A New Micro-physical Interpretation of the Variability of Z-R Relationships

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Abstract

It has been established by past studies that the variability of Z-R relationships ($Z=aR^b$) is related to different characteristics of raindrop size distribution (DSD). A new and simple micro-physical method based on the DSD formulation normalized by drop number concentration (Nt in m^{-3}) and drop mean diameter (Dm in mm) is proposed to explain the variation of the coefficient a and exponent b. This method doesn't take any assumption of analytical function on DSD. The exponent b is determined by the correlation between log(Nt) and log(Dm), multiplied by the ratio of the standard deviation of log(Nt) to the standard deviation of log(Dm). The coefficient a depends on value of b, mean log(Nt) and mean log(Dm). Please see the equations (8) and (9).

Introduction

The gamma distribution with three parameters proposed by Ulbrich (1983)

$$N(D)=N_0 D^{\mu}\exp(-\lambda D) \qquad (1)$$

is considered as a good approximation for describing the natural DSD. However, we can't explain the variation in Z-R relationships by the three parameters because 1) they are not physically meaningful; 2) they have some statistical dependencies among them. For example, one can easily get a Z-R relationship from (1) as

$$Z=\left[\frac{10^3}{7.123}\frac{\Gamma(\mu+7)}{\Gamma(\mu+4.67)}\lambda^{-2.33}\right]R. \qquad (2)$$

But the rain intensity (R) can be well related to the parameter (λ) by a power law relationship so the linear relationship (2) is totally artificial. The representation of the DSD by a scaled distribution has been wildly investigated since the 2000s. One (or two) physically meaningful variable(s) is (are) used to describe the whole variation of DSDs. For example, Testud et al. (2001) proposed an expression of the normalization of the DSD as

$$N(D)=\frac{N_t}{D_m}g(x) \quad \text{with} \quad x=\frac{D}{D_m} \qquad (3)$$

where Nt = concentration of raindrops in (m^{-3}), Dm = mean drops diameter in (mm) and g(x) is the general scaled distribution. Under this conception, Steiner et al. (2004) concluded 3 conceptual situations: 1) a linear Z-R relationship with b=1 for the drops number-controlled situation (Dm is constant); 2) b=1.63 for the drops size-controlled situation (Nt is constant); 3) b=1.5 for the Nt/Dm = constant. In this study, the micro-physical interpretation of Steiner et al. (2004) will be extended to a general framework and new conceptual Z-R situations will be completed.

Conclusions

A new interpretation of the variation of Z-R relationship is proposed. It can well explain the Z-R relationships under the "number-controlled", "size-controlled" and "number-size proportional" DSDs. It completes "the number-size independent" and "number-size inverse" situations and gives a simple relationship (8) to determine the exponent b without consideration of the shape parameter of the gamma distribution. This interpretation suggests that b should be strongly related to the dominant micro-physical process of rain because it is related to the correlation and variation of Nt and Dm. The coefficient a can be determined from Nt and Dm only if the b is correctly identified.

Method

Based on the scaled DSD expression (3), one can obtain the expression of Z and R by integration as

$$Z=\int_0^{\infty}N(D)D^6dD=N_tD_m^6\int_0^{\infty}g(x)x^6dx=N_tD_m^6G_6 \qquad (4)$$

$$R=7.123\,x\,10^{-3}\int_0^{\infty}N(D)D^{3.67}dD=7.123\,x\,10^{-3}N_tD_m^{3.67}G_{3.67} \qquad (5)$$

where G_6 and $G_{3.67}$ are two constants, independent to the DSD variation. The power-law Z-R relationship with the two parameters a and b can be write as

$$\log_{10}Z=\log_{10}a+b\log_{10}R. \qquad (6)$$

Replacing the Z and R by the expressions (4) and (5) yields a linear relationship between Nt and Dm as

$$\log_{10}N_t=\frac{3.67b-6}{1-b}\log_{10}D_m+\frac{F(a,b,G_6,G_{3.67})}{1-b} \qquad (7)$$

with

$$F(a,b,G_6,G_{3.67})=\log_{10}a-\log_{10}G_6+b\log_{10}7.123\times10^{-3}+b\log_{10}G_{3.67} \qquad (8)$$

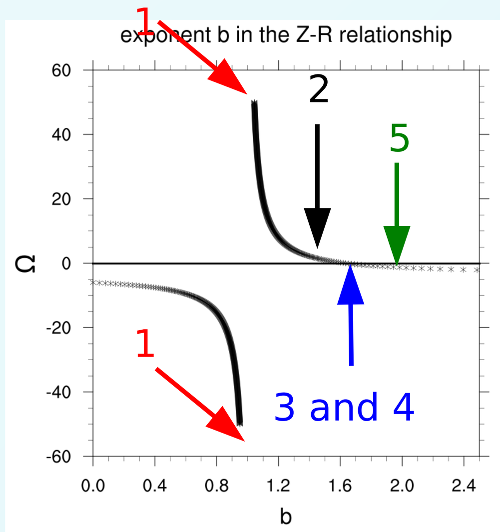
If a, b, $G_{3.67}$ and G_6 are considered as parameters which are independent to the Nt and Dm (or Z and R), the values of a and b can be estimated from the linear regression on (7) as

$$\frac{3.67b-6}{1-b}=correlation[\log_{10}N_t,\log_{10}D_m]\frac{\sigma[\log_{10}N_t]}{\sigma[\log_{10}D_m]}=\Omega \qquad (8)$$

$$\log_{10}a=(1-b)\overline{\log_{10}N_t}-(3.67b-6)\overline{\log_{10}D_m}+\log_{10}G_6+2.147b-b\log_{10}G_{3.67} \qquad (9)$$

The exponent b in the Z-R relationship is explained by the correlation between log(Nt) and log(Dm), the ratio of STD[log(Nt)] to STD[log(Dm)]. It is independent to absolute values of Nt and Dm. The coefficient a depends on the value of b, averaged log(Nt) and log(Dm). However, its relationship with the averaged log(Nt) and log(Dm) in (9) is complicated, depending on b.

The equation (8) is plotted in the following figure with the new variable Ω defined in (8).



The variation of b is usually explained by the shape parameter μ of the gamma distribution (1). The presented graph based on (8) is the same as the plot of Smith and Krajewski (1993) who worked on the μ - b relationship. Therefore the effect of Ω can be the same as μ to explain the variation of b.

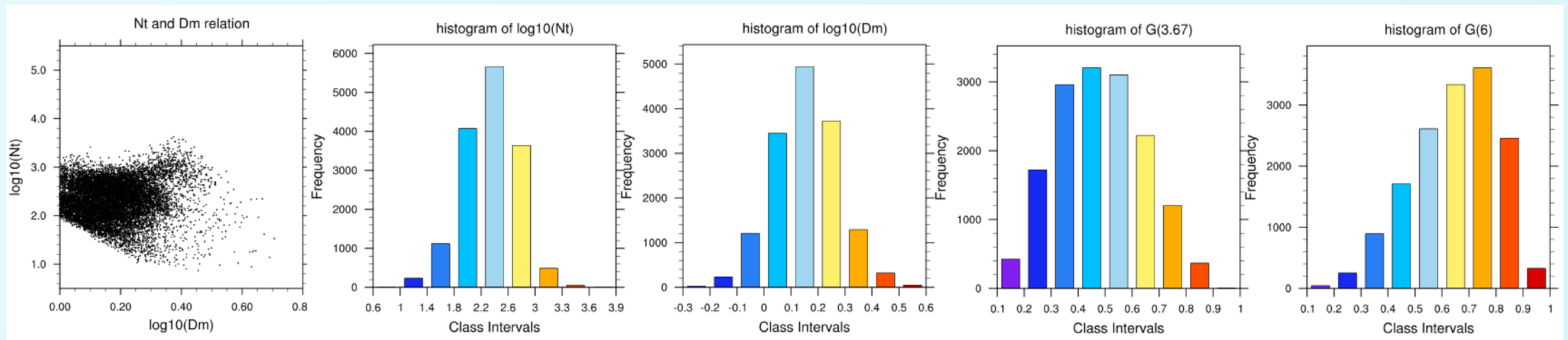
Micro-physical interpretations

Because the standard deviations in (8) are always > 0, the sign of Ω is only determined by the correlation between log(Nt) and log(Dm). If the correlation[log(Nt),log(Dm)] > 0, b should be 1.0 < b < 1.63. This range corresponds to common situations found in many radar-gauges comparisons. If log(Nt) and log(Dm) are totally independent, b is equal to 1.63. This may be so-called "climatological" situation when we mix long-term DSDs observations to get a climatological Z-R relationship. If the correlation[log(Nt),log(Dm)] < 0, b should be > 1.63 for -3.67< Ω <0; or b < 1.0 for Ω < -3.67. More specifically, we have

Situation	Conditions on Nt and Dm	Value of Ω	b	a
1 Drops number-controlled situation	STD[log(Nt)] >> STD[log(Dm)] Correlation[log(Nt), log(Dm)] ≠ 0	+ infinitive or - infinitive	1.0	log(a) is proportional to 2.33 x avg(log(Dm)) and independent to log(Nt)
2 Number-size proportional situation	Nt = k x Dm with k>0 (ZPHI)	1	1.5	log(a) is proportional to -0.5 x avg(log(Nt)) + 0.5 x avg(log(Dm))
3 Drops size-controlled situation	STD[log(Nt)] << STD[log(Dm)]	0	1.63	log(a) is proportional to -0.63 x avg(log(Nt)) and independent to log(Dm)
4 Number-size independent situation	Correlation[log(Nt), log(Dm)] = 0	0	1.63	log(a) is proportional to -0.63 x avg(log(Nt)) and independent to log(Dm)
5 Number-size inverse situation	Nt=k / Dm with k>0 (Illingworth et al. 2004)	-1	1.87	log(a) is proportional to -0.87 x avg(log(Nt)) - 0.86 x avg(log(Dm))

Application on the observed DSD data

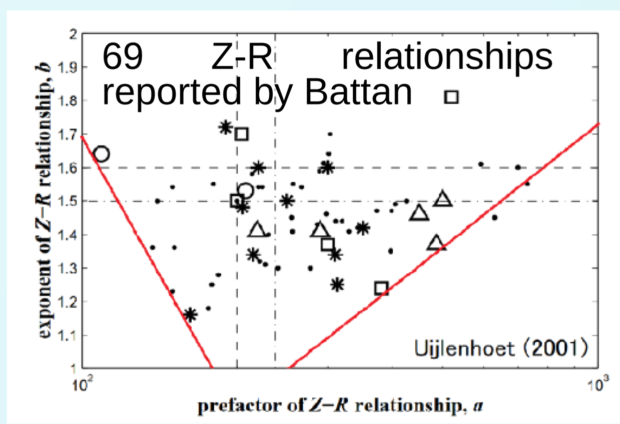
12-month DSD data (15278 1-min DSDs) collected by a Parsivel in the south of France during the OHMCV (Boudevillain et al. 2014) is used to illustrate the correlation of log(Nt) and log(Dm), the distribution of log(Nt), the distribution of log(Dm=M4/M3), the distribution of $G_{3.67}$ and the distribution of G_6.

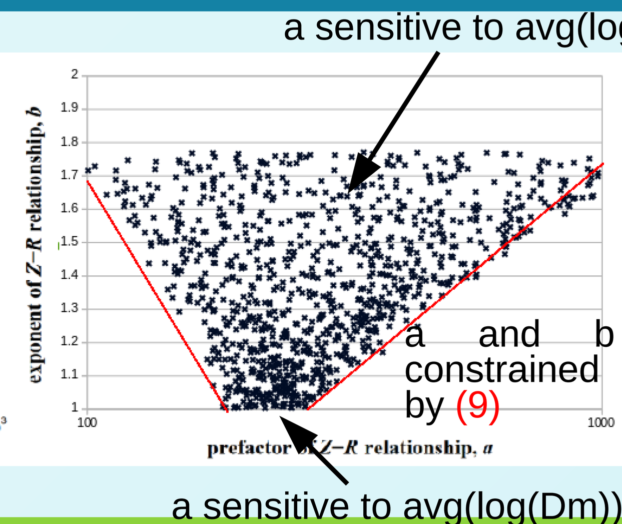


Correlation[log(Nt),log(Dm)] = -0.079
STD[log(Nt)] = 0.394
STD[log(Dm)] = 0.433
Avg[log(Nt)] = 2.340
Avg[log(Dm)] = 0.156
Median($G_{3.67}$) = 0.477
Median(G_6) =0.655
$Z=233R^{1.648}$

Even if the Parsivel can have a large error in Nt measurement, a good Z-R relationship ($Z=233R^{1.648}$) is obtained from (8) and (9) with Nt and Dm. A direct linear regression on log(Z) → log(R) yields a very similar Z-R relationship ($Z=241R^{1.645}$). The study of estimation of a and b from (8) and (9) based on dual-pol radar measurements and numerical simulations is ongoing.

Interpretation of the 69 Z-R relationships reported by Battan (1973)





For the 69 Z-R relationships reported by Battan (left panel), the variation of a seems to be related to the value of b. When b -> 1, the variation of a is small. This corresponds to the "drops number-controlled situation" and a depends on the averaged log(Dm); When b -> 1.63, the variation of a is large. This corresponds to the "drops size-controlled situation" or "number-size independent situation" and a depends on the averaged log(Nt).

An illustration of a and b constrained by (9) is illustrated (right panel) with $G_{3.67}$=0.5 and G_6=0.66. b ranges from 1.0 to 1.75. The log10(Nt) is randomly selected between -0.2 and 2.8. The log10(Dm) is randomly selected between -0.07 and 0.07.