Propagation and backscattering challenges for planar polarimetric phased array radars

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Abstract
Estimating the polarimetric variables with planar phased array radars requires calibration at every pointing direction. In the principal planes calibration is relatively simple. This is because to first order the transmitted fields from two radiators that determine the polarization are orthogonal and if the array is in the vertical plane there is no coupling through propagation and backscattering by rain. This means that separate calibration of each channel can be made. If the pointing direction is outside the principal planes and/or the plane of the array is tilted, the polarization of the transmitted wave in general has both the horizontal (H) and the vertical (V) components, even if only the H or V port is excited. In that case, the estimates of the polarimetric variables incur a geometrically induced bias which is affected by transmitted wave characteristics, propagation, and backscattering. The fundamental challenge is to devise a planar polarimetric PAR that can overcome this geometrically induced bias. Design alternatives that mitigate this bias are presented herein. These are: a) Antennas for which the V radiating element is an electric dipole and the horizontal one is a magnetic dipole. b) Combined use of the antenna ports so that the transmitted field is composed of equal horizontal and vertical component. c) Constrained measurements within a narrow range of directions close to the principal planes. d) Alternate transmission but simultaneous reception through the two ports. e) Phase coding of the transmitted signals in each port. The maturities of these alternatives as well as the relative merits are discussed.

1. Introduction

We consider a planar phased array (PAR) radar antenna with dual polarization. Specifically we assume that the Port 1 produces the intended horizontally (H) polarized field and the Port 2 generates the intended vertically (V) polarized field. It is impossible to produce pure horizontal or vertical field within the beam at all pointing direction with elements of the same type. The distribution of the cross-polar field within the beam depends on the physical structure of the radiating element and the direction of the beam.

If the elements are of the same type then the cross-polar field in the principal planes would ideally be zero. If the co-polar beam is in the principal plane then the cross-polar field at beam center is zero. But if the beam is steered out of the principal plane the cross-polar field will have one prominent peak at or near beam center (i.e., the cross-polar beam's axis nearly coincides with the axis of the copolar beam). This peak is geometrically induced. It has been demonstrated (Zrnic et al., 2010) that it can cause significant bias in the polarimetric variables.

2. Designs to mitigate geometric bias (tilted array)

If the plane of the array is not vertical the intended H polarized fields will be at an angle with respect to the horizontal axis and the intended V field will no longer lie in vertical planes.

This will cause bias in the polarimetric variables and various methods aimed at reducing the effect of this bias are discussed next.

a) *Collinear magnetic and electric dipoles*

First consider the dipole array is not tilted. The vertically oriented electric dipoles produce an electric field oriented along the meridional lines of the radiation sphere that has a vertical polar axis centered on the array. Thus the electric field lies in vertical planes containing the polar axis. Although the field orientation is truly vertical only at the zero elevation angle, this is not detrimental to polarimetric measurements at low elevation angles where the small departure from the vertical causes insignificant bias in differential reflectivity. Vertical magnetic dipoles produce a field parallel to horizontal planes. If the electric and magnetic dipoles are collinear the intended H and V fields at every pointing direction are orthogonal. Therefore, the PAR antenna comprised of collinear magnetic and electric dipoles will produce orthogonal H and V fields in all pointing directions (Crain and Staiman 2009). Nonetheless if the array is tilted the intended H field (out of the principal vertical plane) will not be horizontal and similarly the intended V filed will not be "vertical". The physical layout of such dipoles is three dimensional and the developments so far were exploratory.

b) *Antennas with patch radiators*

Excitation of Port 1 for radiating the H field (by the vertical sides of the patch) also causes radiation of the V field from the horizontal sides (Bhardwaj and Rahmat-Samii 2014). This radiation creates a cross-polar pattern which at broad side has four symmetric peaks of equal intensity but opposite sign (two of the same sign along each diagonal cut). In this case if the cross-polar peaks are at least 25 dB weaker than the co-polar peak, the first order bias (proportional to the one way cross-polar field) in the polarimetric variables is null and the Z_{DR} bias is insignificant (Zrnic et al., 2010). For beams in the principal planes (away from broadside) a pair of these cross-polar peaks shifts towards the beam center location and is symmetrically split with respect to the principal plane of the scan; the other pair diminishes in intensity. Again if the peaks' intensity is more than 25 dB below the co-polar peak the bias is insignificant. If the beam is steered away from the principal planes a single cross-polar peak collinear with the copolar one is created causing significant bias. Special designs might reduce the cross-pol radiation so that its effects on the polarimetric variables are acceptable. Regardless if this can be achieved, the cross-polar fields generated by the cross-polar radiating sides are not considered here. We concentrate on the "geometrically" induced cross-pol radiation associated with the copolar radiating sides.

In Fig. 1 the orientation of the electric field at beam center (θ_0, φ_0) is plotted for an array tilted by γ deg.

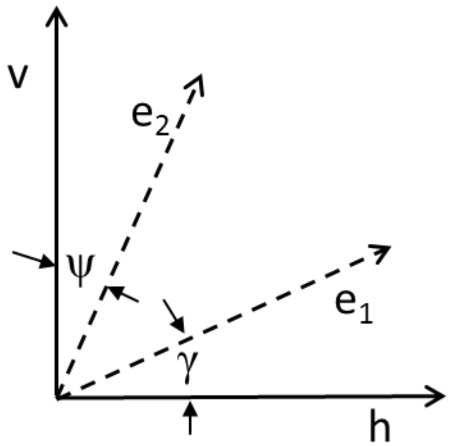

Fig. 1. The horizontal (h) and vertical (v) axis in the polarization plane of a propagating EM wave. The axis h is parallel to the ground; the axis v lies in a vertical plane. The axis $e_1(\theta_0,\varphi_0)$ and $e_2(\theta_0,\varphi_0)$ indicate the direction of the fields E_1, E_2 at beam center generated by the ports 1 and 2.

Rather than using the copolar (one-way) field pattern functions of Zrnic, et al., [2010]

$$F_{11}(\theta_o,\phi_o)=\sqrt{g_{11}}\,f_{11}(\theta_o,\phi_o)\text{, etc.} \tag{1a}$$

we use

$$g_1^{1/2}=\sqrt{g_{11}}\,f_{11}(\theta_o,\phi_o) \tag{1b}$$

and

$$g_2^{1/2}=\sqrt{g_{22}}\,f_{22}(\theta_o,\phi_o)\,, \tag{1c}$$

to designate the magnitudes of the electric fields (in the far field region) at beam center. For weather observations in which the radiation sphere's polar axis is aligned with the vertical direction, the gain in any direction (θ,ϕ) also depends on the beam direction (θ_0,ϕ_0).

We assume (with no loss in substance) that in the broadside direction the two ports produce the same field magnitudes (this needs to be calibrated in the backend of the antenna) but have a difference in phase (on transmission) β. This difference may vary with the pointing angle. If a phase code $c(n)$ is applied to mitigate the effects of coupling (Zrnic et al. 2014) this difference would be a function of nT_s and can be expressed as $\beta(n) = \beta_o + c(n)$. Furthermore, assume propagation through media of oriented oblate scatterers produces differential phase Φ_{DP}. For compactness let's use $S_h = \sum_{V_6} s_{hi}$ and similarly $S_v = \sum_{V_6} s_{vi}$ where the summation is over the scatters (i index in the resolution volume V_6). The s_{hi} is the element of the backscattering matrix for the horizontally polarized incident field; assume oblate spheroids (no depolarization) hence in the usual notation (a symmetry axis, b long axis) the $s_{hi} = s_{bi}$. Moreover, the range dependence and reshuffling are implicit in the summation but omitted for brevity sake. The matrix transformation **P** relating the variables e_1,e_2 to the variables h,v (Fig. 1) is

$$\mathbf{P}=\begin{vmatrix}\cos\gamma & \sin\psi\\ \sin\gamma & \cos\psi\end{vmatrix},\quad \text{therefore}\quad \begin{vmatrix}\mathrm{h}\\ \mathrm{v}\end{vmatrix}=\mathbf{P}\begin{vmatrix}\mathrm{e}_1\\ \mathrm{e}_2\end{vmatrix}. \tag{2}$$

Then the received voltage V_1 (corresponding to the H field) and V_2 (corresponding to V field) are

$$\begin{vmatrix} V_1 \\ V_2 \end{vmatrix} = C \begin{vmatrix} g_1^{1/2}(\theta,\phi) & 0 \\ 0 & C_R g_2^{1/2}(\theta,\phi) e^{j\xi} \end{vmatrix} \boldsymbol{P}^{\mathrm{t}} \begin{vmatrix} S_{\mathrm{h}} & 0 \\ 0 & S_{\mathrm{v}} \end{vmatrix} \boldsymbol{P} \begin{vmatrix} g_1^{1/2}(\theta,\phi) W_1 \\ g_2^{1/2}(\theta,\phi) C_T W_1 e^{j\beta} \end{vmatrix} =$$

$$C \begin{vmatrix} \left(S_{\mathrm{h}}\cos^2\gamma + S_{\mathrm{v}}\sin^2\gamma\right) g_1 W_1 + \left(S_{\mathrm{h}}\cos\gamma\sin\psi + S_{\mathrm{v}}\sin\gamma\cos\psi\right) g_1^{1/2} g_2^{1/2} C_T e^{j\beta} W_1 \\ \left(S_{\mathrm{h}}\cos\gamma\sin\psi + S_{\mathrm{v}}\sin\gamma\cos\psi\right) g_1^{1/2} g_2^{1/2} C_R e^{j\xi} W_1 + \left(S_{\mathrm{h}}\sin^2\psi + S_{\mathrm{v}}\cos^2\psi\right) g_2 C_T C_R e^{j(\beta+\xi)} W_1 \end{vmatrix}. \quad (3)$$

The following explains various assumptions in (3). S_{v} is real but $S_{\mathrm{h}} = |S_{\mathrm{h}}|\exp(-j\Phi_{\mathrm{DP}})$ so that the effects on differential phase Φ_{DP} from propagation and backscatter are accounted for. C is the calibration parameters. W_1 is the voltage at Port 1 generating the field E_1 and $W_2 = C_T e^{j\beta} W_1$ is the voltage at Port 2 generating E_2. The phase difference on reception between the outputs from Port 2 and Port 1 is ξ and the ratio of amplitudes (Port 2 to Port 1) equals C_R.

Regroup the terms in (3) as follows

$$CW_1 \begin{vmatrix} \left(\cos^2\gamma g_1 + \cos\gamma\sin\psi g_1^{1/2} g_2^{1/2} C_T e^{j\beta}\right) S_{\mathrm{h}} + \left(\sin^2\gamma g_1 + \sin\gamma\cos\psi g_1^{1/2} g_2^{1/2} C_T e^{j\beta}\right) S_{\mathrm{v}} \\ \left(\cos\gamma\sin\psi g_1^{1/2} g_2^{1/2} + \sin^2\psi g_2 C_T e^{j\beta}\right) C_R e^{j\xi} S_{\mathrm{h}} + \left(\sin\gamma\cos\psi g_1^{1/2} g_2^{1/2} + \cos^2\psi g_2 C_T e^{j\beta}\right) C_R e^{j\xi} S_{\mathrm{v}} \end{vmatrix}$$

$$= \begin{vmatrix} V_1 \\ V_2 \end{vmatrix}. \quad (4)$$

Equations (3) and (4) apply at beam center. But if the beam is narrow and is not intersecting the principal planes these equations apply to all points within the beam. Furthermore it is assumed that the cross-polar beam is coaxial with the co-polar beam and both have the same shape. Thus integrating products of copolar and cross-polar beam over the resolution volume would yield various polarimetric variables (e.g., Zrnic et al. 2010) but will not be made here as our interest is in quantifying geometric effects using values at the center of the beam. *It can be deduced from Eq.(23) of Zrnic et al. (2010) that if the co-polar and cross-polar beams are similar and coaxial (as it is for PPPAR beams steered away from the principal planes), there is no need for integration!* In principle one can invert (4) to express the S coefficients in terms of the voltages. This requires knowledge of the orientation angles, ψ and γ, the phase differences β and ξ, and the Port 2 to Port 1 scaling factors C_T and C_R, as well as the power gains g_1 and g_2 at every pointing direction. This amounts to eight numbers that need to be known plus the calibration parameter C for calculation of voltages and hence reflectivity.

The voltages and S parameters change from pulse to pulse; say as function of the sample number n of the time series data. Thus the pulse to pulse inversion of (4) would generate two time sequences one for the $S_{\mathrm{h}}(n)$ the other for the $S_{\mathrm{v}}(n)$. From the average powers $\langle |S_h|^2 \rangle$ and $\langle |S_v|^2 \rangle$, Z_{h} and Z_{DR} can be computed. From the correlation of the two sequences the differential phase Φ_{DP} and correlation coefficient ρ_{hv} can be computed. Although promising results of inversion on a small one dimensional array in a laboratory set up have been obtained (Fulton and Chappell 2010) there have been no demonstrations on larger arrays yet.

A different way to compute the second order moments ($\left\langle |S_{\mathrm{h}}|^2 \right\rangle$, $\left\langle |S_{\mathrm{v}}|^2 \right\rangle$, $\left\langle S_{\mathrm{h}}^* S_{\mathrm{v}} \right\rangle$) is from the powers of the returned signals at the Port 1 and Port 2, and the correlation of these two signals (i.e., the power estimates from the first row of (4) summed over *M* samples, similar power estimate from the second row and the estimates of the correlation between the first row and second row signals). The number of electric parameters that need to be known is also nine.

c) *Phase coding*

Phase coding can simplify somewhat this computation. Suppose that the 0^o, 180^o phase code is applied to the Port 2. This can be represented as $e^{j\beta(\mathrm{n})}$ where $\beta(n)$ changes between 0^o, 180^o. Fourier transform of the first row in (3) generates two spectra: one from the first term in row 1 is centered at the Doppler velocity the other (second term in row 1) is offset by the unambiguous velocity. Thus one can separate these two terms as follows:

$$CW_1\left(S_\mathrm{h}\cos^2\gamma + S_\mathrm{v}\sin^2\gamma\right)g_1 = V_1(\overline{v}) \qquad (5a)$$

$$CW_1\left(S_\mathrm{h}\cos\gamma\sin\psi + S_\mathrm{v}\sin\gamma\cos\psi\right)g_1^{1/2}g_2^{1/2}C_T e^{j\beta} = V_1(\overline{v}+v_a)\,, \qquad (5b)$$

where $V_1(\overline{v})$ is the sequence (measurement) obtained from the inverse Fourier transform of the spectral components corresponding to one half the Nyquist interval centered on the mean Doppler velocity $\overline{v}$. $V_1(\overline{v}+v_a)$ is the sequence corresponding to the spectrum offset by the unambiguous velocity from $\overline{v}$. Similarly the two terms in the second row of (3) can be separated so that the number of sequences is four. But the sequences corresponding to the off diagonal terms (i.e., cross polar) differ by a complex multiplying factor. Therefore there are three sequences which can be used to form powers and cross products. Of the three the one corresponding to the diagonal term is redundant hence might be useful for checking consistency or determining the initial transmitting phase.

The powers and cross product of separated diagonal sequences can be used to generate three complex equations in which the unknown terms are $\left\langle |S_\mathrm{h}|^2\right\rangle$, $\left\langle |S_\mathrm{v}|^2\right\rangle$, $\left\langle S_\mathrm{h}^* S_\mathrm{v}\right\rangle$. The third term has the differential phase. It should be expressed as one complex number. In doing so one needs to track this number and its conjugate until the last step in the solution of the three complex equations. The angles ψ and γ, the differential phases β_o and ξ, the gains and amplitude calibration of the two channels on transmit and receive need to be known in addition to *C*.

d) *Alternate transmission of the H and V field* (*AHV*)

The AHV mode is analogous to the phased coding except the four term in the matrix of (3) are separated if the cross polar component is recorded. The cross-polar signal at Port 2 (if only Port 1 is active) is the 21 term in (3) and if the Port 1 is active it is the 12 term in (3). These components are redundant but might be helpful to check stability of the system. Computations of the second order moments are made using the main diagonal terms in (3) which are estimated (measured) sequentially. Therefore the correlation term includes the Doppler effect which needs to be eliminated (Zrnic et al. 2011).

e) *Measurements at directions close to the principal planes*

From the expression (3) the maximum values of the angles ψ and γ for which the bias in the polarimetric variables is acceptable can be determined.

3. Vertically oriented array

If the array is oriented vertically the angle γ=0 and corrections and computations become simpler. Herein we provide more details for this geometry about the computations than is listed in section 2.

a) *Relations*

The governing relation (3) expressed as the two equations is

$$V_1 = CW_1\left(g_1 + \sin\psi g_1^{1/2} g_2^{1/2} C_T e^{j\beta}\right) S_\mathrm{h} \tag{6a}$$

$$V_2 = CW_1\left[\left(\sin\psi g_1^{1/2} g_2^{1/2} + \sin^2\psi g_2 C_T e^{j\beta}\right) C_R e^{j\xi} S_\mathrm{h} + \cos^2\psi g_2 C_R e^{j\xi} C_T e^{j\beta} S_\mathrm{v}\right]. \tag{6b}$$

Multiplying (6a) with $\sin\psi g_2^{1/2} C_R e^{j\xi} / g_1^{1/2}$ (from pulse to pulse) and subtracting from (6b) solves for the second term in (6b) which is

$$CW_1 \cos^2\psi g_2 C_R e^{j\xi} C_T e^{j\beta} S_\mathrm{v} = V_2 - \frac{\sin\psi g_2^{1/2} C_R e^{j\xi}}{g_1^{1/2}} V_1 \tag{6c}$$

Therefore the polarimetric variables can be estimated from (6a) and (6c).

By inspection it can be seen that if β = 0 and ξ =0, the differential phase can be computed directly by correlating the conjugate of (6a) with (6c) and that ρ_hv would equal to the magnitude of the corresponding correlation coefficient. Besides the implicit dependence on the direction angle ψ of the intended V field the polarimetric variables from (6) depend explicitly on this angle through the values of g_1 and g_2.

Note that Z_h and Z_v [from (6a), (6b)] depend explicitly and implicitly (through 1b, 1c) on ψ. The Z_DR depends on the same variables and is independent of C as it is proportional to the ratio (6a) to (6c).

The alternate way to compute the polarimetric variables is from the powers of (6a) and (6b) and the correlation between (6a) and (6b). From these, first the $\langle |S_\mathrm{h}|^2\rangle$, $\langle |S_\mathrm{v}|^2\rangle$, $\langle S_\mathrm{h}^* S_\mathrm{v}\rangle$ and Φ_DP are found and combined to generate the polarimetric variables. Thus, take the power estimates as average of M samples:

$$\sum_{i=1}^{M} |V_{1i}|^2 / M = |CW_1|^2 \left|g_1 + g_1^{1/2} g_2^{1/2} \sin\psi C_T e^{j\beta}\right|^2 \left\langle |S_\mathrm{h}|^2 \right\rangle \tag{7a}$$

$$\sum_{i=1}^{M} |V_{2i}|^2 / M = |CW_1|^2 \left\{ \begin{aligned} &\left|g_1^{1/2} g_2^{1/2} \sin\psi + g_2 \sin^2\psi C_T e^{j\beta}\right|^2 C_R^2 \left\langle |S_\mathrm{h}|^2 \right\rangle + \left(\cos^2\psi\right)^2 g_2^2 C_R^2 C_T^2 \left\langle |S_\mathrm{v}|^2 \right\rangle \\ &+2\,\mathrm{Re}\left[\left(g_1^{1/2} g_2^{1/2} \sin\psi + g_2 \sin^2\psi C_T e^{-j\beta}\right)\left(\cos^2\psi e^{j\beta}\right) C_R^2 C_T e^{j\beta}\right] \left\langle S_\mathrm{h}^* S_\mathrm{v} \right\rangle \end{aligned} \right\} \tag{7b}$$

$$\sum_{i=1}^{M} V_{1i}^{*}V_{2i} / M = \left|CW_1\right|^2 \left[\begin{array}{l} \left(g_1 + \sin\psi g_1^{1/2} g_2^{1/2} C_T e^{-j\beta}\right)\left(\sin\psi g_1^{1/2} g_2^{1/2} + \sin^2\psi g_2 C_T e^{j\beta}\right) C_R e^{j\xi} \left\langle \left|S_{\mathrm{h}}\right|^2 \right\rangle \\ +\left(g_1 + \sin\psi g_1^{1/2} g_2^{1/2} C_T e^{-j\beta}\right)\cos^2\psi g_2 C_R e^{j\xi} C_T e^{j\beta} \left\langle S_{\mathrm{h}}^{*} S_{\mathrm{v}} \right\rangle \end{array} \right]. \quad (7c)$$

From these equations it is evident that all the moments depend on the pointing direction explicitly and also implicitly through the equations (1b, 1c). Equation (7a) is not coupled to the other equations hence to compute Z_{h} one only needs to integrate over the beam the width of which depends on the pointing direction. Again note that adjusting β and ξ to 0 simplifies the solving process. Assuming that the calibration is acceptable (the C, C_T, C_{R} and (1b) and (1c) are known as well as ψ) it is in principle relatively easy to solve the set (7) for the polarimetric variables. For example (7b) can be properly scaled and added to (7c) so that the first term in (7c) is eliminated. Then the cross product $\left\langle S_{\mathrm{h}}^{*} S_{\mathrm{v}} \right\rangle$ can be computed. Subsequent substitution in (7b) yields $\left\langle \left|S_{\mathrm{v}}\right|^2 \right\rangle$. Next we examine the number of parameters needed for calibration.

Backend (behind the antenna): The gain C and the differential gain on transmission C_T and the differential phase β and ξ add to 4 numbers (compare that to 2 gains for the dish antenna and the system differential phase $\beta + \xi$ which can be obtained from data). Similar holds in the receiver channels, the differential gain C_R and the differential phase ξ; note that the overall system calibration C lumps together all the gains and losses in both receiver and transmitter chain. This amounts to 2 more numbers (two gains in the receiver are needed for the dish antenna). We expect that these 5 numbers would be independent of the pointing direction.
Antenna: The gains $g_1(\theta_o, \phi_o)$ and $\mathrm{g}_2(\theta_o, \phi_o)$, the corresponding beamwidth (the two patterns should have the same elliptical shape beam cross sections otherwise the weather PAR is dead on arrival), and the pointing direction ψ. This totals 4 but it may be safe to assume that the beamwidths (two needed for the elliptical shape) would be computed from the known pointing direction and the computed (calibrated via measurements) gains. That would reduce the number of “independent” variables to 3 for each pointing direction. To cover 90 degrees in azimuth and 15 elevations with a planar array, 1350 beam positions are needed. This translates to 4050 calibration numbers. Because of viewing symmetry (the left field of view is symmetric to the right one) the actual number to calibrate might reduce by a factor of 2, to 2025. Some other reductions in complexity are expected in and near the principal planes (at about less than 300 points).

b) *Phase coding*

To condense notation the signal centered on the mean Doppler in Port 1 is written as V_{11} the one in Port 2 is V_{22} and the cross-port signals (offset by the unambiguous velocity) are V_{12} and V_{21}. Furthermore for consistency with the previous results, the relative calibration (ratio of) Port 2 to Port 1 voltage on transmission is denoted as $W_2 = C_T W_1 e^{jn\pi}$; this implies that $\beta(n) = \beta_o + e^{jn\pi}$. Upon reconstruction (separation of the components) the $e^{jn\pi}$ term is not present. Set γ=0 in (5a) and (5b) to express V_{11} and V_{12} as

$$V_{11} = CW_1 g_1 S_{\mathrm{h}}, \tag{8a}$$

$$V_{12} = CW_1 \left(\sin\psi C_T e^{j\beta_o} \right) g_1^{1/2} g_2^{1/2} S_{\mathrm{h}}. \tag{8b}$$

Separation of spectra from Port 2 (6b) isolates the first term spectrum from the other two terms. These two terms become

$$V_{21} = CW_1 \sin\psi g_1^{1/2} g_2^{1/2} C_R e^{j\xi} S_{\mathrm{h}}, \tag{8c}$$

$$V_{22} = CW_1 \left(\sin^2\psi S_{\mathrm{h}} + \cos^2\psi S_{\mathrm{v}} \right) g_2 C_T C_R e^{j(\beta_o + \xi)}. \tag{8d}$$

It is understood that (8) is a shorthand notation for these voltages at any one sample time and that the four complex sequences (or four complex spectra) consist of *M* samples each.

The powers and correlation of the first three terms are

$$\left\langle |V_{11}|^2 \right\rangle = C^2 |W_1|^2 g_1^2 \left\langle |S_{\mathrm{h}}|^2 \right\rangle, \tag{9a}$$

$$\left\langle |V_{12}|^2 \right\rangle = C^2 |W_1|^2 g_1 g_2 \sin^2\psi C_T^2 \left\langle |S_{\mathrm{h}}|^2 \right\rangle, \tag{9b}$$

$$\left\langle |V_{21}|^2 \right\rangle = C^2 |W_1|^2 g_1 g_2 \sin^2\psi C_R^2 \left\langle |S_{\mathrm{h}}|^2 \right\rangle, \tag{9c}$$

$$\left\langle V_{11}^* V_{12} \right\rangle = C^2 |W_1|^2 g_1^{3/2} g_2^{1/2} \sin\psi C_T e^{j\beta_o} \left\langle |S_{\mathrm{h}}|^2 \right\rangle, \tag{9d}$$

$$\left\langle V_{11}^* V_{21} \right\rangle = C^2 |W_1|^2 g_1^{3/2} g_2^{1/2} \sin\psi C_R e^{j\xi} \left\langle |S_{\mathrm{h}}|^2 \right\rangle, \tag{9e}$$

Inspection reveals that the five calibration parameters can be obtained easily. The phases β_o and ξ from the arguments of (9d) and (9c); the $\sin\psi$, C_T, and C_R by dividing (9b), (9c), and (9d) with (9a). Dividing (9e) with (9a) will generate a redundant relation that should be consistent with the other relations. This redundancy may be helpful in diagnosis of system stability. The reader is alerted that in these expressions the system differential phase $\Phi_{\mathrm{DPsys}} = \beta_o + \xi$. This further simplifies checking calibrations. Phase coding is more effective if the unambiguous velocity interval is larger than the spectrum width of the weather signal so that the two components can be well separated in the frequency domain. Otherwise the original and offset spectra of weather signals would overlap precluding clean separation.

c) *Alternate transmission AHV*

The second order moments in case of the AHV mode are given also by the (9) except the terms (voltages) with the first subscript 1 are obtained from one transmitted pulse and the terms with the first subscript 2 are obtained from the subsequent transmission. There is a Doppler effect between these alternating terms consisting of mean offset and spread. These need to be removed prior to computing the polarimetric variables (see Zrnic et al. 2011).

d) Principal horizontal plane

Calibration for the principal horizontal plane simplifies considerably as can be seen from the equations (6a) and (6b) after substituting $\gamma=0$ and $\psi=0$. The simplified form becomes

$$V_1 = CW_1 g_1 S_{\mathrm{h}} \tag{10a}$$

$$V_2 = CW_1 g_2 C_R e^{j\xi} C_T e^{j\beta} S_{\mathrm{v}} \ . \tag{10b}$$

Because it is assumed that coupling is insignificant the individual terms in (10) do not need to be calibrated (known). Rather their products have to be known as function of azimuth and elevation. Therefore calibration becomes analogous to the one on the parabolic dish except it needs to be done each azimuthal position because both g_1 and g_2 depend on the pointing direction (azimuth).

e) *The relation between azimuth and elevation angles and the orientation angle* ψ

It can be shown that the orientation angle ψ is related to the azimuth angle α and the elevation angle e via the following expression

$$\cos\psi = \frac{\cos\alpha}{\sqrt{1-\cos^2(e)\sin^2(\alpha)}} \ . \tag{11}$$

The plots of (10) indicate at $\alpha = 45^{\mathrm{o}}$ the orientation angle ψ is almost equal to the elevation angle e.

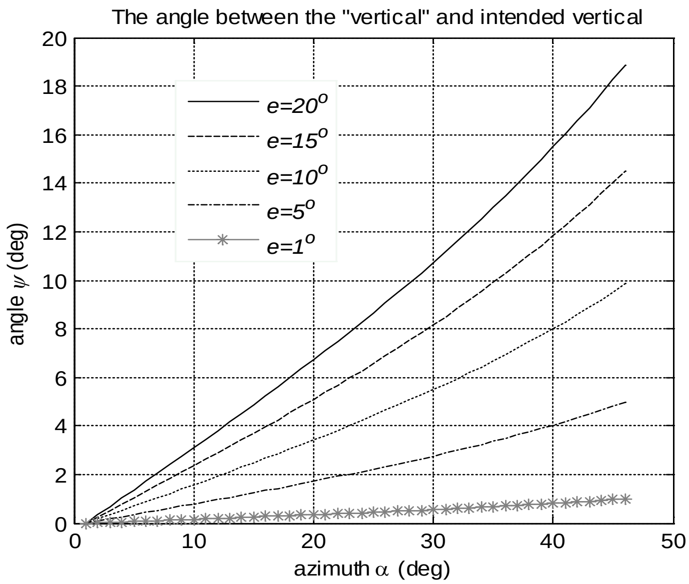

Fig. 2. Dependence of the orientation angle ψ on azimuth for few elevations. The array is in the vertical plane.

4. Conclusions

It is demonstrated that nine calibration parameters are needed to construct a set of linear equations relating second order moments of the received signals (at Ports 1, and 2 corresponding to intended H and V polarizations) to the pertinent elements of the polarimetric covariance matrix. Depending on the orientation of the array and the beam pointing direction the number of parameters and complexity of equations change. The calibration considered here is for the array that consists of patch radiators. It is assumed that calibration can be obtained from the gains at beam center. Furthermore the effects of cross coupling due to cross polar radiation of the patches is insignificant so that the main contribution to biases comes from a) the non orthogonality of intended H and V and b) from non-collinearity of the intended H with the true H direction and/or non-collinearity of the intended V with the true V direction.

Appendix:

Determination of the angle ψ

The intended V field from the patch (radiating are the top and bottom sides) is tangent to the parallels of a sphere in which the pole is along the *y* axis as depicted in Fig. A. In this figure the plane of the array is in the *z*,*y* plane and the array is pointing at azimuth α and elevation *e*. The azimuth in this convention is measured counterclockwise but because the results are symmetric with respect to the vertical principal plane the orientation is immaterial. Our goal is to determine the angle ψ between the parallel (in the sphere with pole along the *y* axis) and the meridian of the sphere that has the pole along the *z* axis. This angle equal to the angle of the tangents to these two curves at the intersection point (Fig. A.1). Start with the vector $\vec{r}$ which is expressed as

$$\vec{r} = r\cos e\cos\alpha\boldsymbol{a}_x + r\cos e\sin\alpha\boldsymbol{a}_y + r\sin e\boldsymbol{a}_z \qquad \text{(a.1)}$$

where the $\boldsymbol{a}_i$s are unit vectors. The unit vector ***e*** tangent to the meridian is

$$\boldsymbol{e} = \frac{\partial \vec{r} / \partial e}{\left| \partial \vec{r} / \partial e \right|} = -\sin e \cos \alpha \boldsymbol{a}_x - \sin e \sin \alpha \boldsymbol{a}_y + \cos e \boldsymbol{a}_z \,. \qquad \text{(a.2)}$$

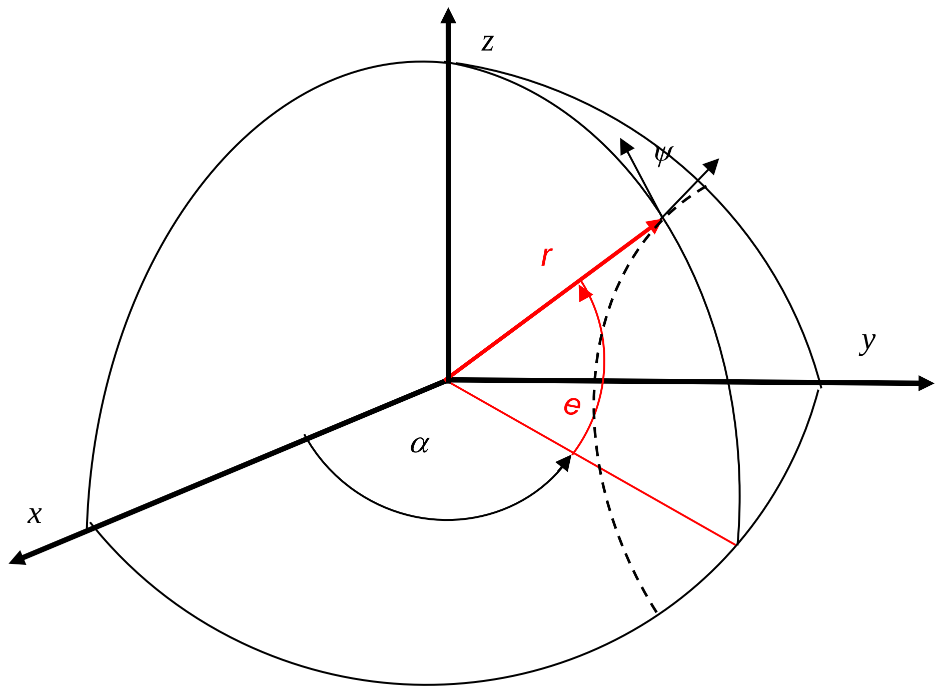

Fig. A. The coordinate system with the antenna in the *z*,*y* plane pointing along the *r* direction. The E field is tangent to the dashed semicircle.

Next consider a spherical system with the pole along the *y* axis and the ϕ' angle is measured in the *x*,*z* plane starting from *x*. The θ' angle is with respect to the *y* axis, in Fig A.1 it is the angle between $\vec{r}$ and the *y* axis (not drawn to avoid cluttering the figure). Then the $\vec{r}$ can be expressed in terms of these two angles

$$\vec{r} = r \sin\theta' \cos\phi' \boldsymbol{a}_x + r\cos\theta' \boldsymbol{a}_y + r \sin\theta' \sin\phi' \boldsymbol{a}_z \,, \qquad \text{(a.2)}$$

and the unit vector tangent to the circle of constant θ' is $\boldsymbol{\phi}'$ expresses as

$$\boldsymbol{\phi}' = \frac{\partial \vec{r} / \partial \phi'}{\left| \partial \vec{r} / \partial \phi' \right|} = -\sin\phi' \boldsymbol{a}_x + \cos\phi' \boldsymbol{a}_z \,. \qquad \text{(a.3)}$$

Then

$$\cos\psi = \boldsymbol{e} \cdot \boldsymbol{\phi}' = \sin e \cos\alpha \sin\phi' + \cos e \cos\phi' \,. \qquad \text{(a.4)}$$

Use the fact that (a.1) must equal (a.2) to equate the appropriate vector components and express the sinusoidal function of *e* and α in terms of the sinusoidal function of θ' and ϕ'. Insertion of these into (a.4) produces the following relation

$$\cos\psi = \frac{\cos\alpha}{\sqrt{1-\cos^2(e)\sin^2(\alpha)}}. \qquad \text{(a.5)}$$

From the plots (Fig. 2) it is clear that at 45^o azimuth the intersection angle is almost equal to the elevation angle *e*.

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