35 A METHOD TO LOCATE AIRBORNE RADAR OBSERVATION DATA

Shubei Wang*, Zhendong Yao
Chengdu University of Information Technology, Chengdu, China

1. INTRODUCTION

Airborne radar as a new meteorological detecting instrument, whose observation data with high spatial and temporal resolution, plays an important role in the research of strong convective weather fine structure and nowcasting. Airborne radar can not only observe the weather system flexibly, but also can detect some regions that ground-based radar is difficult to measure, like sea. Thus, airborne radar makes up the defect. In recent decades, airborne radar devised by NASA and putted on the unmanned aerial vehicle (uav) to observe weather, compared with manned machine, uav has the advantages of security, stability, and the prolonged flight time. But this kind of emerging weather radar is different from the fixed platform of ground-based radar, which carrying platform is moving ,with relative movement to target, and the instability of vehicle itself can also affect the detection position, giving rise to airborne weather radar data [1]processing with more harder than ground radar data processing. In order to apply detection data effectively to atmospheric science, hydrology, nowcasting, and other fields, this paper utilizes airborne radar (named HIWRAP) developed by NASA Goddard Space Flight Center as a research object, then do some researches on its observation data processing. First, The scanning strategy of the airborne weather radar was analyzed, and the location problems involved in radar observation data processing according to observational process are illustrated clearly. Then, this paper analyzes the detection process and clarified relations in relevant spatial locations. Next, a method to locate airborne radar observation data was described in this paper, and its process of locating observation data is applicable to other airborne radar whose scanning strategy is similar with HIWRAP. Finally, this paper discuss the performance of an early version of the method when it is run on data collected by the HIWRAP during GRIP and put forward a simple method to reduce time-consuming.

2. METHODOLOGY

In this part, scanning strategy of the airborne weather radar was analyzed at first, according to observational process, the location problems involved in radar observation data processing are illustrated clearly, namely the coordinate system for observation and data processing is not consistent. Using the coordinate transformation technique to find the actual location of the radar observation data and put all data information into the same coordinate system. Then , aiming at the problem of locating observation data, the

* *Corresponding author address:* Shubei Wang, Chengdu University of Information Technology, Chengdu, China, 610103
E-mail:Wang_Shubei@foxmail.com

detection processes were analyzed and relations in relevant spatial locations were clarified in this part. The homogeneous coordinate method is used to accomplish the transformation of data coordinate. In order to put the data in the same coordinate system, the intermediate coordinate systems are presented. Finally , the processes in detail from radar coordinate system to geodetic coordinate system are analyzed, and the location algorithm of data detected by airborne radar is put forward.

2.1 *Scanning strategy*

The High-altitude Imaging Wind and Rain Airborne Profiler (HIWRAP) is a dual-frequency (Ku- and Ka-band), and dual-look angle (30 & 40 degree incidence angle) Doppler radar system. It was developed under the support of the NASA Instrument Incubator Program (IIP) and flew for the first time on the NASA Global Hawk during Genesis and Rapid Intensification Processes (GRIP) in 2010 with a downward-looking conical scan antenna. From its scanning strategy during GRIP, we can see that HIWRAP scanning mode is similar to the PPI of ground-based radar. However, the scanning direction of HIWRAP is downward and the scanning result is conical when the plane is hovering as shown in Fig.1. In addition, when the plane is moving along a straight line with uniform speed and correct posture, the scanning result is spiral trajectories. Fig.1 also shows the trajectories of a radar beam with an incident angle at two different heights during flight.Fig.2 shows the downward looking trajectories of a radar beam with an incident angle during flight.

Figure 1 Scanning strategy of HIWRAP shows the trajectories of a radar beam with an incident angle at two different heights during flight.

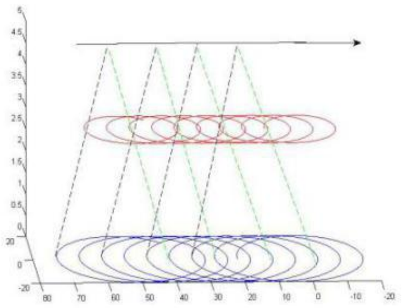


Figure 1 Scanning strategy

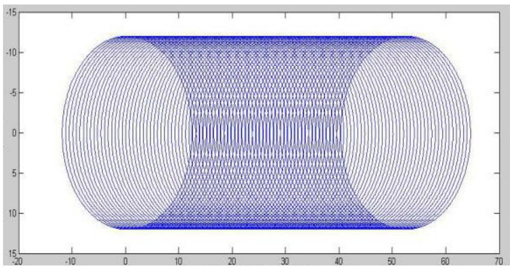


Figure 2 The downward looking trajectories of a radar beam with an incident angle when the plane is moving along a straight line with uniform speed and correct posture.

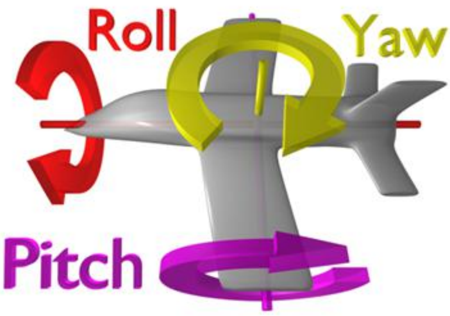
According to the radar system installation standard, the radar is fixed on the plane in the process of the detection. Therefore, the observation data (including azimuth, angle of pitch and distance) of radar is based on its own measurement coordinate system. However, the carrying platform is not stationary, the actual location of the data will change as the platform moves. So in order to seek out the actual location of the probe accurately, we need lead into the position and attitude parameter to complete the positioning. Fortunately, the Global Hawk UAV platform can provides these (including GPS Global Hawk latitude,GPS aircraft longitude,aircraft roll angle,aircraft pitch angle, aircraft track angle, aircraft altitude, aircraft heading).

2.2 *Parameters involved in positioning*

As mentioned and analyzed above, The determination of the detection position requires three steps. Firstly, the position of the aircraft is determined by GPS. And then, the position of the radar platform is confirmed by the attitude of the aircraft. Finally, we can ensure the detection position through radar measurement parameters. The aircraft position is provided by the GPS, including longitude, latitude and height. Radar measurement parameters include the pitch angle, rotation angle and pitch of the radar beam. The aircraft attitude is the state of the three axis of an aircraft in the air relative to a reference line or a reference plane, or a fixed coordinate system. It shows the angular position of an aircraft relative to the ground, as indicated by three angles: roll, pitch, yaw, as shown in Fig.3.

Figure 3 Aircraft attitude parameters.

The roll is the angle between the vertical plane of the longitudinal axis of the body and the symmetry plane, it will be a positive number when the aircraft roll to the



right; The pitch is the angle between the longitudinal axis of aircraft and the horizontal plane,it will be a positive number when the plane is flying up, conversely, it is negative. The yaw is the angle between projection of the vertical axis of the plane on the horizontal plane and the parameter line on the plane, it is positive when the nose yaw to right, conversely, it is negative.

In the course of motion, the fuselage moves around three axes(horizontal axis, vertical axis, vertical axis). The aircraft can move round one axis, or round three axis at the same time. These three angles of the plane reflect relative position of body coordinate system and the ground coordinate system, it responses the attitude of plane relative to the ground.

It can be seen from previous description that the coordinate systems are not the same when referencing these parameters to locate the observation data, it includes GPS and geodetic coordinate system which corresponding to position information, ground coordinate system and body coordinate system which corresponding to attitude information, and the radar coordinate measurement system and so on. In order to process and analyze radar data, all the detected data should be studied in the same inertial coordinate system. In this paper, we consider the geodetic coordinate system as inertial coordinate system. Therefore, in order to process data from the radar's measuring coordinate system to the earth coordinate, we should build the auxiliary intermediate coordinate system and complete the transformation of coordinates.

2.3 *Coordinate transformation*

2.3.1 *Homogeneous coordinate transfor -mation*

The homogeneous coordinate transformation can transform the points in the three-dimensional space to another coordinate system through 4 × 4 coordinate matrix. The the coordinate conversion method is as follows:

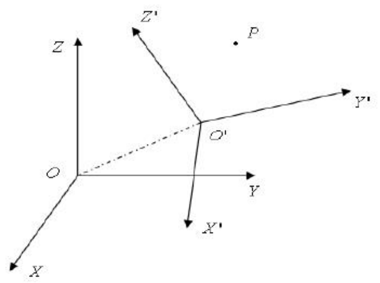
A column vector of n= (x1, x2, x3, x4) ^{T} represent the homogeneous coordinate points in three-dimensional space of point P. The coordinates of point P in space Cartesian coordinate system is (x, y, z), then (x, y, z) can be expressed by n=(x1, x2, x3, x4)^T. The conversion relationship is as follows:x =x1/ x4, y =x2/ x4, z =x3/ x4

In this paper, we use homogeneous

coordinates in three-dimensional space which expressed as:(x1, x2, x3, 1)^T

Firstly, the process of conversion of the Descartes coordinates is described as follows.

Figure4 The position relationship between the coordinates O '-X' Y 'Z' and the coordinate system O-XYZ.



The position relationship between the coordinates O '-X' Y 'Z' and the coordinate system O-XYZ as shown in figure4, P in the coordinates O '-X' Y 'Z' is expressed as (x ', y', z') ^{T}, then the coordinates (x, y, z) ^{T} of point P in the coordinate system O-XYZ can be obtained by the formula:

$$\begin{cases} x = u_1 x' + u_2 y' + u_3 z' + r_1 \\ y = v_1 x' + v_2 y' + v_3 z' + r_2 \\ z = w_1 x' + w_2 y' + w_3 z' + r_3 \end{cases} \tag{1}$$

Where, $(r_1, r_2, r_3)^T$ is the coordinate of O' point in the O-XYZ coordinate system, u_n, v_n, w_n, n=1~3 are the direction cosine of the three coordinate axis of O′-X′Y′Z′ relative to the coordinate system O-XYZ. In this paper, we express P in O-XYZ with homogeneous coordinates (x1, x2, x3, x4)^T and (x1 ', x2', x3 ', x4')^T in O '-X' Y 'Z' . Make x4 '= x4, and the formula above can be written as:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & u_3 & r_1 \\ v_1 & v_2 & v_3 & r_2 \\ w_1 & w_2 & w_3 & r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} \tag{2}$$

The transformation matrix is T, then

$$T = \begin{bmatrix} u_1 & u_2 & u_3 & r_1 \\ v_1 & v_2 & v_3 & r_2 \\ w_1 & w_2 & w_3 & r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

Represented by matrices:

$$T = \begin{bmatrix} L & R \\ 0 & 1 \end{bmatrix} \tag{4}$$

Where L is 3 * 3 matrix, it consists of two coordinate direction cosine, represents the rotation between the coordinate system; R is the column matrix of the relative position of the origin in two coordinate system, represent the shift between the two coordinate systems. The homogeneous transformation matrix T can be decomposed into rotation and translation. Written down as:

$$T = T_r T_m \tag{5}$$

Where, Tr is a rotation transform, and Tm is a translation transformation, the matrix form of them are: $T_r = \begin{bmatrix} E & R \\ 0 & 1 \end{bmatrix}$ $T_m = \begin{bmatrix} L & 0 \\ 0 & 1 \end{bmatrix}$

2.3.2 *The 3D coordinates of the homogeneous coordinate transformation*

Coordinate system changes including rotation, translation and scaling. By using these transformations to complete the conversion of coordinates based on the different coordinate systems, the conversion process becomes intuitive, simple and clear.

a.Scaling.The coordinate system takes the origin as the center of scale, the coordinate transformation formula is as follow:

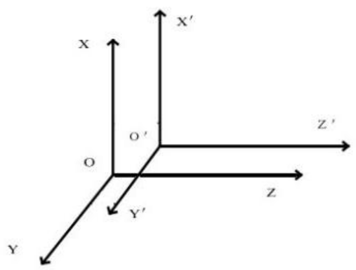
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} B_x & 0 & 0 & 0 \\ 0 & B_y & 0 & 0 \\ 0 & 0 & B_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{4}$$

The procedure indicates that the coordinate system is centered at the origin and amplifies (Bx, By, Bz>1) or shrinks (Bx, By, Bz<1) in the x, y, and z directions to the original Bx, By, and Bz times.

b.Coordinate translation (as shown in Fig.5), assuming that coordinates of O in the new coordinate system O '-X' Y 'Z' is (-e, -f, -g). Then the coordinate transformation formula is:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -e \\ 0 & 1 & 0 & -f \\ 0 & 0 & 1 & -g \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (5)$$

Figure 5 Coordinate translation.



c.Rotation of coordinate systems can be divided into three cases: rotate around X, Y, Z, respectively.

Coordinate rotating ψ around X axis is shown in Figure 6. The coordinate transformation formula is as follow:

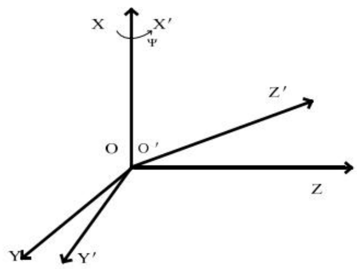


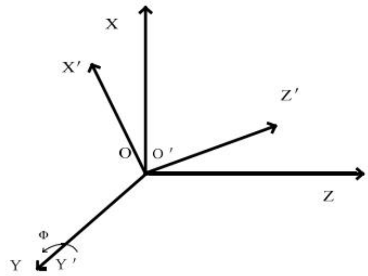
Figure 6

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & \sin\psi & 0 \\ 0 & -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (6)$$

Coordinate rotating Φ around Y axis is shown in Fig.7.

The coordinate transformation formula is as follow:

Figure 7

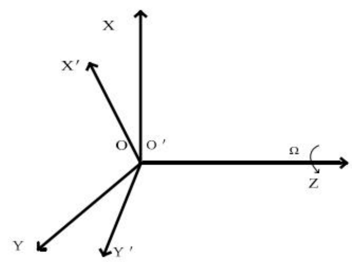


$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Phi & 0 & -\sin\Phi & 0 \\ 0 & 1 & 0 & 0 \\ \sin\Phi & 0 & \cos\Phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (7)$$

Coordinate rotating Ω around Z axis is shown in Fig.8.

Figure 8

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\Omega & \sin\Omega & 0 & 0 \\ -\sin\Omega & \cos\Omega & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (8)$$



2.4 C*oordinate systems*

To sum up, arbitrary three-dimensional space Cartesian coordinate system can be transformed base on the homogeneous coordinate system. In this study, the key to finding the location of data points is to find the location relations of all auxiliary coordinates. And then, convert each auxiliary coordinate system based on the homogeneous coordinate conversion method. Finally, integrate the conversion equation to establish a positioning algorithm.

In the process of deriving the coordinate transformation equation of airborne radar observation data, we use the space Descartes Cartesian coordinate system, which satisfies the right hand system rule. By analyzing the radar scanning strategy and actual conditions of the aircraft platform, this paper establish five related coordinate systems, which are used to establish the transformation relationship from radar to geodetic coordinate. The five coordinate system are radar installation platform coordinate system(RS), aircraft coordinate system(AS), geographical coordinate system(SS), spatial geodetic coordinate system(GS) and geodetic coordinate system(CS). Among them, the first three coordinate systems can realize the transformation of coordinates under different systems according to the method of homogeneous coordinate transformation, but the geodetic coordinate system and spatial geodetic coordinate system are built on the earth ellipsoid model, we will study the conversion relationship between them in the future.Several coordinate systems involved in the conversion are described as follows:

(1) C (Oc – XcYcZc) Geodetic coordinate system

The origin of the WGS84 geodetic coordinate system lies in the center of mass of the earth, and the Zc axis points to the earth's North Pole, Xc axis points from the origin to the intersection of the meridional plane of Greenwich and the equator, Yc axis is perpendicular to the XcOcZc plane and forms the right coordinate system with the Zc axis and the Xc axis. The coordinate of a given point is (B, L, H), it represents the latitude, longitude, and elevation of the points. The sketch map of the earth coordinate is as follows:

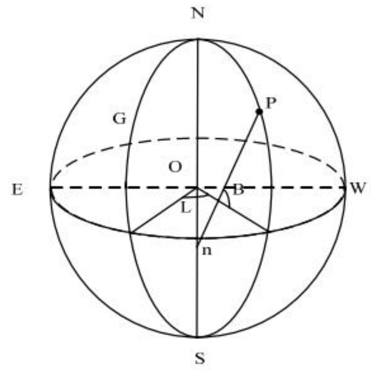


Figure 9 Geodetic coordinate system

The angle(L) between the meridian plane (NPS) of ellipsoidal point P and the prime meridian plane (NGS) is called geodetic longitude, the longitude is positive to east of NGS and negative to the west. The angle (B) between the normal of ellipsoid surface of P point (nP) and equatorial plane is called geodetic latitude of P point, the latitude is positive to the north of EOW and negative to south. The distance from P to ellipsoid surface along the normal direction is geodetic height, the distance is positive when it is outward from the ellipsoid surface.

(2) spatial geodetic coordinate system G(Og –XgYgZg)

The space geodetic coordinate system has the same coordinate axis as the geodetic coordinate system. The difference between them is that the space geodetic coordinate system is the Descartes coordinate system, and the geodetic coordinate system is ellipsoidal coordinate system. The space geodetic coordinate of a point is expressed as (x_g, y_g, z_g).

(3) geographical coordinate system S (Os – XsYsZs)

In this paper, the aircraft platform can be regarded as a point, and its position (B, L, H) is the origin of the geographical coordinate system, it can be expressed as (λ_s, α_s, h_s) through longitude, latitude and altitude. Zs axis points the north direction,Xs axis points from the origin to the zenith direction. The right coordinate system, which is made up of Ys, Zs and Xs, is called the geographic coordinate system, and it is the Descartes coordinate system. The geographic coordinate of one point is expressed as (x_s, y_s, z_s).

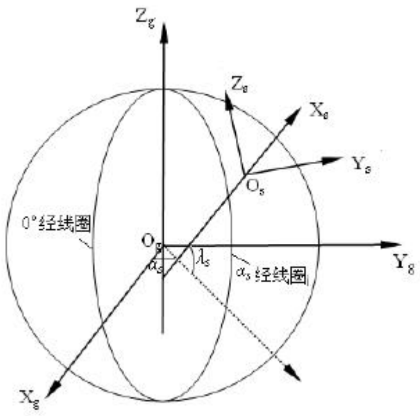


Figure 10 shows that when the aircraft position is regarded as the origin of the geographical coordinate system, spatial geodetic rectangular coordinate system and the geographic coordinate system can be linked by the GPS, and put up a bridge for our position of detection data. Through this bridge, the aircraft position parameters are introduced to complete the location of the platform in the geographical coordinate system, and to pave the way for the subsequent introduction of the coordinate system of the factors affecting the aircraft attitude.

(4) aircraft coordinate system A(Oa – XaYaZa)

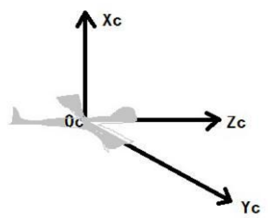
The aircraft coordinate system and the geographic coordinate system have the same origin. The plane coordinate system is equivalent to the coordinate system that the geographic coordinate system rotates through three axes. When the plane attitude angle is zero, the plane coordinate system and the geography coordinate system will coincide. Supposing the three axis attitude angles of the aircraft are θ, κ, λ, and the coordinate system of the aircraft is the coordinate system formed by the geographic coordinate system rotates θ, κ, λ around the Ya (pitch), Za (roll), Xa (yaw), respectively. The angle between the head and the north direction is yaw angle when the plane made its flight toward the target. The aircraft coordinates of a point is represented by (x_s, y_s, z_s).

(5) radar installation platform coordinate system(RS) Oc-XcYcZc

The radar coordinate system Oc-XcYcZc constitutes the Cartesian Cartesian coordinate system that satisfies the right hand system. As shown in the following figure.

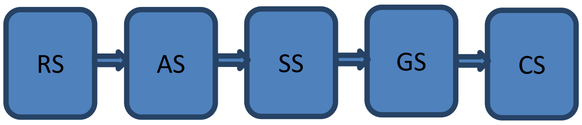
Figure 11 radar installation platform coordinate system(RS)

In this coordinate system, we use radar beam launching points as the origin of radar coordinate systems, the Zc axis is parallel to the axis of the aircraft fuselage,the direction of the origin pointing along the nose is positive, Xc, Yc axis plane perpendicular to the Zc axis, the Yc axis is positive along the origin to the right wing. The Zc axis perpendicular to the plane, and it is positive when it points to the top of the plane.



2.5 *The process of location and method*

Through the foregoing, we have specified required parameters and coordinate systems. The parameter are longitude, latitude, altitude which provided by GPS; aircraft navigation attitude (yaw, pitch, roll); radar scanning parameters (pitch, azimuth, distance). Using these parameters, the data can be processed under system from the radar's own measuring coordinate system to the geodetic coordinate system. The whole process is as follows. **Figure 12**



2.5.1 *Initial coordinate*

According to the azimuth, pitch and radial distance of the observation data points, the Initial coordinates in the radar coordinate system are obtained.

In this coordinate system, the coordinates of a point P (x, y, z) in the space can be obtained by the following formula:

y=rsinθcosϕ

z=rsinθsinϕ

x=rcosθ

Where r is the distance between origin O and P; θ is the angle between OP and positive X axis, which can be calculated by pitching, in this radar, pitch = π - beam incident angle; ϕ is azimuth which is from the positive X axis view turn counterclockwise from the Y axis to the corner of the OM, where M is the projection point on the surface of P in ZOY plane. r，θ，ϕ can all be read from radar observation data, clearly, variation range of r，θ，ϕ are r ∈ [0, +∞)，θ ∈ [0, π]， ϕ ∈ [0, 2π]. The geometric meaning of them are as follow : r is radial distance between P and the origin; θ is the incident angle between the OP and the positive X axis; ϕ is azimuth between the line of projection of the line segment of origin to point P in the Y-Z plane and Y axis.

T= (X_C, Y_C, Z_C, 1) represents data point position in radar coordinate, the Xc, Y_C and Z_C can calculated by the formula above. Where, the radial distance, the incident angle and azimuth can read from the radar observation data file.

2.5.2 *The coordinate transformation process of RS to the AS*

The direction of the three axis of the carrier coordinate system is the same as the three orthogonal axes of the radar coordinate system, but the point of origin of the coordinate system is aircraft, not the radar. So, this transformation is the translation of the origin of the coordinate system, and the three axis remains the same direction. The coordinate system is converted from radar coordinate system Of-XYZ to carrier coordinate system Oa-XYZ. Suppose that the coordinates of the aircraft particles in the original radar coordinate system is (e, f, g), and the coordinates of the original radar system are represented by a matrix $F = [x_f, y_f, z_f, 1]^T$, and the carrier coordinates is $A = [x_a, y_a, z_a, 1]^T$, Then the transition relationship is as follow:

$$\begin{bmatrix} x_f \\ y_f \\ z_f \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & e \\ 0 & 1 & 0 & f \\ 0 & 0 & 1 & g \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_a \\ y_a \\ z_a \\ 1 \end{bmatrix} \quad (9)$$

This is the transition process from the carrier coordinate system to the radar coordinate system, where, in the transformation, matrix Q_1 is

$$Q_1 = \begin{bmatrix} 1 & 0 & 0 & e \\ 0 & 1 & 0 & f \\ 0 & 0 & 1 & g \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Therefore, the conversion of radar coordinate system to carrier coordinate system is: $A = Q_1^{-1} F$ that is:

$$\begin{bmatrix} x_a \\ y_a \\ z_a \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -e \\ 0 & 1 & 0 & -f \\ 0 & 0 & 1 & -g \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_f \\ y_f \\ z_f \\ 1 \end{bmatrix} \quad (10)$$

2.5.3 *Conversion of AS to SS S (Os –*

XsYsZs)

Firstly, the transformation of geographical coordinate system to carrier coordinate system is deduced.

Take the aircraft platform as an example, the aircraft platform can be regarded as a point, and its position (B, L, H) is the origin of the geographical coordinate system, it can be expressed as (λs，αs，hs) through longitude, latitude and altitude. Zs axis points the north direction,Xs axis points from the origin to the zenith direction. The right coordinate system, which is made up of Ys, Zs and Xs, is called the geographic coordinate system, and it is the Descartes coordinate system. The geographic coordinate of one point is expressed as (xs，ys，zs).

The transform from the geographic coordinate system S to the carrier coordinate system A as follows.

A.Rotate α around the X axis, where the transformation matrix Q4 is

$$Q_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where α is heading angle, the angle start from the North, and Rotate as anticlockwise.

B.Rotate β around the Y axis, where the transformation matrix Q4 is

$$Q_3 = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where β is Pitch.

C.Rotate γ around the Z axis , the transformation matrix Q2 is

$$Q_2 = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where γ is roll.

The coordinates of the data points in the geographical coordinate system are represented as matrices $S = [x_s, y_s, z_s, 1]^T$, and the coordinates in the plane coordinates are taken as matrices $A = [x_a, y_a, z_a, 1]^T$, then the conversion process from the geographical coordinate system to the aircraft coordinate system is $A = Q_2 Q_3 Q_4 S$.

So, the translation from the carrier coordinate system to the geographic coordinate system is:

$$S = Q_4^{-1} Q_3^{-1} Q_2^{-1} A \tag{11}$$

2.5.4 *Conversion of SS to GS*

The conversion process of geodetic space rectangular coordinate system to geographical coordinate system is simple and clear. It can be analyzed first and then backward deduction to obtain the conversion of geodetic coordinates system to the rectangular space coordinate system.

The conversion process of geodetic space rectangular coordinate system to geographical coordinate system as follows.

A. Move OgOs' ($O_g O_s' = Ne^2 \sin\lambda_s$) along the negative direction of Z axis. where Transformation matrix Q8 is

$$Q_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & O_g O_s' \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B.Rotate αs (longitude)around the Z axis. Where Transformation matrix Q7 is

$$Q_7 = \begin{bmatrix} \cos\alpha_s & \sin\alpha_s & 0 & 0 \\ -\sin\alpha_s & \cos\alpha_s & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

C.Rotate $-\lambda_s$ (λ_s is latitude) around the Y axis . where Transformation matrix Q6 is

$$Q_6 = \begin{bmatrix} \cos\lambda_s & 0 & \sin\lambda_s & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\lambda_s & 0 & \cos\lambda_s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

D.Move $O_sO_s^{'}$ (H+h, height) along the positive direction of X axis .where Transformation matrix Q5 is

$$Q_5 = \begin{bmatrix} 1 & 0 & 0 & O_sO_s^{'} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The coordinates of the data points in the geodetic Cartesian coordinate system are represented as matrices $G = [x_g, y_g, z_g, 1]^T$, and the coordinates in the geographical coordinate system are represented as matrices $S = [x_s, y_s, z_s, 1]^T$, the conversion process from the geodetic Cartesian coordinate system to the geographical coordinate system is $S = Q_5Q_6Q_7Q_8G$.

so, the transformation from the geographic coordinate system to the spatial geodetic Cartesian coordinate system is:

$$G = Q_8^{-1}Q_7^{-1}Q_6^{-1}Q_5^{-1}S \quad (12)$$

In summary, the transformation the carrier coordinate system to the geodetic Cartesian coordinate system is:

$$G = Q_8^{-1}Q_7^{-1}Q_6^{-1}Q_5^{-1}Q_4^{-1}Q_3^{-1}Q_2^{-1}A \quad (13)$$

Due to $A = Q_1^{-1}F$, so the transformation from radar coordinate system to spatial Geodetic coordinate system is:

$$G = Q_8^{-1}Q_7^{-1}Q_6^{-1}Q_5^{-1}Q_4^{-1}Q_3^{-1}Q_2^{-1}Q_1^{-1}F \quad (14)$$

Where,

$$\begin{bmatrix} x_a \\ y_a \\ z_a \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -e \\ 0 & 1 & 0 & -f \\ 0 & 0 & 1 & -g \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_f \\ y_f \\ z_f \\ 1 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 1 & 0 & 0 & e \\ 0 & 1 & 0 & f \\ 0 & 0 & 1 & g \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_5 = \begin{bmatrix} 1 & 0 & 0 & O_sO_s^{'} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_6 = \begin{bmatrix} \cos\lambda_s & 0 & \sin\lambda_s & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\lambda_s & 0 & \cos\lambda_s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_7 = \begin{bmatrix} \cos\alpha_s & \sin\alpha_s & 0 & 0 \\ -\sin\alpha_s & \cos\alpha_s & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & O_g O_s^{'} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Namely,

$$\begin{bmatrix} x_g \\ y_g \\ z_g \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & O_g O_s^{'} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\alpha_s & \sin\alpha_s & 0 & 0 \\ -\sin\alpha_s & \cos\alpha_s & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\lambda_s & 0 & \sin\lambda_s & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\lambda_s & 0 & \cos\lambda_s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & O_s O_s^{'} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha & 0 \\ 0 & -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & 0 \\ -\sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & e \\ 0 & 1 & 0 & f \\ 0 & 0 & 1 & g \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_f \\ y_f \\ z_f \\ 1 \end{bmatrix}$$

The homogeneous coordinate transformation matrix is an orthogonal matrix, and the inverse matrix of the orthogonal matrix is equal to its transpose matrix, so the formula can be written as:

$$\begin{bmatrix} x_g \\ y_g \\ z_g \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -O_g O_s^{'} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha_s & -\sin\alpha_s & 0 & 0 \\ \sin\alpha_s & \cos\alpha_s & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\lambda_s & 0 & -\sin\lambda_s & 0 \\ 0 & 1 & 0 & 0 \\ \sin\lambda_s & 0 & \cos\lambda_s & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -O_s O_s^{'} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 & 0 \\ \sin\gamma & \cos\gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e \\ 0 & 1 & 0 & -f \\ 0 & 0 & 1 & -g \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_f \\ y_f \\ z_f \\ 1 \end{bmatrix}$$

Thus, the process in detail from radar coordinate system to geodetic coordinate system is described, and the location algorithm of data detected by airborne radar is put forward.

Transformation formulation:

$G=Q_8^{-1}Q_7^{-1}Q_6^{-1}Q_5^{-1}Q_4^{-1}Q_3^{-1}Q_2^{-1}Q_1^{-1}F$

Where, F is a matrix represented the detected data position under radar coordinate system, it consist of range from radar,rotation angle and incidence angle. Q1 is a transformation matrix transformed data coordinates from aircraft coordinate system to radar coordinate system, it includes the spatial position of origin of radar coordinate system relative to the spatial position of aircraft particle. Q2Q3Q4 are transformation matrix transformed data coordinates from aircraft intermediate coordinate system to aircraft coordinate system, it includes the aircraft attitude parameters (roll angle, pitch angle, track angle and heading). Q5Q6Q7Q8 are transformation matrix transformed data coordinates from space rectangular coordinate system to aircraft intermediate coordinate system, it includes GPS information(aircraft latitude, longitude and altitude) and geometric parameters of the ellipsoid model based on earth. G is a matrix represented the observation data coordinate under geodetic coordinate system.

3. VERIFICATION METHODOLOGY AND RESULT

3.1 *Analysis*

The real data collected during GRIP were chosen to test the algorithm. Figure of observation data position mapped by using method are consistent with the ideal figure according to the radar scanning strategy. The measures (including tracks) and their comparisons show that the algorithm performs well.

Those pictures are mapped by using data collected by HIWRAP on 17th Step. 2017. figures are all Inner Beam in Chirp mode.

Figure 13 Data for the previous 200 time points

Fig. 13 (1)

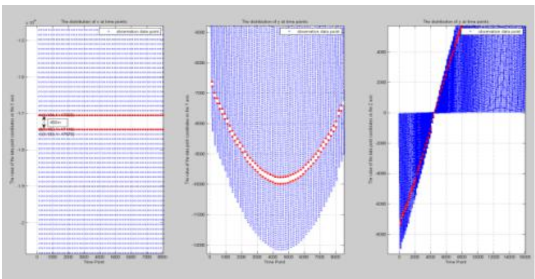


Fig. 13 (2)

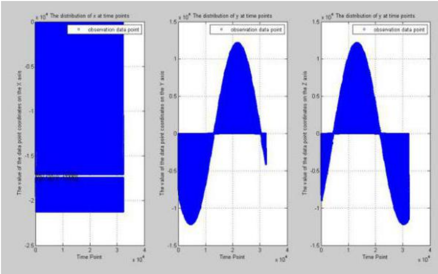


Fig. 13(3)

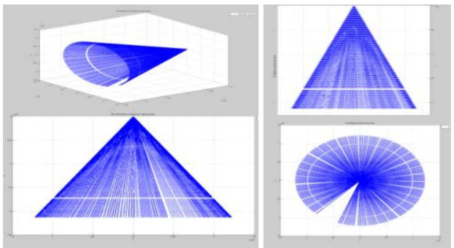
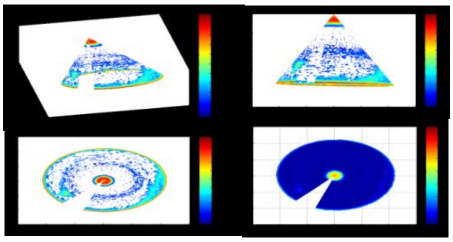


Fig. 13(4)



(1)reading the data, it was found that in this document, named kainnerchirp_20100917_000000-002430, all the data had a large interval sampling gap between 131 and 132 gate with a length of three ranges of gate spacing, which was consistent with the red part of Fig. 13 (1).Where, Spacing of large gap is 150 * 3 = 450 m.

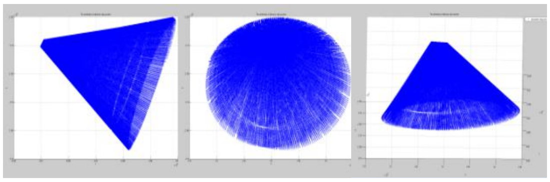
(2)According to the conical scanning strategy, and the inner beam incident angle of 30°, scan zone can be seen in the figure 13(3), this imaging on 200 time data points is roughly consist with the characteristics of cone, its three view drawing express more clearly.

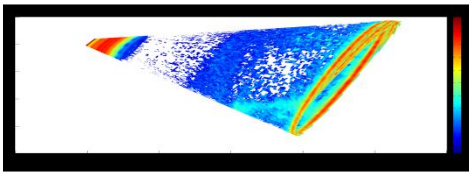
(3)The further analysis, the aircraft is in moving forward, so it is spiral scan forward, therefore, its three view drawing, graphics and positive side graphics are not the same as is hovering present circular plane, but are approximately same, vertical view shows approximate circular, pictured above, in accordance with the actual situation.

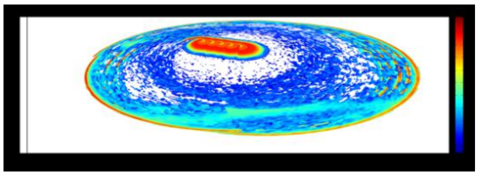
(4) In the process of the aircraft in flight, spiral, his motion equation is roughly in line with the helix equation, containing sine function, according to Fig. 13 (2) in the x, y, z coordinates in the distribution of the point in time, as you can see, they generally has the characteristics of sine function, in accordance with the theory.

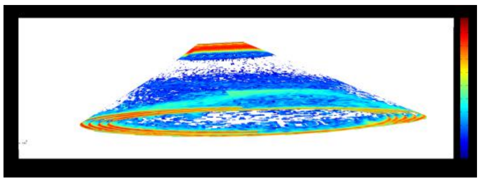
(5)Fig. 13(4) shows the spatial three-dimensional distribution of dBZ, and the position of the data is consistent with the position in figure. Among them, the last picture is the distribution of pwr, we can see that its distribution is also consistent with dBZ and position.

Fig.14 Data for the previous 1000 time points









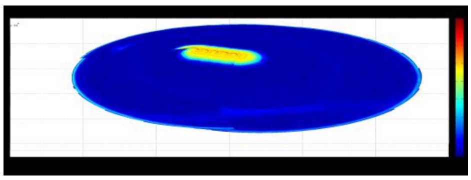
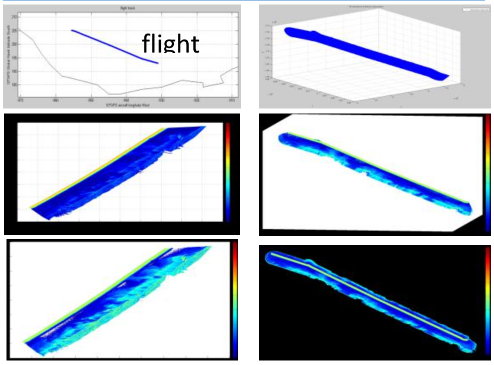


Fig.14 further verify the above conclusions. Figures are mapped by 1,000 time points, compared to the previous 200 time points, with obvious scanning space flight trajectory moves, and a close look at figure, we found that there is obvious spiral "information", in accordance with the theory. Through this method, has realized the detection of the mobile platform. And according to data and figures, the results are preliminary match expected.

Finally, Fig.15 shows entire file (88152 time points) 3d display graphics from three different views by using this method.As we can see, the flight path and the echo position are corresponding, and the dBZ intensity distribution is consistent with the pwr strength distribution.

Fig.15 All time points(88152)



Those figures shows that method is feasible, but at the time of implementation also has some problems, such as the time is too long, this paper took the detection data from the different time (including sampling time and total time) with the method of trial, found that all need to spend a lot of time, sometimes also can cause computer pause, the following is used for the data file time consuming situation analysis.

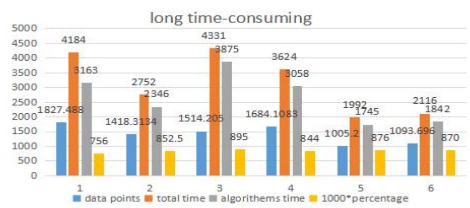


Fig.16 Time required for processes

The blue is collecting time of each data file, the orange is the total time of each data consist of location and display, the gray is spending time by using method proposed in this paper in each data file, the yellow is the algorithm accounted for the proportion of total time, Fig.16 shows that this algorithm is very time consuming, accounted for more than seventy-five percent in each run, took too long and easy to cause computer pause even crash that is very unfavorable. So, the improvement of the algorithm is very necessary, a preliminary analysis of this method found that program need to call the positioning function in every time point, and each data file’s collected time point from HIWRAP are too many, and positioning function made of 8 matrices of 4×4, so the preliminary way to reduce running time is reducing matrix order. According to this idea, the homogeneous coordinates and the positioning method are analyzed, an improved algorithm is proposed as follows:G=A7A6(A5A4A3F+A1)

$$\begin{bmatrix} x_g \\ y_g \\ z_g \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha_s & -\sin\alpha_s & 0 \\ \sin\alpha_s & \cos\alpha_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\lambda_s & 0 & -\sin\lambda_s \\ 0 & 1 & 0 \\ \sin\lambda_s & 0 & \cos\lambda_s \end{bmatrix} \left(\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ h \end{bmatrix} \right)$$

3.2 *Summary and future work*

This method can locate the actual location of the radar observation data and put all data information into the same coordinate system.

There are plans to improve the algorithm for time-consuming. A method of reducing the dimension matrix is presented to decrease the computing time. And it needs further analysis and verification.

There are plans to analyse the influence of location factor on location accuracy and discuss the possibility of correcting error and method of improving accuracy.

There are plans to develop integrated software for airborne weather radar data to achieve diversification of observation data information display.

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