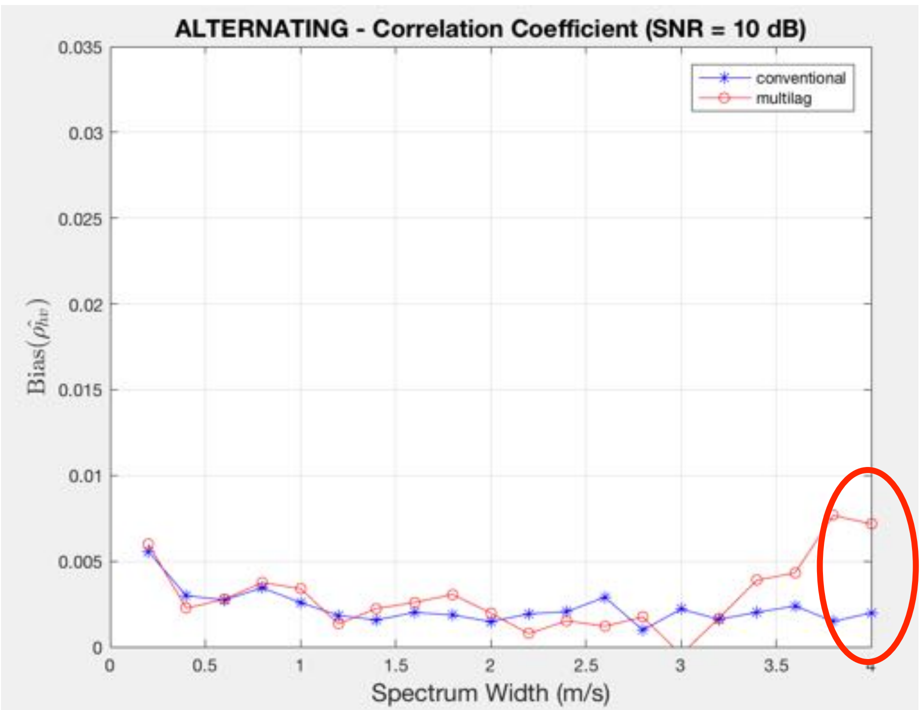




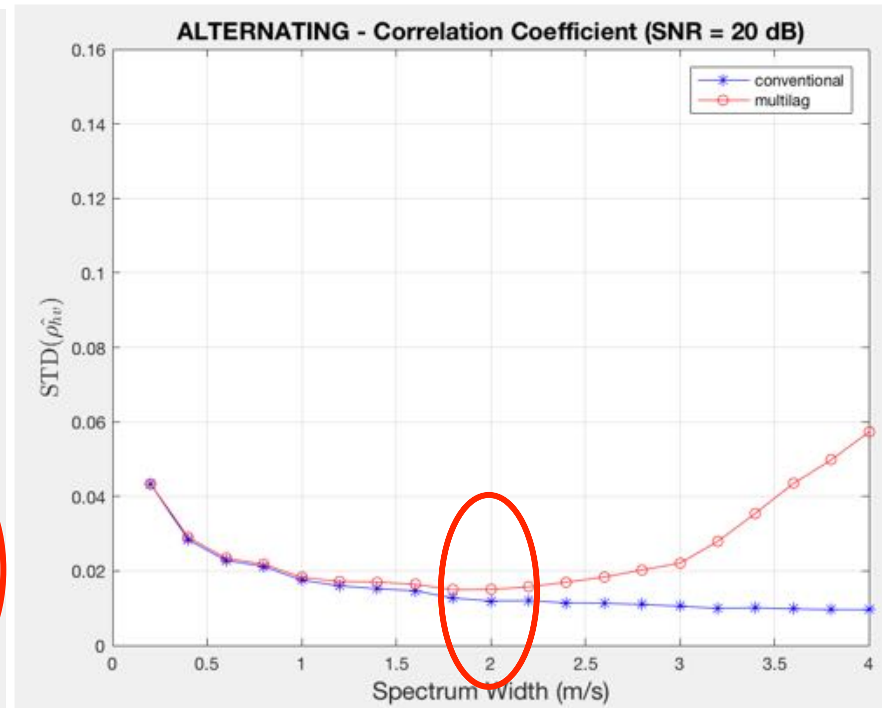


Simulation Results

- **Simulation Parameters**: ***M* = 128 pulses/dwell, Z_{dr} = 1, ρ_{hv} = 0.97; frequency = 9.41 GHz (X-band), PRF = 3750 Hz (40 km range)**
- **Comparing the ρ_{hv} estimators to the SENSR (2016) requirements**



SNR = 10 dB, W = 4 m/s → Bias(rhohv) < 0.006
Conventional bias = 0.002, multilag bias = 0.007

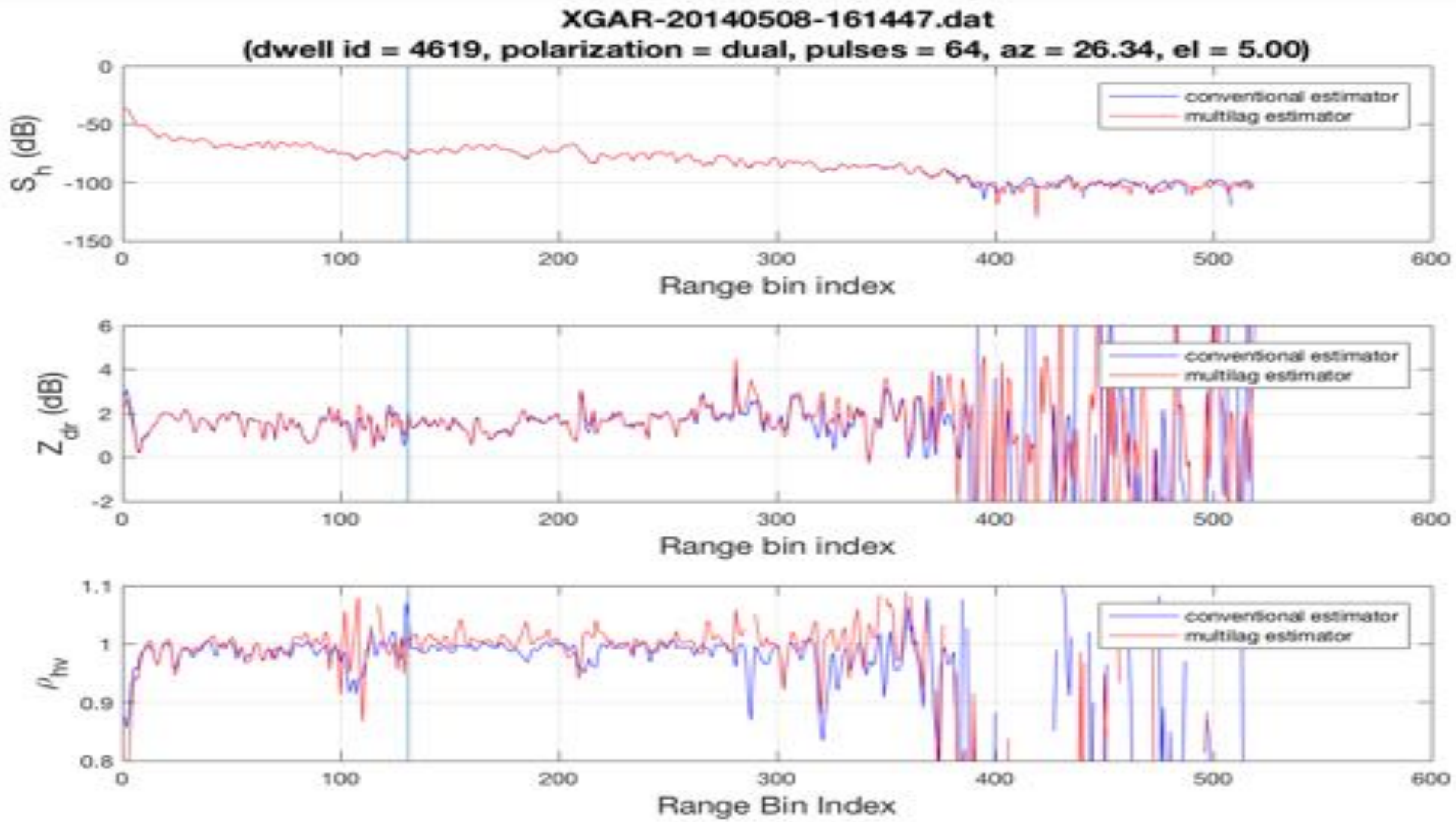


SNR = 20 dB, W = 2 m/s → STD(rhohv) < 0.006
Conventional STD = 0.012, multilag STD = 0.015

- ***Spectrum Efficient National Surveillance Radar* (SENSR): *Preliminary Performance Requirements*, Version 0.9, October 7, 2016.**

Real-World Results

- **X-band phased-array radar, ALTERNATING mode, *M* = 64 pulses/dwell, PRF = 3900 Hz, 30 km range**
  - Conventional estimator in blue, multilag estimator in red



Field Test Results

- X-band phased-array radar, ALTERNATING mode, *M* = 64 pulses/dwell, PRF = 3900 Hz, 30 km range
  - Conventional estimator on top, multilag estimator on the bottom