KDP Estimation Based on Spline Smoothing with the Self-Consistency Principle

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Background and Problem Formulation

- In dual-polarized weather radars, K_{DP} [deg/km] is useful since it is **strongly correlated with rain rate and robust to miscalibration and rain attenuation.**

- However, K_{DP} cannot be calculated directly from observed IQ signal $x_{\mathrm{h},n}[l]$ and $x_{\mathrm{v},n}[l]$ $(l = 1,2,\ldots,L)$. (h, v: horizontal, vertical, n: range bin, l: pulse)

- First, **total differential phase** Ψ_{DP} [deg] is given from $x_{\mathrm{h},n}[l]$ and $x_{\mathrm{v},n}[l]$ as

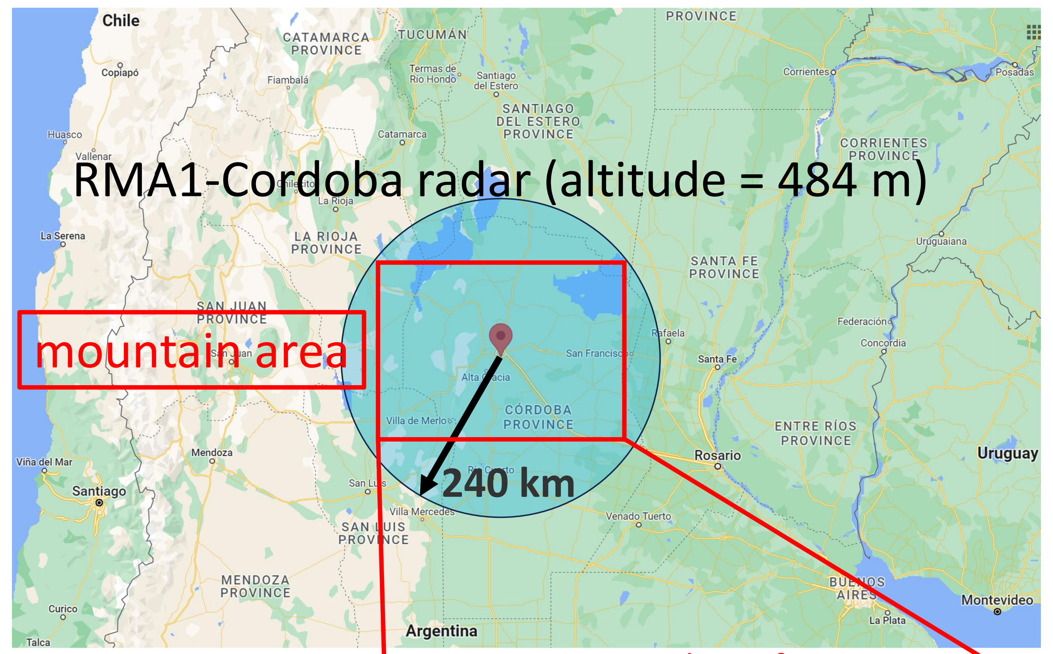
$$\Psi_{\mathrm{DP}}^{\mathrm{wrap}}(r_n) = \Psi_{\mathrm{DP},n}^{\mathrm{wrap}} = \frac{180}{\pi}\arg\left(\frac{1}{L}\sum_{l=1}^{L} x_{\mathrm{h},n}^*[l]x_{\mathrm{v},n}[l]\right) \in [0,360).$$

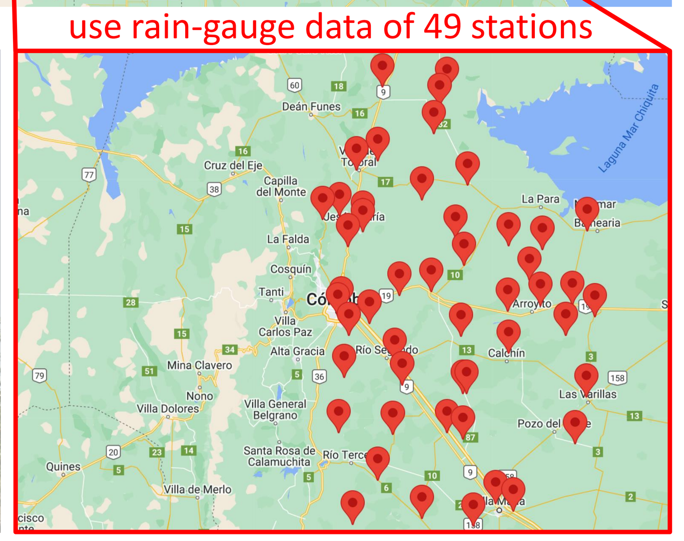
wrapped in less than 360 degrees

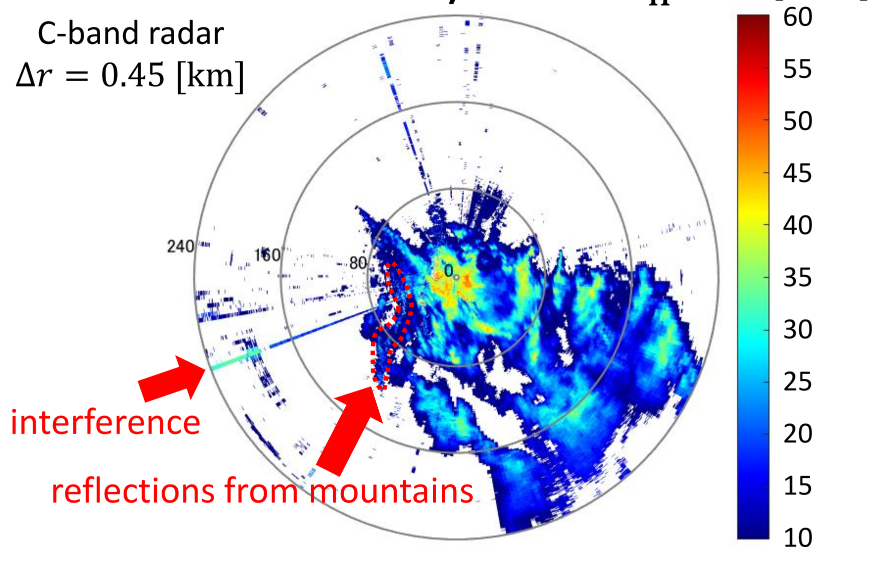
- After phase unwrapping (unfolding) of $\Psi_{\mathrm{DP},n}^{\mathrm{wrap}}$, we estimate K_{DP} from

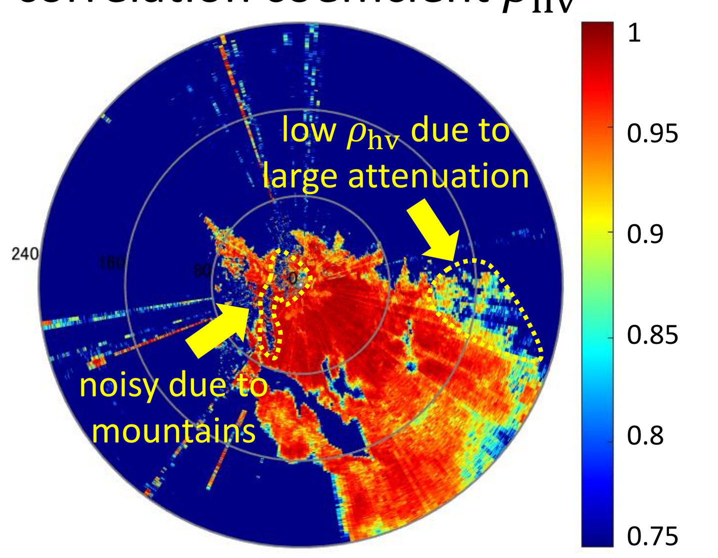
$$\underbrace{\Psi_{\mathrm{DP},n}}_{\text{observed phase}} = \underbrace{\Phi_{\mathrm{DP},n}}_{\text{differential propagation phase including initial phase}} + \underbrace{\delta_n}_{\text{differential backscatter phase}} + \underbrace{\varepsilon_n}_{\text{observation noise}} = 2\int_0^{r_n} \underbrace{K_{\mathrm{DP}}(r)}_{\text{specific differential phase to be estimated}}\,\mathrm{d}r + \underbrace{\Phi_{\mathrm{DP},0}}_{\text{initial phase including radome attenuation}} + \delta_n + \varepsilon_n.$$

- Argentine C-band dual-polarized radar data has partially noisy regions due to WLAN interference, partial beam blockage, and reflections from mountains.









Conventional and Proposed Methods

Local Linear (Polynomial) Fitting: US NWS Operational Algorithm

- Define gate length g (g is odd) and obtain a smoothed total differential phase $\Psi_{\mathrm{DP}}^{\mathrm{filt}}$ by g-gate moving average filter (= remove high-frequency noise of observed Ψ_{DP}).
- For each gate, create a local linear (polynomial) model $\Phi_{\mathrm{DP}}(r) \approx 2K_{\mathrm{DP},n}r + b_n$ by

$$\underset{K_{\mathrm{DP},n},\, b_n}{\text{minimize}} \underbrace{\sum_{m=-(g-1)/2}^{(g-1)/2} \left|\Psi_{\mathrm{DP},n+m}^{\mathrm{filt}} - (2K_{\mathrm{DP},n}r_{n+m} + b_n)\right|^2}_{\text{squared error}}.$$

Computation is very fast
Phase unwrapping is needed in advance
Continuity of $\Phi_{\mathrm{DP}}(r)$ is not considered

- Use short gate $K_{\mathrm{DP},n}$ ($g = 9$) if $Z_{\mathrm{h},n} \geq Z_{\mathrm{thr}}$ and long gate $K_{\mathrm{DP},n}$ ($g = 25$) otherwise.

Complex-Valued Spline Smoothing: Algorithm of Wang and Chandrasekar

- Estimate angular Φ_{DP} profile $e^{j\frac{\pi}{180}\Phi_{\mathrm{DP}}(r)}$ (not $\Phi_{\mathrm{DP}}(r)$) as a complex-valued cubic spline function (complex-valued smooth piecewise polynomial of degree 3) $s(r)$ by

$$\underset{s(r)}{\text{minimize}} \underbrace{\sum_{n=1}^{N} w_n \left|e^{j\frac{\pi}{180}\Psi_{\mathrm{DP},n}^{\mathrm{wrap}}} - s(r_n)\right|^2}_{\text{squared error}} + \underbrace{\sum_{n=1}^{N-1} q_n \int_{r_n}^{r_{n+1}} |s''(r)|^2\,\mathrm{d}r}_{\text{roughness penalty}}$$

$$\text{subject to} \quad s''(r_1) = s''(r_N) = 0 \quad (\text{or} \quad s'(r_1) = s'(r_N) = 0).$$

Computation is fast
Phase unwrapping is not needed
Continuity of $\Phi_{\mathrm{DP}}(r)$ is considered
w_n and q_n are adaptively adjusted
Use of self-consistency is difficult

- $K_{\mathrm{DP},n} = \frac{180}{2\pi}\mathrm{Im}\left[\frac{s'(r_n)}{s(r_n)}\right]$ and $\Phi_{\mathrm{DP},n}$ is estimated from $\frac{180}{\pi}\arg\left(s(r) \approx e^{j\frac{\pi}{180}\Phi_{\mathrm{DP}}(r)}\right)$.

Proposed Framework: Nonnegative K_{DP} Estimation with Self-Consistency (SC)

1. creation of target mask by QC and creation of weights w_n for $\Psi_{\mathrm{DP},n}^{\mathrm{wrap}}$
2. Φ_{DP} estimation with the algorithm of Wang and Chandrasekar
3. height setting of melting layer; $\Delta\Phi_{\mathrm{DP}}$ estimation; phase unwrapping of $\Psi_{\mathrm{DP},n}^{\mathrm{wrap}}$
4. calculation of attenuation-corrected reflectivity $Z_{\mathrm{h},n}^{\mathrm{cor}}$ and corresponding $K_{\mathrm{DP}}^{\mathrm{SC}}(Z_{\mathrm{h},n}^{\mathrm{cor}})$ by **ZPHI method** with α-adjustment
5. **edge-preserving denoising** of $K_{\mathrm{DP}}^{\mathrm{SC}}(Z_{\mathrm{h},n}^{\mathrm{cor}})$ with median filter
6. creation of weights q_n and w_n & w_n^{SC} for $\Psi_{\mathrm{DP},n}$ & $K_{\mathrm{DP},n}^{\mathrm{SC}}$
7. **Monotonically Increasing Spline Smoothing** (Final Re-Fitting)

w_n are adaptively set mainly from $\rho_{\mathrm{hv},n}$

Phase unwrapping is not needed
Computation is fast (with constant q_n)
We use Φ_{DP} estimates rather than K_{DP} estimates

$\Delta\Phi_{\mathrm{DP}}$ is stably given and phase unwrapping is easy thanks to Φ_{DP} estimates satisfying continuity

SC: $A_{\mathrm{h}} = a[Z_{\mathrm{h}}^{\mathrm{cor}}]^b$ and $A_{\mathrm{h}} = \alpha K_{\mathrm{DP}}$ (attenuation coefficient) **are incorporated**
$K_{\mathrm{DP}}^{\mathrm{SC}}(Z_{\mathrm{h},n}^{\mathrm{cor}}) = A_{\mathrm{h},n}/\alpha$ **is robust to undesired bias** in Z_{h}

Outlier removal and azimuth direction smoothing are achieved (**up to this point, it takes about 5 seconds**)

w_n, w_n^{SC}, and q_n are set from $\rho_{\mathrm{hv},n}$ and $K_{\mathrm{DP},n}^{\mathrm{SC}}$ so that $K_{\mathrm{DP},n}^{\mathrm{SC}}$ ($\Psi_{\mathrm{DP},n}$) is emphasized in light (heavy) rain areas

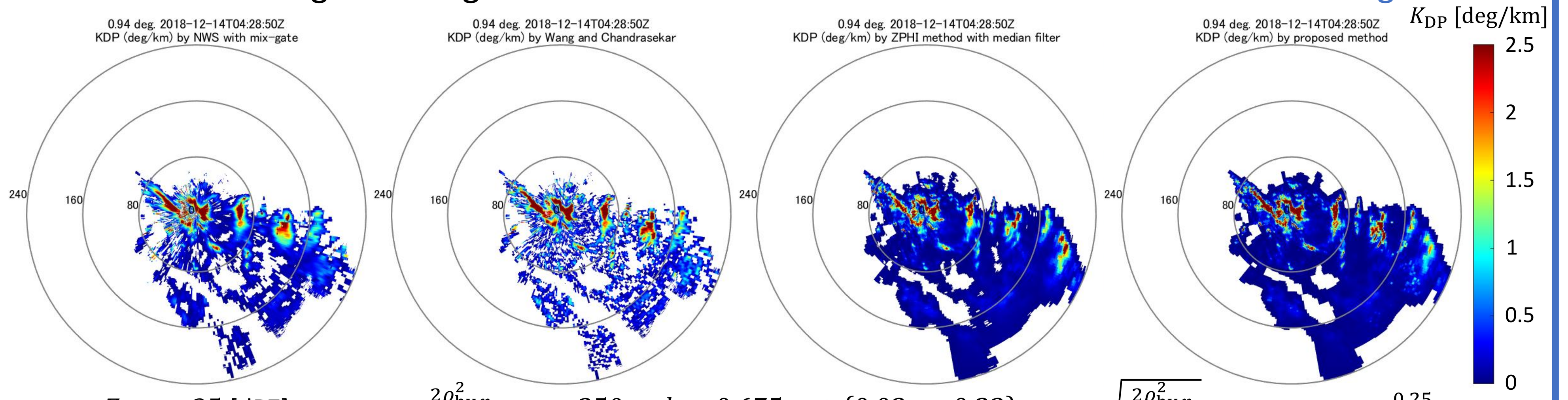
rain rate R is directly estimated from R-K_{DP} relation

$$\underset{\substack{s(r)\\ \text{real-valued spline function}}}{\text{minimize}} \underbrace{\sum_{n=1}^{N} w_n \left|\Psi_{\mathrm{DP},n} - \underbrace{s(r_n)}_{\text{estimate of }\Phi_{\mathrm{DP},n}}\right|}_{\text{absolute error}} + \underbrace{\sum_{n=1}^{N} w_n^{\mathrm{SC}} \left|\underbrace{K_{\mathrm{DP},n}^{\mathrm{SC}}}_{\text{denoised ZPHI}} - \underbrace{\frac{s(r_n+\Delta r/2)-s(r_n-\Delta r/2)}{2\Delta r}}_{\text{estimate of }K_{\mathrm{DP},n}}\right|}_{\text{absolute error}} + \underbrace{\sum_{n=1}^{N} q_n \int_{r_n-\Delta r/2}^{r_n+\Delta r/2} |s''(r)|^2\,\mathrm{d}r}_{\text{roughness penalty}}$$

$$\text{subject to} \quad s'(r) \geq 0 \text{ if } \mathrm{Height}(r) \leq \mathrm{Height}_{\mathrm{melt}} \quad \text{and} \quad \underbrace{K_{\mathrm{DP},n}^{\mathrm{min}} \leq \frac{s(r_n+\Delta r/2)-s(r_n-\Delta r/2)}{2\Delta r} \leq K_{\mathrm{DP},n}^{\mathrm{max}}}_{\text{optional } K_{\mathrm{DP}} \text{ constraint}}$$

monotonically increasing = nonnegativity of $K_{\mathrm{DP}}(r)$ (below the melting layer) ➡ **quadratic programming problem**

Results and Conclusion

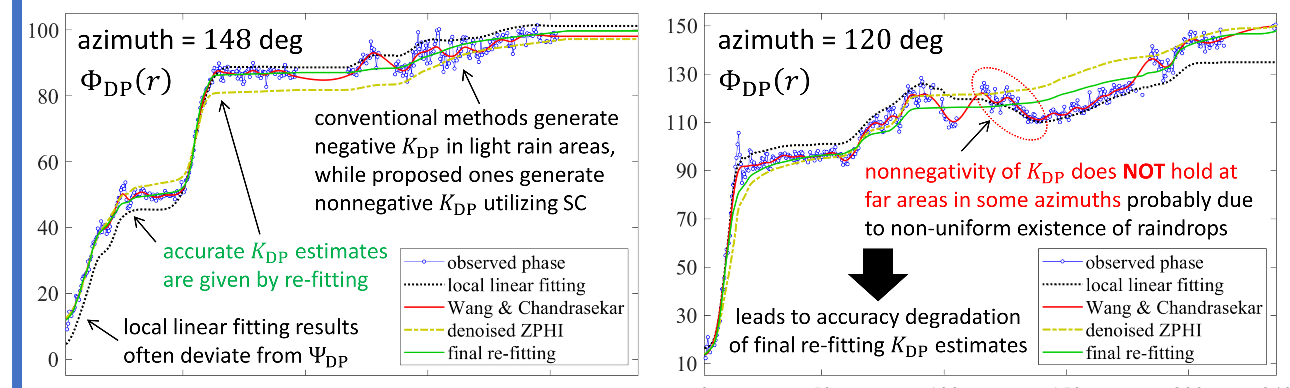


$Z_{\mathrm{thr}} = 35$ [dBZ] $\quad w_n = \frac{2\rho_{\mathrm{hv},n}^2}{1-\rho_{\mathrm{hv},n}^2}$ $\quad q_n = 250$ $\quad b = 0.675$ $\quad \alpha \in \{0.03,\ldots,0.23\}$ $\quad w_n = \sqrt{\frac{2\rho_{\mathrm{hv},n}^2}{1-\rho_{\mathrm{hv},n}^2}}$ $\quad q_n = \frac{0.25}{\min(1,\max(\sqrt{0.05},K_{\mathrm{DP},n}^{\mathrm{SC,filt}})^2)}$

$$w_n^{\mathrm{SC}} = 5\sqrt{\frac{1}{\min(30,\max(1,3*K_{\mathrm{DP},n}^{\mathrm{SC,filt}}))}} \times \sqrt{\frac{1}{\max(0.0025,(2|K_{\mathrm{DP}}^{\mathrm{SC}}(Z_{\mathrm{h},n}^{\mathrm{cor}})-K_{\mathrm{DP},n}^{\mathrm{SC}}|+K_{\mathrm{DP},n}^{\mathrm{SC}})/4)}}$$

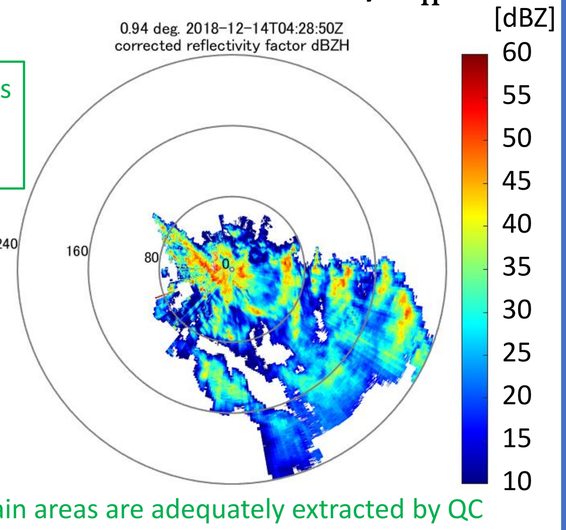
$$K_{\mathrm{DP},n}^{\mathrm{min}} = 2.5(1-e^{-\max(0,K_{\mathrm{DP},n}^{\mathrm{SC}}-0.0015)/3})$$

$K_{\mathrm{DP},n}^{\mathrm{max}}$ is not used and degree of $s(r)$ is 5



corrected reflectivity $Z_{\mathrm{h}}^{\mathrm{cor}}$

$K_{\mathrm{DP},n}^{\mathrm{SC}}$ corresponds to attenuation-corrected $Z_{\mathrm{h},n}^{\mathrm{cor}}$

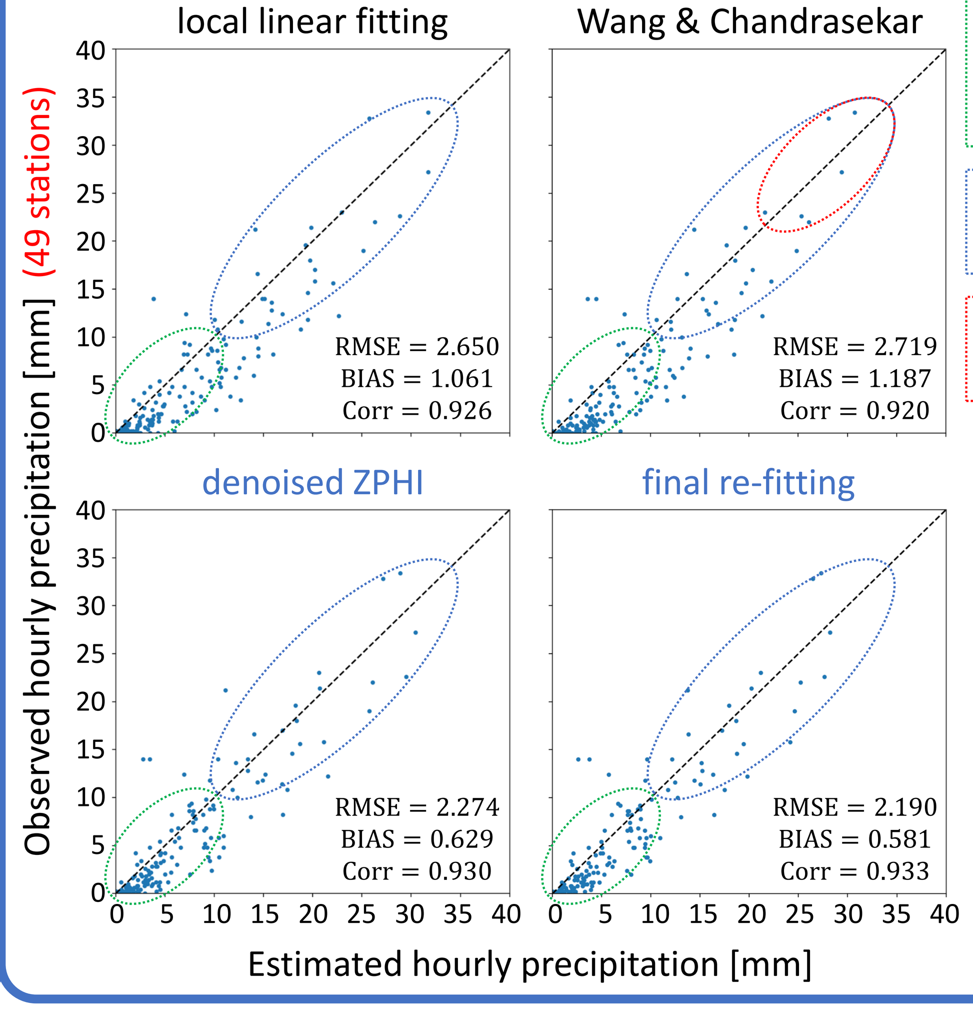


rain areas are adequately extracted by QC

- Conventional methods without SC have difficulty in estimating K_{DP} in light rain areas and generate many negative K_{DP} estimates.
- Proposed methods (denoised ZPHI & final re-fitting) can generate high-resolution nonnegative K_{DP} estimates incorporating SC for $Z_{\mathrm{h}}^{\mathrm{cor}}$.
- Nonnegativity of K_{DP} estimates is useful in subsequent QPE step.

QPE results for 7 hours (51 PPI of convective rain) from 1 am to 8 am on Dec. 14, 2018 in Cordoba

R-K_{DP} relation [mm/h]
$R = 22.87K_{\mathrm{DP}}^{0.834}$



Both proposed methods (denoised ZPHI & final re-fitting) **improved RMSE, estimation bias, and correlation coefficient** **mainly in light rain areas by introducing the self-consistency**

In the proposed framework, the **accuracy for heavy rain areas can be improved from** denoised ZPHI results by final re-fitting

Algorithm of Wang & Chandrasekar had the lowest average score in this case, but high accuracy for very heavy rain areas

We also confirmed that the proposed methods work stably in partial beam blockage regions of Buenos Aires radar as follows